

ON NANO b - OPEN SETS IN NANO TOPOLOGICAL SPACES

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ABSTRACT. The purpose of this paper is to define and study a new class of sets called Nano b -open sets in nano topological spaces. Basic properties of nano b -open sets are analyzed. We also used this set to introduce the new type of continuous functions called nano- b continuous function and its properties are investigated.

1. INTRODUCTION

Nano Topology in engineering and medical provides an interdisciplinary forum uniquely focused on conveying advancements in nano science and applications of nano structures and nano materials to the creative conception, design, development, analysis, control and operation of devices and technologies in engineering, medical and life science systems.

In 1963, Levine [8] introduced the notation of semi-open sets and semi-continuity in topological spaces. In 1963, Mashhour [1] introduced pre-open sets in topological spaces. While in 1996, Andrijevic [2] introduced and studied a class of generalised open sets in a topological space called b -open sets. This class of sets contained in the class of β open sets [5] and contains all semi-open sets [8] and all pre-open sets [1]. The notation of nano topology was introduced by Lellis Thivagar [7] which was

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defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it and also defined nano closed sets, nano interior and nano closure. In this paper, we introduced a new class of sets on nano topological spaces called nano b-open sets and the relation of this new sets with existing sets.

2. PRELIMINARIES

Definition 2.1. [8] A subset A of a topological space (X, τ) is called a semi-open set if $A \subseteq \text{cl}(\text{int}(A))$. The complement of a semi-open set of a space X is called semi-closed set in X .

Definition 2.2. [1] A subset A of a topological space (X, τ) is called a pre-open set if $A \subseteq \text{int}(\text{cl}(A))$. The complement of a pre-open set of a space X is called pre-closed set in X .

Definition 2.3. [6] A subset A of a topological space (X, τ) is called a α -open set if $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$. The complement of a α -open set of a space X is called α -closed set in X .

Definition 2.4. [4] A subset A of a topological space (X, τ) is called a regular-open set if $A = \text{int}(\text{cl}(A))$. The complement of a regular-open set of a space X is called regular-closed set in X .

Definition 2.5. [2] The b-closure of a subset A of a space X is the intersection of all b closed sets containing A and is denoted by $\text{bcl}(A)$. The b-interior of a subset A of a space X is the union of all b-open sets contained in A and is denoted by $\text{bint}(A)$.

Definition 2.6. [7] Let U be a non-empty finite set of all objects called the universe and R be an equivalence relation on U named as in discernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence

class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$. Then,

- (1) The lower approximation of X with respect to R is the of all objects which can be for certain classified as X with respect to R and is denoted by $L_R(X)$.
 $L_R(X) = \cup\{R(X) : R(X) \subseteq X, x \in U\}$ where $R(X)$ denotes the equivalence class determined by $x \in U$.
- (2) The upper approximation of X with respect to R is the set of all objects which can be possibly classified as X with respect to R and is denoted by
 $U_R(X) = \cup\{R(X) : R(X) \cap X \neq \phi, x \in U\}$.
- (3) The boundary region of X with respect to R is the set of all objects which can be classified neither as X nor as not $- X$ with respect to R and is denoted by $B_R(X)$. $B_R(X) = U_R(X) - L_R(X)$.

Definition 2.7. [7] If (U, R) is an approximation space and $X, Y \subseteq U$, then

- (1) $L_R(X) \subseteq X \subseteq U_R(X)$
- (2) $L_R(\phi) = U_R(\phi) = \phi$
- (3) $L_R(U) = U_R(U) = U$
- (4) $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$
- (5) $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$
- (6) $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$
- (7) $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$
- (8) $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$
- (9) $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$
- (10) $U_R[U_R(X)] = L_R[U_R(X)] = U_R(X)$
- (11) $L_R[L_R(X)] = U_R[L_R(X)] = L_R(X)$.

Definition 2.8. [7] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then by Definition 2.7, $\tau_R(X)$ satisfies the following axioms:

- (1) U and $\phi \in \tau_R(X)$.
- (2) The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.
- (3) The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

Then $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X . $(U, \tau_R(X))$ is called the nano topological space. Elements of the nano topology are known as nano open sets in U and the complement of nano open set is called a nano closed set. Elements of $[\tau_R(X)]^c$ are called dual nano topology of $\tau_R(X)$.

Definition 2.9. [7] If $\tau_R(X)$ is the nano topology on U with respect to X , then the set $B = \{U, L_R(X), B_R(X)\}$ is the basis for $\tau_R(X)$.

Definition 2.10. [7] If $(U, \tau_R(X))$ is a nano topological space with respect to X where $X \subseteq U$ and if $A \subseteq U$, then

- (1) The nano interior of the set A is defined as the union of all nano open subsets contained in A and is denoted by $NInt(A)$. $NInt(A)$ is the largest nano open subset of A .
- (2) The nano closure of the set A is defined as the intersection of all nano closed sets containing A and is denoted by $NCl(A)$. $NCl(A)$ is the smallest nano closed set containing A .

Definition 2.11. Let $(U, \tau_R(X))$ be a nano topological space and $A \subseteq U$. Then A is said to be

- (1) Nano semi-open[7] if $A \subseteq NCl[NInt(A)]$ and Nano semi-closed[7] if $NInt[NCl(A)] \subseteq A$.

- (2) Nano pre-open[7] if $A \subseteq \text{NInt}[\text{NCl}(A)]$ and Nano pre-closed[7] if $\text{NCl}[\text{NInt}(A)] \subseteq A$.
- (3) Nano α -open[7] if $A \subseteq \text{NInt}[\text{NCl}(\text{NInt}(A))]$ and Nano α -closed[7] if $\text{NCl}[\text{NInt}(\text{NCl}(A))] \subseteq A$.
- (4) Nano regular-open[7] if $A = \text{NInt}[\text{NCl}(A)]$ and Nano regular-closed[7] if $\text{NCl}[\text{NInt}(A)] = A$.
- (5) Nano semi pre-open[3] if $A \subseteq \text{NCl}[\text{NInt}(\text{NCl}(A))]$ and Nano semi pre-closed[3] if $\text{NInt}[\text{NCl}(\text{NInt}(A))] \subseteq A$.

$\text{NSO}(U, X)$, $\text{NPO}(U, X)$, $\text{NRO}(U, X)$, $\text{NSPO}(U, X)$ and $\text{N}\alpha\text{O}(U, X)$ respectively denote the families of all nano semi-open, nano pre-open, nano regular-open, nano semi pre-open and nano α -open subsets of U .

3. NANO b - OPEN SETS

Throughout this paper, $(U, \tau_R(X))$ is a nano topological space with respect to X where $X \subseteq U$, R is an equivalence relation on U . Then U/R denotes the family of equivalence classes of U by R . In this section, we define and study the forms of nano b -open sets.

Definition 3.1. Let $(U, \tau_R(X))$ is a nano topological space and $A \subseteq U$. Then A is said to be nano b -open (briefly Nb-open) set if $A \subseteq \text{NCl}(\text{NInt}(A)) \cup \text{NInt}(\text{NCl}(A))$. The complement of nano b -open set is called nano b -closed set (briefly Nb-closed).

Example 3.1. Let $U = \{a, b, c, d\}$ with $U/R = \{a\}, \{c\}, \{b, d\}$ and $X = \{a, b\}$. Then the nano topology $\tau_R(X) = \{U, \phi, \{a\}, \{a, b, d\}, \{b, d\}\}$ and nano b -open sets are $U, \phi, \{a\}, \{b\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}$.

Theorem 3.1. Every nano open set is nano b -open.

Proof. Let A be a nano open set in $(U, \tau_R(X))$ since $A \subseteq \text{NCl}(A)$ and $A = \text{NInt}(A)$,

$NInt(A) \subseteq NInt(NCl(A))$ and then $NInt(A) \subseteq NCl(NInt(A))$ which implies $NInt(A) \subseteq NCl(NInt(A)) \cup NInt(NCl(A))$. Hence $A \subseteq NInt(A) \subseteq NCl(NInt(A)) \cup NInt(NCl(A))$ and A is nano b -open in $(U, \tau_R(X))$.

The converse of the above theorem need not be true as shown by the following example.

Example 3.2. Let $U = \{a, b, c, d\}$ with $U/R = \{a\}, \{c\}, \{b, d\}$ and $X = \{a, b\}$. Then the nano topology $\tau_R(X) = \{U, \phi, \{a\}, \{a, b, d\}, \{b, d\}\}$. Here $\{a, b\}$ is nano b -open but it is not a nano open.

Theorem 3.2. Every nano semi open set is nano b -open.

Proof. Let A be a nano semi open set in $(U, \tau_R(X))$. Then $A \subseteq NCl(NInt(A))$.

Hence

$A \subseteq NCl(NInt(A)) \cup NInt(NCl(A))$ and A is nano b -open in $(U, \tau_R(X))$.

The converse of the above theorem need not be true as shown by the following examples.

Example 3.3. Let $U = \{a, b, c, d\}$ with $U/R = \{a\}, \{c\}, \{b, d\}$ and $X = \{a, b\}$. Then the nano topology $\tau_R(X) = \{U, \phi, \{a\}, \{a, b, d\}, \{b, d\}\}$. Here $\{a, b\}$ is nano b -open but it is not a nano semi-open.

Theorem 3.3. Every nano-pre open set is nano b -open.

Proof. Let A be a nano pre-open set in $(U, \tau_R(X))$. Then $A \subseteq NInt(NCl(A))$. Hence $NCl(A) \subseteq NCl(NInt(A)) \cup NInt(NCl(A))$ and A is nano b -open in $(U, \tau_R(X))$.

Example 3.4. Let $U = \{a, b, c, d\}$ with $U/R = \{a\}, \{c\}, \{b, d\}$ and $X = \{a, b\}$. Then the nano topology $\tau_R(X) = \{U, \phi, \{a\}, \{a, b, d\}, \{b, d\}\}$. Here $\{a, c\}$ is nano b -open but it is not a nano pre-open.

Theorem 3.4. *Every nano regular-open set is nano b -open.*

Proof. Let A be a nano regular-open set in $(U, \tau_R(X))$. Then $A = NInt(NCl(A))$. Hence $NCl(A) = NCl(Int(A)) \cup NInt(Cl(A))$ but $A \subseteq NCl(A)$. Therefore $A \subseteq NCl(NInt(A)) \cup NInt(NCl(A))$ and A is nano b -open in $(U, \tau_R(X))$.

The converse of the above theorem need not be true as shown by the following examples.

Example 3.5. Let $U = \{a, b, c, d\}$ with $U/R = \{a\}, \{c\}, \{b, d\}$ and $X = \{a, b\}$. Then the nano topology $\tau_R(X) = \{U, \phi, \{a\}, \{a, b, d\}, \{b, d\}\}$. Here $\{a, c\}$ is nano b -open but it is not a nano regular open.

Theorem 3.5. *Every nano b -open set is nano semi pre-open.*

Proof. Let A be a nano b -open set in $(U, \tau_R(X))$. Then $A \subseteq NCl(Int(A)) \cup NInt(Cl(A))$. Hence

$A \subseteq NCl(NInt(A)) \cup NInt(NCl(A))$, $NCl(NInt(A)) \cup NInt(NCl(A)) \subseteq NCl(NInt(NCl(A)))$ and A is nano semi pre-open in $(U, \tau_R(X))$. The converse of the above theorem need not be true as shown by the following examples.

Example 3.6. Let $U = \{a, b, c, d\}$ with $U/R = \{a\}, \{c\}, \{b, d\}$ and $X = \{a, b\}$. Then the nano topology $\tau_R(X) = \{U, \phi, \{a\}, \{a, b, d\}, \{b, d\}\}$. Here $\{a, c, d\}$ is nano semi pre-open but it is not a nano b -open.

Theorem 3.6. *Every nano α -open set is nano b -open.*

Proof. Let A be a nano α -open set in $(U, \tau_R(X))$. Then $A \subseteq NInt(NCl(NInt(A)))$. Hence $A \subseteq NInt(NCl(NInt(A))) \subseteq NCl(NInt(A)) \subseteq NCl(NInt(A)) \cup NInt(NCl(A))$ and A is nano b -open in $(U, \tau_R(X))$. The converse of the above theorem need not be true as shown by the following examples.

Example 3.7. Let $U = \{a, b, c, d\}$ with $U/R = \{a\}, \{c\}, \{b, d\}$ and $X = \{a, b\}$. Then the nano topology $\tau_R(X) = \{U, \phi, \{a\}, \{a, b, d\}, \{b, d\}\}$. Here $\{a, c\}$ is nano b -open but it is not a nano α -open.

Theorem 3.7. (1) Arbitrary union of two nano b -open sets in $(U, \tau_R(X))$ is not a nano b -open sets in $(U, \tau_R(X))$.

(2) Finite intersection of two nano b -open sets may fail to be nano b -open.

Proof.

(1) Let A and B be two nano b -open sets. Then, $A \subseteq NCl(NInt(A)) \cup NInt(NCl(A))$ and $B \subseteq NCl(NInt(B)) \cup NInt(NCl(B))$. Then $A \cup B \subseteq [NCl(NInt(A)) \cup NInt(NCl(A))] \cup [NCl(NInt(B)) \cup NInt(NCl(B))]$
 $\subseteq NCl(NInt(NCl(A)) \cup NInt(NCl(B)))$
 $\subseteq NCl(NInt(NCl(A) \cup NCl(B))) \subseteq NCl(NInt(NCl(A \cup B)))$

. Therefore $A \cup B$ is nano b -open.

(2) In Example 3.2 $\{a, c\}, \{b, c, d\}$ are nano b -open sets, but their intersection $\{c\}$ is not a nano b -open set.

Theorem 3.8. (1) Arbitrary intersection of two nano b -closed sets in $(U, \tau_R(X))$ is not a nano b -closed sets in $(U, \tau_R(X))$.

(2) Finite union of two nano b -closed sets may fail to be nano b -closed.

Proof.

(1) This follows immediately from Example 3.2

(2) In Example 3.2 $\{a\}, \{b, d\}$ are nano b -closed sets, but their union $\{a, b, d\}$ is not a nano b -closed set.

Definition 3.2. The nano b -closure of a set A , denoted by $NCl_b(A)$, is the intersection of nano closed b -closed sets including A . The nano b -interior of a set A , denoted by $NInt_b(A)$, is the union of nano b -open sets included in A .

Remark.3.1 It is clear that $NInt_b(A)$ is a nano b -open set and $NCl_b(A)$ is a nano b -closed set.

Theorem 3.9. (1) $A \subseteq NCl_b(A)$ and $A = NCl_b(A)$ iff A is a nano b -closed set;
 (2) $NInt_b(A) \subseteq A$ and $A = NInt_b(A)$ iff A is a nano b -open set;
 (3) $X - NInt_b(A) = NCl_b(A)(X - A)$;
 (4) $X - NCl_b(A) = NInt_b(A)(X - A)$;

Proof. Obvious.

Proposition 3.1. *The intersection of a nano α -open set and a nano b -open set is a nano b -open set.*

4. NANO b - CONTINUITY

Definition 4.1. Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be nano topological spaces. Then a mapping $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is nano continuous on U if the inverse image of every nano b -open set in V is a nano open in U .

Example 4.1. Let $U = \{a, b, c, d\}$ with $U/R = \{\{a, c\}, \{b\}, \{d\}\}$ and let $X = \{a, d\}$. Then the nano topology $\tau_R(X) = \{U, \phi, \{d\}, \{a, c, d\}, \{a, c\}\}$. Let $V = \{x, y, z, w\}$ with $V/R' = \{\{x\}, \{y, z\}, \{w\}\}$ and let $Y = \{x, z\}$. Then the nano topology $\tau_{R'}(Y) = \{V, \phi, \{x\}, \{x, y, z\}, \{y, z\}\}$. Define $f : U \rightarrow V$ as $f(a) = y, f(b) = w, f(c) = z, f(d) = x$. $f^{-1}(\{x\}) = \{d\}$, $f^{-1}(\{x, y, z\}) = \{a, c, d\}$ and $f^{-1}(\{y, z\}) = \{a, c\}$. That is, the inverse image of every nano b -open set in V is cannot be a nano open in U . Therefore, f is not a nano b -continuous.

The following theorem characterizes nano- b continuous functions in terms of nano b -closed sets.

Theorem 4.1. *A function $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is nano b -continuous if and only if the inverse image of every nano- β closed set in V is nano closed in U .*

Proof. Let f be nano b -continuous and F be nano b -closed in V . That is, $V-F$ is nano b -open in V . Since f is nano continuous, $f^{-1}(F)$ is nano closed in U . thus the inverse image of every nano b -closed set in V is nano closed in U , if f is nano continuous in U . Conversely, let the inverse image of every nano b -closed set be nano closed. Let g be nano b -open in V . Then $V - G$ is nano b -closed in V . then $f^{-1}(V-G)$ is nano closed in U . That is, $U - f^{-1}(G)$ is nano closed in U . Therefore $f^{-1}(G)$ is nano open in U . Thus, the inverse image of every nano b -open set in V is nano open in U . That is, f is nano b -continuous on U .

Theorem 4.2. The following theorem, we establish a characterization of nano b -continuous functions in terms of nano b -closure. A function $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is nano b -continuous if and only if $f(NCl_b(A)) \subseteq NCl_b(f(A))$ for every subset A of U .

Proof. Let f be nano b -continuous and $A \subseteq U$. Then $f(A) \subseteq V$. $NCl_b(f(A))$ is nano b -closed in V . Since f is nano b -continuous, $f^{-1}(NCl_b(f(A)))$ is nano closed in U . Since $f(A) \subseteq NCl_b(f(A))$, $A \subseteq f^{-1}(NCl_b(f(A)))$. Thus $f^{-1}(NCl_b(f(A)))$ is nano b -closed set containing A . Therefore $NCl_b(A) \subseteq f^{-1}(NCl_b(f(A)))$. That is, $f(NCl_b(A)) \subseteq NCl_b(f(A))$. Conversely, let $f(NCl_b(A)) \subseteq NCl_b(f(A))$ for every subset A of U . If F is nano b -closed in V , since $f^{-1}(F) \subseteq U$, $f(NCl_b(f^{-1}(F))) \subseteq NCl_b(f(f^{-1}(F))) \subseteq NCl_b(F)$. That is, $NCl_b(f^{-1}(F)) \subseteq f^{-1}(NCl_b(f(f^{-1}(F))))$. Therefore, $NCl_b(f^{-1}(F)) = f^{-1}(F)$. Therefore, $f^{-1}(F)$ is nano closed in U for every nano- b closed set F in V . That is f is nano b -continuous.

Theorem 4.3. Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be two nano topological spaces where $X \subseteq U$ and $Y \subseteq V$. Then $\tau_{R'}(Y) = \{V, \phi, L_{R'}(Y), U_{R'}(Y), B_{R'}(Y)\}$ and its basis is given by $B_{R'} = \{V, L_{R'}(Y), B_{R'}(Y)\}$. A function $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is nano b -continuous if and only if the inverse image of every member of $B_{R'}$ is nano open in U .

Proof. Let f be a nano b -continuous on U . Let $B \in B_{R'}$. Then B is nano b -open in V . That is, $B \in \tau_{R'}(Y)$. Since f is nano b -continuous, $f^{-1}(B) \in \tau_R(X)$. That is, the inverse image of every member of $B_{R'}$ is nano b -open in U . Conversely, let the inverse image of every member of $B_{R'}$ be nano b -open in U . Let G be a nano b -open in V . Then $G = \cup\{B : B \in B_1\}$, where $B_1 \subseteq B_{R'}$. Then $f^{-1}(G) = f^{-1}(\cup\{B : B \in B_1\}) = \cup\{f^{-1}(B) : B \in B_1\}$, where each $f^{-1}(B)$ is nano open in U and hence their union, which is $f^{-1}(G)$ is nano open in U . Thus f is nano b -continuous on U . The above theorem characterizes nano b -continuous functions in terms of basis elements. In the following theorem, we characterize nano b -continuous functions in terms of inverse image of nano closure.

Theorem 4.4. A function $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is nano b -continuous if and only if $NCl_b(f^{-1}(B)) \subseteq f^{-1}(NCl_b(B))$ for every subset B of V .

Proof. Let f be a nano b -continuous and $B \in V$, $NCl_b(B)$ is nano b -closed in V and hence $f^{-1}(NCl_b(B))$ is nano closed in U . Therefore, $N[Cl_b(f^{-1}(NCl_b(B)))] = f^{-1}(NCl_b(B))$. Since $B \subseteq NCl_b(B)$, $f^{-1}(B) \subseteq f^{-1}(NCl_b(B))$. Therefore, $NCl(f^{-1}(B)) \subseteq N_b(f^{-1}(NCl_b(B))) = f^{-1}(NCl_b(B))$. That is, $NCl_b(f^{-1}(B)) \subseteq f^{-1}(NCl_b(B))$ for every $B \in V$. Let B be nano b -closed in V . Then $NCl_b(B) = B$. By assumption, $NCl_b f^{-1}(B) \subseteq f^{-1}(NCl_b(B)) = f^{-1}(B)$. Thus, $NCl_b f^{-1}(B) \subseteq f^{-1}(B)$. But $f^{-1}(B) \subseteq NCl_b(f^{-1}(B))$. Therefore, $NCl_b(f^{-1}(B)) = f^{-1}(B)$. That is, $f^{-1}(B)$ is nano closed in U for every nano b -closed set B in V . Therefore, f is nano b -continuous on U .

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