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SOFT GROUP BASED ON SOFT ELEMENT

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Abstract. Using the notion of soft element [13], in this paper, we define a binary operation on the set of all nonempty soft elements of a given soft set to introduce soft groupoid. Then we give the definition of soft group based on soft elements and establish necessary and sufficient conditions for a soft set to be a soft group. Also we compare some properties like commutative property, cyclic property of soft group with those of given parameter set and initial universe set.

1. Introduction

To deal with imprecise data, vague concepts in various fields like economics, engineering, medical science and crucially in the area of artificial intelligence, many concepts like fuzzy sets [14], rough sets [11], multi sets, soft sets [10] etc. have been developed. Among them, soft set theory is more generalized tool to deal with uncertainty because fuzzy sets, rough sets, multi sets etc. can be considered as particular types of soft sets. So many research work have been done in the field of soft set theory, also in hybrid structures of soft set theory with fuzzy set theory and rough set theory [2]-[8] to deal with problems having different uncertainties. In 2007, Aktas and Cagman [2] defined a soft group as a parameterized family of subgroups. Extending this notion of soft group, many authors also defined soft (ring, field, ideal), fuzzy soft (group, ring, field, ideal) (see [1, 3, 5]) etc.

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In this paper, we consider the definition of soft element, given by Wardowski [13] and define a binary operation on the set of all nonempty soft elements of a given soft set with the help of a binary operation defined in a parameter set as well as in a universal set. Then we give a condition in order that a soft set forms a soft groupoid. Also we discuss various properties like soft identity element, soft inverse element with suitable examples. Lastly we define soft group and compare some properties like commutative property, cyclic property of soft group with those of given parameter set and initial universe set. Then we discuss the properties of union and intersection of two soft groups with suitable examples.

2. Preliminaries

Throughout this paper unless otherwise stated, let U be the universal set, E the set of parameters with respect to U and $P(U)$ the power set of U. In this section, we recall some basic definitions in soft set theory which will be needed in the sequel.

Definition 2.1. [10] Let $A \subseteq E$. A soft set F_A on U is a set of the form

$$F_A = \{(e, F(e)) : e \in A\}$$

where F is a mapping given by $F : A \to P(U)$.
The collection of all soft sets on U will be denoted by $S(U)$.

Definition 2.2. [4] Let $A, B \subseteq E$ and $F_A, G_B \in S(U)$. Then F_A is called a soft subset of G_B, denoted by $F_A \,\widetilde{\subseteq}\, G_B$, if
(i) $A \subseteq B$,
(ii) $F(e) \subseteq G(e)$ for all $e \in A$.

Definition 2.3. [12][9] Let F_A and G_B be two soft sets over a common universe U and A, $B \subseteq E$.
(1) The intersection of F_A and G_B, denoted by $F_A \,\widetilde{\cap}\, G_B$, is defined as the soft set

H_C, where $C = A \cap B$ and $H(e) = F(e) \cap G(e)$ for all $e \in C$.

(2) The union of F_A and G_B, denoted by $F_A \widetilde{\cup} G_B$, is defined as the soft set H_C, where $C = A \cup B$ and $\forall\, e \in C$,

$$H(e) = \begin{cases} F(e), & \text{if } e \in A \setminus B, \\ G(e), & \text{if } e \in B \setminus A, \\ F(e) \cup G(e), & \text{if } e \in A \cap B. \end{cases}$$

Note 2.1. *Let A, $B \subseteq E$ and F_A, $G_B \in S(U)$ such that $F_A \widetilde{\subseteq} G_B$. Then $F_A \widetilde{\cap} G_B = F_A$ and $F_A \widetilde{\cup} G_B = G_B$.*

Example 2.1. [13] *Let $U = \{u_1, u_2, u_3, u_4\}$, $E = \{p_1, p_2, p_3\}$ and $A = \{p_1, p_2\}$, $B = \{p_1, p_3\}$. For the soft sets of the form $F_A = \{(p_1, \{u_1, u_2\}), (p_2, \{u_2, u_3\})\}$, $G_B = \{(p_1, \{u_1, u_4\}), (p_3, \{u_3, u_4\})\}$, we have $F_A \widetilde{\cap} G_B = \{(p_1, \{u_1\})\}$ and $F_A \widetilde{\cup} G_B = \{(p_1, \{u_1, u_2, u_4\}), (p_2, \{u_2, u_3\}), (p_3, \{u_3, u_4\})\}$.*

Definition 2.4. (i) For a soft set F_A, the set $Supp(F_A) = \{e \in A : F(e) \neq \phi\}$ is called the support of the soft set F_A;

(ii) A soft set F_A is said to be non-null if $Supp(F_A) \neq \phi$, otherwise F_A is called null soft set;

(iii) A soft set F_A is said to be full soft set if $Supp(F_A) = A$.

The collection of all full soft sets on U will be denoted by $S_f(U)$.

Definition 2.5. [13] Let $A \subseteq E$ and $F_A \in S(U)$. We say that $(e, \{u\})$ is a nonempty soft element of F_A if $e \in A$ and $u \in F(e)$. The pair (e, ϕ), where $e \in A$, will be called an empty soft element of F_A. The fact that $(e, \{u\})$ is a soft element of F_A will be denoted by $(e, \{u\}) \widetilde{\in} F_A$.

Note 2.2. *We denote the set of all nonempty soft elements of F_A by $F_A^{\bullet}$. Also note that a soft element $(e, \{u\})$ belongs to $F_A^{\bullet}$ will be denoted by $(e, \{u\}) \widetilde{\in} F_A^{\bullet}$.*

Proposition 2.1. [13] *For each* $F_A \in S(U)$, *the following holds:*

$$F_A = \widetilde{\bigcup}_{(e_i,\{u_j\})\,\widetilde{\in}\,F_A} \{(e_i, \{u_j\})\}$$

Note 2.3. *For each* $F_A \in S_f(U)$, *the following also holds:*

$$F_A = \widetilde{\bigcup}_{(e_i,\{u_j\})\,\widetilde{\in}\,F_A^\bullet} \{(e_i, \{u_j\})\}.$$

Example 2.2. *Let* $U = \{u_1, u_2\}$, $E = \{e_1, e_2, e_3\}$ *and* $F_A \in S_f(U)$ *be of the form* $F_A = \{(e_2, \{u_1, u_2\}), (e_3, \{u_2\})\}$. *Hence all the soft elements of* F_A *are* $(e_2, \{u_1\}), (e_2, \{u_2\}), (e_2, \phi), (e_3, \{u_2\}), (e_3, \phi)$. *Then the soft elements of* $F_A^\bullet$ *are* $(e_2, \{u_1\}), (e_2, \{u_2\}), (e_3, \{u_2\})$. *Therefore*

$$\begin{aligned} F_A &= \{(e_2, \{u_1\})\}\,\widetilde{\cup}\,\{(e_2, \{u_2\})\}\,\widetilde{\cup}\,\{(e_2, \phi)\}\,\widetilde{\cup}\,\{(e_3, \{u_2\})\}\,\widetilde{\cup}\,\{(e_3, \phi)\} \\ &= \{(e_2, \{u_1\})\}\,\widetilde{\cup}\,\{(e_2, \{u_2\})\}\,\widetilde{\cup}\,\{(e_3, \{u_2\})\} \\ &= \widetilde{\bigcup}_{(e_i,\{u_j\})\,\widetilde{\in}\,F_A^\bullet} \{(e_i, \{u_j\})\}. \end{aligned}$$

Now let $G_B \in S(U)$ *be of the form* $G_B = \{(e_1, \phi), (e_2, \{u_1, u_2\})\}$. *Then the soft elements of* $G_B^\bullet$ *are* $(e_2, \{u_1\}), (e_2, \{u_2\})$.
Therefore $\{(e_2, \{u_1\})\}\,\widetilde{\cup}\,\{(e_2, \{u_2\})\} = \{(e_2, \{u_1, u_2\})\} \neq G_B$.

3. Soft groupoid

Throughout this section, let $(E, \circ)$ and $(U, *)$ be two groupoids and $A \subseteq E$. Also let $F_A \in S_f(U)$, i.e., F_A be a full soft set on U, i.e., for each parameter $e \in A$, there exists at least one nonempty soft element of F_A. We define a binary composition $\widetilde{*}$ on $F_A^\bullet$ by

$$(e_i, \{u_k\})\,\widetilde{*}\,(e_j, \{u_l\}) = (e_i \circ e_j, \{u_k * u_l\}) \tag{3.1}$$

for all $(e_i, \{u_k\}), (e_j, \{u_l\})\,\widetilde{\in}\,F_A^\bullet$.
$F_A^\bullet$ is said to be closed under the binary composition $\widetilde{*}$ if and only if

$(e_i \circ e_j, \{u_k * u_l\}) \,\widetilde{\in}\, F_A^{\bullet}$ for all $(e_i, \{u_k\}), (e_j, \{u_l\}) \,\widetilde{\in}\, F_A^{\bullet}$ i.e., if and only if $e_i \circ e_j \in A$ and $u_k * u_l \in F(e_i \circ e_j)$ for all $(e_i, \{u_k\}), (e_j, \{u_l\}) \,\widetilde{\in}\, F_A^{\bullet}$.

Definition 3.1. If $F_A^{\bullet}$ is closed under the binary composition $\widetilde{*}$, then the algebraic system $(F_A^{\bullet}, \widetilde{*})$ is said to be a soft groupoid over (E, U).

Theorem 3.1. *Let $F_A \in S_f(U)$. Then $(F_A^{\bullet}, \widetilde{*})$ forms a soft groupoid over (E, U) if and only if*
(i) A is a subgroupoid of E, i.e., $e_i \circ e_j \in A$ for all $e_i, e_j \in A$ and
(ii) for $e_i, e_j \in A$, $u_k \in F(e_i)$, $u_l \in F(e_j) \Rightarrow u_k * u_l \in F(e_i \circ e_j)$.

Proof. Suppose $(F_A^{\bullet}, \widetilde{*})$ is a soft groupoid over (E, U). Let $e_i, e_j \in A$. Since $F_A \in S_f(U)$, there exist some $u_k, u_l \in U$ such that $(e_i, \{u_k\}), (e_j, \{u_l\}) \,\widetilde{\in}\, F_A^{\bullet}$. Hence $(e_i, \{u_k\}) \,\widetilde{*}\, (e_j, \{u_l\}) \,\widetilde{\in}\, F_A^{\bullet}$. This implies $(e_i \circ e_j, \{u_k * u_l\}) \,\widetilde{\in}\, F_A^{\bullet} \Rightarrow e_i \circ e_j \in A$ and $u_k * u_l \in F(e_i \circ e_j)$, by Definition 2.5. Therefore A is a subgroupoid of E and for $e_i, e_j \in A$, $u_k \in F(e_i)$, $u_l \in F(e_j) \Rightarrow u_k * u_l \in F(e_i \circ e_j)$.
Conversely, suppose that the given two conditions hold. Now let $(e_i, \{u_k\}), (e_j, \{u_l\}) \,\widetilde{\in}\, F_A^{\bullet}$. This implies that $e_i, e_j \in A$ and $u_k \in F(e_i)$, $u_l \in F(e_j)$.
By hypothesis (i), $e_i, e_j \in A \Rightarrow e_i \circ e_j \in A$.
By hypothesis (ii), $u_k \in F(e_i)$, $u_l \in F(e_j) \Rightarrow u_k * u_l \in F(e_i \circ e_j)$.
Therefore $(e_i \circ e_j, \{u_k * u_l\}) \,\widetilde{\in}\, F_A^{\bullet}$. So, $F_A^{\bullet}$ is closed under the binary composition $\widetilde{*}$. Hence $(F_A^{\bullet}, \widetilde{*})$ forms a soft groupoid over (E, U). □

Example 3.1. *Let $(E, \circ)$ be the klein's 4-group and $(U, *)$ the symmetric group S_3, where $E = \{e_1, e_2, e_3, e_4\}$ be the set of parameters and U the set of all permutations on the set $\{1, 2, 3\}$ i.e., $U = \{\rho_0, \rho_1, \rho_2, \rho_3, \rho_4, \rho_5\}$, where $\rho_0 =$ the identity permutation, $\rho_1 = (1,2,3)$, $\rho_2 = (1,3,2)$, $\rho_3 = (2,3)$, $\rho_4 = (1,3)$, $\rho_5 = (1,2)$. Take $A = \{e_1, e_3\}$ and define a soft set $F : A \to P(U)$ by $F(e_1) = \{\rho_0, \rho_1, \rho_2\}$, $F(e_3) = \{\rho_3, \rho_4, \rho_5\}$. So, the soft elements of F_A are (e_1, ϕ), $(e_1, \{\rho_0\})$, $(e_1, \{\rho_1\})$, $(e_1, \{\rho_2\})$,*

(e_3, ϕ), $(e_3, \{\rho_3\})$, $(e_3, \{\rho_4\})$, $(e_3, \{\rho_5\})$. *Hence the elements of* $F_A^\bullet$ *are* $(e_1, \{\rho_0\})$, $(e_1, \{\rho_1\})$, $(e_1, \{\rho_2\})$, $(e_3, \{\rho_3\})$, $(e_3, \{\rho_4\})$, $(e_3, \{\rho_5\})$. *Then the binary composition* $\widetilde{*}$ *on* $F_A^\bullet$ *is given by*

$$(e_i, \{\rho_k\}) \,\widetilde{*}\, (e_j, \{\rho_l\}) \;=\; (e_i \circ e_j, \{\rho_k * \rho_l\})$$

for all $(e_i, \{\rho_k\})$, $(e_j, \{\rho_l\}) \,\widetilde{\in}\, F_A^\bullet$.
Here, it is easy to verify that $\rho_k * \rho_l \in F(e_i \circ e_j)$ *for all* $(e_i, \{\rho_k\})$, $(e_j, \{\rho_l\}) \,\widetilde{\in}\, F_A^\bullet$. *Hence* $(F_A^\bullet, \widetilde{*})$ *is a soft groupoid.*

Theorem 3.2. *If* $(F_A^\bullet, \widetilde{*})$ *is a soft groupoid over* (E, U) *then* $\bigcup_{e_i \in A} F(e_i)$ *is a subgroupoid of* U.

Proof. Since for each $e_i \in A$, $F(e_i) \subseteq U$, then $\bigcup_{e_i \in A} F(e_i) \subseteq U$. Let $u_k, u_l \in \bigcup_{e_i \in A} F(e_i)$. This implies that $\exists\, e_i, e_j \in A$ such that $u_k \in F(e_i)$, $u_l \in F(e_j)$. Then by Theorem 3.1, $u_k * u_l \in F(e_i \circ e_j)$. Since A is a subgroupoid of E, then $e_i \circ e_j \in A$. Hence $u_k * u_l \in \bigcup_{e_i \in A} F(e_i)$. So, $\bigcup_{e_i \in A} F(e_i)$ is a subgroupoid of U. □

Remark 1. The converse of Theorem 3.2 may not be true, which is justified by the following Example.

Example 3.2. *Let* $E = \{e_1, e_2, e_3, e_4\}$ *be the group as in Example 3.1 and* $(U, .)$ *the abelian group, where* $U = \{1, \omega, \omega^2\}$, *the set of all cube roots of unity. Take* $A = \{e_1, e_2\}$ *and define a soft set* $F : A \to P(U)$ *by* $F(e_1) = \{1, \omega\}$, $F(e_2) = \{\omega^2\}$. *So, the elements of* $F_A^\bullet$ *are* $(e_1, \{1\})$, $(e_1, \{\omega\})$, $(e_2, \{\omega^2\})$. *Then the binary composition* $\widetilde{*}$ *on* $F_A^\bullet$ *is given by* $(e_i, \{\omega_k\}) \,\widetilde{*}\, (e_j, \{\omega_l\}) = (e_i \circ e_j, \{\omega_k \,.\, \omega_l\})$ *for all* $(e_i, \{\omega_k\})$, $(e_j, \{\omega_l\}) \,\widetilde{\in}\, F_A^\bullet$. *Here* $(e_1, \{\omega\}) \,\widetilde{*}\, (e_1, \{\omega\}) = (e_1, \{\omega^2\}) \,\widetilde{\notin}\, F_A^\bullet$. *So,* $F_A^\bullet$ *is not closed under the binary composition* $\widetilde{*}$. *Hence* $(F_A^\bullet, \widetilde{*})$ *is not a soft groupoid but* A *is a subgroupoid of* E *and* $\bigcup_{e_i \in A} F(e_i) = \{1, \omega, \omega^2\}$ *is a subgroupoid of* U.

Definition 3.2. Let $(F_A^\bullet, \widetilde{*})$ be a soft groupoid over (E, U), where the binary composition $\widetilde{*}$ is defined in equation 3.1. Then $\widetilde{*}$ is said to be

(i) commutative if $(e_i, \{u_j\}) \,\widetilde{*}\, (e_k, \{u_l\}) = (e_k, \{u_l\}) \,\widetilde{*}\, (e_i, \{u_j\})$;

(ii) associative if

$[(e_i, \{u_j\}) \,\widetilde{*}\, (e_k, \{u_l\})] \,\widetilde{*}\, (e_m, \{u_n\}) = (e_i, \{u_j\}) \,\widetilde{*}\, [(e_k, \{u_l\}) \,\widetilde{*}\, (e_m, \{u_n\})]$

for all $(e_i, \{u_j\}), (e_k, \{u_l\}), (e_m, \{u_n\}) \,\widetilde{\in}\, F_A^\bullet$.

Definition 3.3. A soft element $(e, \{u\}) \,\widetilde{\in}\, F_A^\bullet$ is said to be a soft identity element in a soft groupoid $(F_A^\bullet, \widetilde{*})$ if for all $(e_i, \{u_j\}) \,\widetilde{\in}\, F_A^\bullet$,

$$(e, \{u\}) \,\widetilde{*}\, (e_i, \{u_j\}) = (e_i, \{u_j\}) = (e_i, \{u_j\}) \,\widetilde{*}\, (e, \{u\}).$$

Theorem 3.3. *Let $(F_A^\bullet, \widetilde{*})$ be a soft groupoid over (E, U).*

(i) If the composition $\circ$ on A and the composition $*$ on U are associative (commutative) then the composition $\widetilde{*}$ on $F_A^\bullet$ is associative (commutative).

(ii) If $F_A^\bullet$ contains the soft identity element $(e, \{u\})$ then e is the identity element of A and u is the identity element of $\bigcup_{e_i \in A} F(e_i)$.

Proof. Since the binary composition $\widetilde{*}$ on $F_A^\bullet$ is given by

$$(e_i, \{u_k\}) \,\widetilde{*}\, (e_j, \{u_l\}) = (e_i \circ e_j, \{u_k * u_l\})$$

for all $(e_i, \{u_k\}), (e_j, \{u_l\}) \,\widetilde{\in}\, F_A^\bullet$.

(i) It is easy to verify that $\widetilde{*}$ is associative and commutative on $F_A^\bullet$.

(ii) Since $(F_A^\bullet, \widetilde{*})$ is a soft groupoid, then by Theorem 3.2, A is a subgroupoid of E and $\bigcup_{e_i \in A} F(e_i)$ is a subgroupoid of U. Suppose $(e, \{u\}) \,\widetilde{\in}\, F_A^\bullet$ be the soft identity element i.e., for all $(e_i, \{u_j\}) \,\widetilde{\in}\, F_A^\bullet$, we have

$(e, \{u\}) \,\widetilde{*}\, (e_i, \{u_j\}) = (e_i, \{u_j\}) = (e_i, \{u_j\}) \,\widetilde{*}\, (e, \{u\})$

$\Rightarrow (e \circ e_i, \{u * u_j\}) = (e_i, \{u_j\}) = (e_i \circ e, \{u_j * u\})$

$\Rightarrow e \circ e_i = e_i = e_i \circ e$ and $u * u_j = u_j = u_j * u$.

Since $(e_i,\, \{u_j\})$ is arbitrary, $e_i \in A$ and $u_j \in F(e_i) \subseteq \bigcup_{e_i \in A} F(e_i)$ are arbitrary. So, $e \in A$ and $u \in F(e)$ are identity element of A and $\bigcup_{e_i \in A} F(e_i)$ respectively. □

Remark 2. The converse of Theorem 3.3 (ii) may not be true. If e is the identity element of A and u is the identity element of $\bigcup_{e_i \in A} F(e_i)$, then $(e, \{u\})$ may not be the soft identity element of $F_A^\bullet$, because u may not belong to $F(e)$.

Example 3.3. *Let* $E = \{e_1,\, e_2\}$ *and* $U = \{\overline{0},\, \overline{1},\, \overline{2},\, \overline{3},\, \overline{4},\, \overline{5}\}$, *the classes of residues of integers modulo 6. The composition table of* $\circ$ *on* E *is given by*

$\circ$	e_1	e_2
e_1	e_1	e_2
e_2	e_2	e_2

Then $(E, \circ)$ *is a commutative monoid with* e_1 *as the identity element and* $(U,\, \times_6)$ *the commutative monoid with* $\overline{1}$ *as the identity element, where* $\times_6$ *is the multiplication modulo 6. Take* $A = E$ *and define a soft set* $F : A \to P(U)$ *by* $F(e_1) = \{\overline{2},\, \overline{4}\}$, $F(e_2) = \{\overline{1},\, \overline{2},\, \overline{4}\}$. *So, the soft elements of* $F_A^\bullet$ *are* $(e_1,\, \{\overline{2}\})$, $(e_1,\, \{\overline{4}\})$, $(e_2,\, \{\overline{1}\})$, $(e_2,\, \{\overline{2}\})$, $(e_2,\, \{\overline{4}\})$. *The binary composition* $\widetilde{*}$ *on* $F_A^\bullet$ *is given by*

$$(e_i,\, \{u_j\})\, \widetilde{*}\, (e_k,\, \{u_l\}) = (e_i \circ e_k,\, \{u_j \times_6 u_l\})$$

for all $(e_i,\, \{u_j\}),\, (e_k,\, \{u_l\})\, \widetilde{\in}\, F_A^\bullet$. *Here it is easy to verify that* $F_A^\bullet$ *is a commutative soft groupoid with respect to* $\widetilde{*}$ *without soft identity element. We also note that* e_1 *is the identity element of* A *and* $\overline{1}$ *is the identity element of* U *but* $(e_1,\, \{\overline{1}\})\, \widetilde{\notin}\, F_A^\bullet$.

The converse of Theorem 3.3 (ii) can also be true if we add an additional condition to the hypothesis of this Theorem 3.3 (ii) and it is given by the following Theorem.

Theorem 3.4. *Let* $(F_A^\bullet, \widetilde{*})$ *be a soft groupoid over* (E, U). *If* e *is the identity element of* A *and* u *is the identity element of* $\bigcup_{e_i \in A} F(e_i)$ *such that* $u \in F(e)$, *then* $(e, \{u\})$ *is the soft identity element of* $F_A^\bullet$.

Proof. Since $e,\, u$ are identity elements of $A,\, \bigcup_{e_i \in A} F(e_i)$ respectively and $u \in F(e)$, then $(e,\, \{u\})\, \widetilde{\in}\, F_A^{\bullet}$. Hence it is easy to prove that $(e, \{u\})$ is the soft identity element of $F_A^{\bullet}$. □

Definition 3.4. Let $(F_A^{\bullet},\, \widetilde{*})$ be a soft groupoid with soft identity element $(e, \{u\})$. A soft element $(e_i,\, \{u_j\})\, \widetilde{\in}\, F_A^{\bullet}$ is said to be invertible if there exists a soft element $(e_i^{'},\, \{u'_j\})\, \widetilde{\in}\, F_A^{\bullet}$ such that $(e_i,\, \{u_j\})\, \widetilde{*}\, (e_i^{'},\, \{u'_j\}) = (e, \{u\}) = (e_i^{'},\, \{u'_j\})\, \widetilde{*}\, (e_i,\, \{u_j\})$. Then $(e_i^{'},\, \{u'_j\})$ is called the soft inverse of $(e_i,\, \{u_j\})$.

The soft inverse of a soft element $(e_i,\, \{u_j\})\, \widetilde{\in}\, F_A^{\bullet}$ is denoted by $(e_i,\, \{u_j\})^{-1}$.

Definition 3.5. (i) A soft groupoid $(F_A^{\bullet},\, \widetilde{*})$ is said to be a soft semigroup if $\widetilde{*}$ is associative;
(ii) A soft semigroup $(F_A^{\bullet},\, \widetilde{*})$ containing soft identity element is said to be a soft monoid.

Theorem 3.5. *(i) If a soft groupoid $(F_A^{\bullet},\, \widetilde{*})$ contains a soft identity element, then that element is unique.*
(ii) In a soft monoid $(F_A^{\bullet},\, \widetilde{*})$, if a soft element $(e_i,\, \{u_k\})$ be invertible then it has a unique soft inverse.

Proof. Proofs are same as classical algebra. □

Theorem 3.6. *Let $(F_A^{\bullet},\, \widetilde{*})$ be a soft groupoid with soft identity element $(e, \{u\})$. If a soft element $(e_i,\, \{u_j\})\, \widetilde{\in}\, F_A^{\bullet}$ is invertible then e_i is invertible in E and $u_j \in F(e_i)$ is invertible in U.*

Proof. Suppose $(e_i,\, \{u_j\})\, \widetilde{\in}\, F_A^{\bullet}$ is invertible. Then there exist a soft element $(e_i^{'},\, \{u_j^{'}\})$ $\widetilde{\in}\, F_A^{\bullet}$ such that $(e_i,\, \{u_j\})\, \widetilde{*}\, (e_i^{'},\, \{u_j^{'}\}) = (e, \{u\}) = (e_i^{'},\, \{u'_j\})\, \widetilde{*}\, (e_i,\, \{u_j\})$
$\Rightarrow (e_i \circ e_i^{'},\, \{u_j * u_j^{'}\}) = (e, \{u\}) = (e_i^{'} \circ e_i,\, \{u_j^{'} * u_j\})$
$\Rightarrow e_i \circ e_i^{'} = e = e_i^{'} \circ e_i$ and $u_j * u_j^{'} = u = u_j^{'} * u_j$.

Since $(e, \{u\})$ is the soft identity element of $F_A^\bullet$, then by Theorem 3.3, e is the identity element of A and u is the identity element of $\bigcup_{e_i \in A} F(e_i)$.
Hence $e_i \circ e_i' = e = e_i' \circ e_i \Rightarrow e_i$ is invertible in $A \subseteq E$ and
$u_j * u_j' = u = u_j' * u_j \Rightarrow u_j$ is invertible in $\bigcup_{e_i \in A} F(e_i) \subseteq U$. □

Note 3.1. *In a soft groupoid* $(F_A^\bullet, \widetilde{*})$ *with soft identity element, if a soft element* $(e_i, \{u_j\})$ *is invertible then* $(e_i, \{u_j\})^{-1} = (e_i^{-1}, \{u_j^{-1}\})$.

Remark 3. Converse of the Theorem 3.6 is not necessarily true. In a soft groupoid $(F_A^\bullet, \widetilde{*})$ with soft identity element, if e_i is invertible in E and $u_j \in F(e_i)$ is invertible in U then $(e_i, \{u_j\}) \widetilde{\in} F_A^\bullet$ is not necessarily invertible in $F_A^\bullet$.

Example 3.4. *Let* $E = \{e_1, e_2\}$ *and* U *be the set of all* 2×2 *real non-singular matrices such that* $(E, \circ), (U, .)$ *be two groups, where the composition* $'.'$ *is the matrix multiplication and the composition* $'\circ'$ *is given by:*

$\circ$	e_1	e_2
e_1	e_1	e_2
e_2	e_2	e_1

Define a soft set $F : E \to P(U)$ *by* $F(e_1) = \{I_2, A, A^2, A^3, \cdots\}$ *and*
$F(e_2) = \{A, A^2, A^3, \cdots\}$, *where* $A = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ *and* $I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.
So, the soft elements of $F_E^\bullet$ *are* $(e_1, \{I_2\}), (e_1, \{A\}), (e_1, \{A^2\}), \cdots,$
$(e_2, \{A\}), (e_2, \{A^2\}), (e_2, \{A^3\}), \cdots$ *Obviously,* $(F_E^\bullet, \widetilde{*})$ *is a soft groupoid with soft identity element* $(e_1, \{I_2\})$. *Also* e_2, A *are invertible in* E, U, *respectively. Suppose* $(e_2, \{A\})$ *is invertible in* $F_E^\bullet$. *Then by Note 3.1, soft inverse of* $(e_2, \{A\})$ *must be* $(e_2^{-1}, \{A^{-1}\})$. *But* $e_2^{-1} = e_2$ *and* $A^{-1} = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}$. *Hence* $A^{-1} \notin F(e_2^{-1})$, *i.e.,* $(e_2^{-1}, \{A^{-1}\}) \widetilde{\notin} F_E^\bullet$. *Therefore* $(e_2, \{A\})$ *is not invertible in* $F_E^\bullet$.

Converse of the Theorem 3.6 can also be true if we add an additional condition with the hypothesis of this theorem 3.6 and it is given by the following Theorem.

Theorem 3.7. *Let* $(F_A^{\bullet}, \widetilde{*})$ *be a soft groupoid with soft identity element* $(e, \{u\})$*. If* $e_i \in A$ *is invertible in* A *and* $u_j \in F(e_i)$ *is invertible in* U *such that* $u_j^{-1} \in F(e_i^{-1})$ *then* $(e_i, \{u_j\}) \widetilde{\in} F_A^{\bullet}$ *is invertible in* $F_A^{\bullet}$.

Proof. Since $u_j^{-1} \in F(e_i^{-1})$, $(e_i^{-1}, \{u_j^{-1}\}) \widetilde{\in} F_A^{\bullet}$ and it is easy to prove that $(e_i^{-1}, \{u_j^{-1}\})$ is the soft inverse of $(e_i, \{u_j\}) \widetilde{\in} F_A^{\bullet}$. □

4. Soft group

In this section, let $(E, \circ)$, $(U, *)$ be two groups, A, $B \subseteq E$ and F_A, $G_B \in S_f(U)$. Here we define a soft group based on the concept of soft element and discuss some of the properties of a soft group.

Definition 4.1. A soft groupoid $(F_A^{\bullet}, \widetilde{*})$ over (E, U) is said to be a soft group if
(i) $\widetilde{*}$ is associative,
(ii) there exists a soft element $(e, \{u\}) \widetilde{\in} F_A^{\bullet}$ such that

$$(e, \{u\}) \,\widetilde{*}\, (e_i, \{u_j\}) = (e_i, \{u_j\}) \,\widetilde{*}\, (e, \{u\}) = (e_i, \{u_j\})$$

for all $(e_i, \{u_j\}) \in F_A^{\bullet}$,
(iii) for each soft element $(e_i, \{u_j\}) \widetilde{\in} F_A^{\bullet}$, there exists a soft element $(e_i^{'}, \{u_j^{'}\}) \widetilde{\in} F_A^{\bullet}$ such that $(e_i, \{u_j\}) \,\widetilde{*}\, (e_i^{'}, \{u_j^{'}\}) = (e_i^{'}, \{u_j^{'}\}) \,\widetilde{*}\, (e_i, \{u_j\}) = (e, \{u\})$.

Here $(e, \{u\})$ is said to be the soft identity element and the soft element $(e_i^{'}, \{u_j^{'}\})$ is said to be the soft inverse of $(e_i, \{u_j\})$.

Note 4.1. *By Theorem 3.3,* $\widetilde{*}$ *is associative on* $F_A^{\bullet}$*. Hence condition (i) of Definition 4.1 can be omitted. Therefore the soft groupoid* $(F_A^{\bullet}, \widetilde{*})$ *is said to form a soft group if and only if the conditions (ii) and (iii) hold.*

Example 4.1. *Let* $E = U = Z$, *the set of all integers. Then* $(E, +)$ *and* $(U, +)$ *are two groups, where* $+$ *is the usual addition of integers. Define a soft set* $F : E \to P(U)$ *by* $F(e) = \{..., -4, -2, 0, 2, 4, ...\}$, *when* e *is an even integer;*
$F(e) = \{..., -5, -3, -1, 1, 3, 5, ...\}$, *when* e *is an odd integer.*
Then $F_E^{\bullet}$ *forms a soft commutative group with respect to* $\widetilde{*}$, *where* $\widetilde{*}$ *is given by* $(e_i, \{u_k\}) \widetilde{*} (e_j, \{u_l\}) = (e_i + e_j, \{u_k + u_l\})$ *for all* $(e_i, \{u_k\}), (e_j, \{u_l\}) \widetilde{\in} F_E^{\bullet}$. *Here* $(0, \{0\})$ *is the soft identity element.*

Theorem 4.1. *If* $(F_A^{\bullet}, \widetilde{*})$ *is a soft group over* (E, U) *then*
(i) A *is a subgroup of* E, *and*
(ii) $\bigcup_{e_i \in A} F(e_i)$ *is a subgroup of* U.

Proof. By Theorem 3.1 and Theorem 3.2, $A, \bigcup_{e_i \in A} F(e_i)$ are subgroupoids of E, U respectively. Since $(F_A^{\bullet}, \widetilde{*})$ is a soft group, then it contains the soft identity element, say $(e, \{u\})$. Then by Theorem 3.3, e is the identity element of A and u is the identity element of $\bigcup_{e_i \in A} F(e_i)$.
(i) Now let $e_i \in A$. Since $F_A \in S_f(U)$, $(e_i, \{u_j\}) \widetilde{\in} F_A^{\bullet}$ for some $u_j \in U$. Since each $(e_i, \{u_j\}) \widetilde{\in} F_A^{\bullet}$ is invertible, then by Theorem 3.6, $e_i \in A$ is invertible in A. Therefore A is a subgroup of E.
(ii) Suppose $u_k \in \bigcup_{e_i \in A} F(e_i)$. Then $u_k \in F(e_l)$ for some $e_l \in A$. Hence $(e_l, \{u_k\}) \widetilde{\in} F_A^{\bullet}$. Since $(e_l, \{u_k\})$ is invertible, then by Theorem 3.6, u_k is invertible in $\bigcup_{e_i \in A} F(e_i)$. Therefore $\bigcup_{e_i \in A} F(e_i)$ is a subgroup of U. □

Example 4.2. *In Example 3.1, we see that* $(E, \circ), (U, *)$ *are two groups and it is easy to verify that* $(e_1, \{\rho_0\})$ *is the soft identity element of* $(F_A^{\bullet}, \widetilde{*})$. *Also it can be checked that* $(e_1, \{\rho_0\})^{-1} = (e_1, \{\rho_0\}), (e_1, \{\rho_1\})^{-1} = (e_1, \{\rho_2\}), (e_1, \{\rho_2\})^{-1} = (e_1, \{\rho_1\}), (e_3, \{\rho_3\})^{-1} = (e_3, \{\rho_3\}), (e_3, \{\rho_4\})^{-1} = (e_3, \{\rho_4\}), (e_3, \{\rho_5\})^{-1} = (e_3, \{\rho_5\})$. *Therefore* $(F_A^{\bullet}, \widetilde{*})$ *is a soft group. Moreover* A *is a subgroup of* E *and* $\bigcup_{e_i \in A} F(e_i) = U$ *is a subgroup of* U.

Remark 4. The converse of Theorem 4.1 may not be true, which is justified by the following example.

Example 4.3. *Continuing Example 3.2, we can see that* A, $\bigcup_{e_i \in A} F(e_i) = \{1,\, w,\, w^2\}$ *are subgroups of* E, U *respectively. But* $F_A^\bullet$ *is not a soft groupoid with respect to* $\widetilde{*}$.

Theorem 4.2. *Let* $F_A \in S_f(U)$. *Then* $(F_A^\bullet, \widetilde{*})$ *is a soft group over* (E, U) *if and only if the following conditions hold:*
(i) A *is a subgroup of* E;
(ii) *for each* $e_i, e_j \in A$, $u_k \in F(e_i)$, $u_l \in F(e_j) \Rightarrow u_k * u_l \in F(e_i \circ e_j)$;
(iii) *for each* $e_i \in A$, $u_k \in F(e_i) \Rightarrow u_k^{-1} \in F(e_i^{-1})$.

Proof. At first suppose that $(F_A^\bullet, \widetilde{*})$ is a soft group over (E, U). Then by the Theorem 4.1 and Theorem 3.1, conditions (i) and (ii) hold respectively. Since each soft element $(e_i, \{u_k\}) \widetilde{\in} F_A^\bullet$ have a soft inverse, by Note 3.1, $(e_i, \{u_k\})^{-1} = (e_i^{-1}, \{u_k^{-1}\})$ $\widetilde{\in} F_A^\bullet \Rightarrow u_k^{-1} \in F(e_i^{-1})$. Hence for each $e_i \in A$, $u_k \in F(e_i) \Rightarrow u_k^{-1} \in F(e_i^{-1})$, i.e., condition (iii) holds.
Conversely, let the given three conditions hold. We have to prove that $(F_A^\bullet, \widetilde{*})$ is a soft group over (E, U). By Theorem 3.1, conditions (i), (ii) implies that $(F_A^\bullet, \widetilde{*})$ is a soft groupoid over (E, U). Since $\circ, *$ are associative on E, U respectively, then by Theorem 3.3, $\widetilde{*}$ is associative on $F_A^\bullet$. Suppose e, u are identity elements of E, U respectively. By condition (iii), for each $e_i \in A$, $u_k \in F(e_i) \Rightarrow u_k^{-1} \in F(e_i^{-1})$. Hence by condition (ii), $u_k * u_k^{-1} \in F(e_i \circ e_i^{-1}) \Rightarrow u \in F(e)$. So, by Theorem 3.4, $(e, \{u\})$ is the soft identity element of $F_A^\bullet$. For each $(e_i, \{u_k\}) \widetilde{\in} F_A^\bullet$, $(e_i, \{u_k\}) \widetilde{*} (e_i^{-1}, \{u_k^{-1}\}) =$ $(e_i \circ e_i^{-1}, \{u_k * u_k^{-1}\}) = (e, \{u\})$. Hence $(e_i^{-1}, \{u_k^{-1}\})$ is the soft inverse of the soft element $(e_i, \{u_k\}) \widetilde{\in} F_A^\bullet$. Therefore $(F_A^\bullet, \widetilde{*})$ is a soft group over (E, U). □

Theorem 4.3. *Let* $F_A \in S_f(U)$. *Then* $(F_A^\bullet, \widetilde{*})$ *is a soft group over* (E, U) *if and only if the following conditions hold:*

(i) A *is a subgroup of* E;

(ii) *for each* $e_i,\, e_j \in A,\, u_k \in F(e_i),\, u_l \in F(e_j) \Rightarrow u_k * u_l^{-1} \in F(e_i \circ e_j^{-1})$.

Note 4.2. *If* $(E, \circ), (U, *)$ *be two commutative groups then the soft group* $(F_A^\bullet, \widetilde{*})$ *is also commutative.*

Definition 4.2. Let $(F_A^\bullet, \widetilde{*})$ be a soft group over $(E,\, U)$. For all positive integer n, we define the integral power of each soft element $(e_i,\, \{u_j\}) \widetilde{\in} F_A^\bullet$ by
$(e_i,\, \{u_j\})^n = (e_i,\, \{u_j\}) \widetilde{*} (e_i,\, \{u_j\}) \widetilde{*} \cdots \widetilde{*} (e_i,\, \{u_j\})$ (n factors) $= (e_i^n,\, \{u_j^n\})$;
$(e_i,\, \{u_j\})^{-n} = (e_i,\, \{u_j\})^{-1} \widetilde{*} (e_i,\, \{u_j\})^{-1} \widetilde{*} \cdots \widetilde{*} (e_i,\, \{u_j\})^{-1}$ (n factors) $= (e_i^{-n},\, \{u_j^{-n}\})$;
and $(e_i,\, \{u_j\})^0 = (e, \{u\})$, where $(e, \{u\})$ is the soft identity element of $(F_A^\bullet, \widetilde{*})$.

Definition 4.3. Let $(F_A^\bullet, \widetilde{*})$ be a soft group over $(E,\, U)$. The order of a soft element $(e_i,\, \{u_j\}) \widetilde{\in} F_A^\bullet$, denoted by $O(e_i,\, \{u_j\})$, is the least positive integer n such that $(e_i,\, \{u_j\})^n = (e, \{u\})$, where $(e, \{u\})$ is the soft identity element of $F_A^\bullet$. If such positive integer exist then $(e_i,\, \{u_j\})$ is said to be of finite order otherwise $(e_i,\, \{u_j\})$ is said to be of infinite order.

Theorem 4.4. *Let* $(F_A^\bullet, \widetilde{*})$ *be a soft group over* $(E,\, U)$. *Then for any soft element* $(e_i,\, \{u_j\}) \widetilde{\in} F_A^\bullet$,

$$O(e_i,\, \{u_j\}) = lcm[\, O(e_i),\, O(u_j)\,].$$

Proof. Let $O(e_i,\, \{u_j\}) = k,\, O(e_i) = m,\, O(u_j) = n$ and $lcm(m,n) = l$. Also let, $(e,\, \{u\})$ be the soft identity element of $F_A^\bullet$. Then by Theorem 3.3, e is the identity element of A and u is the identity element of U. Hence $(e_i,\, \{u_j\})^k = (e,\, \{u\}),\, (e_i)^m = e$ and $(u_j)^n = u$. Now $lcm(m,n) = l \Rightarrow m|l$ and $n|l \Rightarrow \exists$ positive integers $m_1,\, n_1$ such that $l = mm_1,\, l = nn_1$. Therefore $(e_i,\, \{u_j\})^l = (e_i^l,\, \{u_j^l\}) = (e_i^{mm_1},\, \{u_j^{nn_1}\}) = ((e_i^m)^{m_1},\, \{(u_j^n)^{n_1}\}) = (e^{m_1},\, \{u^{n_1}\}) = (e,\, \{u\})$. Since k is the order of $(e_i,\, \{u_j\})$, then $k|\,l$.
Again $(e_i,\, \{u_j\})^k = (e,\, \{u\}) \Rightarrow (e_i)^k = e,\, (u_j)^k = u$.

Hence $(e_i)^k = e$ and $O(e_i) = m \Rightarrow m|k$.

Similarly, $(u_j)^k = u$ and $O(u_j) = n \Rightarrow n|k$.

Therefore $m|k,\, n|k \Rightarrow l|k$. Hence $k|l,\, l|k \Rightarrow k = l$. □

Definition 4.4. A soft group $(F_A^\bullet, \widetilde{*})$ over (E, U) is said to form a soft cyclic group if there exists a soft element $(a, \{v\}) \widetilde{\in} F_A^\bullet$ such that each soft element of $F_A^\bullet$ can be expressed in the form $(a, \{v\})^m$ for some integer m. The soft element $(a, \{v\})$ is then called a generator of the soft cyclic group $(F_A^\bullet, \widetilde{*})$.

Theorem 4.5. *Let $(F_A^\bullet, \widetilde{*})$ be a soft cyclic group over (E, U) with generator $(a, \{v\})$. Then a is a generator of A and v is a generator of $\bigcup_{e_i \in A} F(e_i)$.*

Proof. Proof is straightforward. □

Remark 5. The Converse of Theorem 4.5 may not be true. If a is a generator of A and v is a generator of $\bigcup_{e_i \in A} F(e_i)$, then $(a, \{v\})$ may not be a generator of $F_A^\bullet$, because v may not belong to $F(a)$. This fact is justified by the following example.

Example 4.4. *Let $E = \{e_1, e_2\}$ and $U = Z_2$, the classes of residues of integers modulo 2, where $(E, \circ)$ be a group defined in Example 3.4 and U be a group with respect to $+_2$, addition (modulo 2). Take $A = E$ and define a soft set $F : A \to P(U)$ by $F(e_1) = \{\overline{0}, \overline{1}\}$ and $F(e_2) = \{\overline{0}\}$. Then $F_A^\bullet = \{(e_1, \overline{0}), (e_1, \overline{1}), (e_2, \overline{0})\}$. Here A is a cyclic group, generated by e_2 and $\bigcup_{e_i \in A} F(e_i) = U$ is a cyclic group, generated by $\overline{1}$ but $(F_A^\bullet, \widetilde{*})$ is a soft group which is not cyclic because $(e_2, \overline{1}) \widetilde{\notin} F_A^\bullet$.*

We now evaluate the intersection and union of $F_A^\bullet$ & $G_B^\bullet$ in the following example.

Example 4.5. *Let $U = \{u_1, u_2, u_3, u_4, u_5\}$, $E = \{e_1, e_2, e_3, e_4\}$ and $F_A, G_B \in S_f(U)$ be defined by $F_A = \{(e_1, \{u_1, u_2\}), (e_2, \{u_2, u_3\})\}$, $G_B = \{(e_1, \{u_3, u_4\}), (e_2, \{u_3, u_5\}), (e_3, \{u_1, u_5\})\}$. Then $F_A \widetilde{\cap} G_B = \{(e_1, \emptyset), (e_2, \{u_3\})\}$, $F_A \widetilde{\cup} G_B = \{(e_1, \{u_1, u_2, u_3, u_4\}), (e_2, \{u_2, u_3, u_5\}), (e_3, \{u_1, u_5\})\}$, and*

$F_A^\bullet = \{(e_1, \{u_1\}), (e_1, \{u_2\}), (e_2, \{u_2\}), (e_2, \{u_3\})\}$,
$G_B^\bullet = \{(e_1, \{u_3\}), (e_1, \{u_4\}), (e_2, \{u_3\}), (e_2, \{u_5\}), (e_3, \{u_1\}), (e_3, \{u_5\})\}$,
$F_A^\bullet \cap G_B^\bullet = \{(e_2, \{u_3\})\}$,
$F_A^\bullet \cup G_B^\bullet = \{(e_1, \{u_1\}), (e_1, \{u_2\}), (e_1, \{u_3\}), (e_1, \{u_4\}), (e_2, \{u_2\}), (e_2, \{u_3\}),$
$(e_2, \{u_5\}), (e_3, \{u_1\}), (e_3, \{u_5\})\}$,
$(F_A \widetilde{\cap} G_B)^\bullet = \{(e_2, \{u_3\})\}$,
$(F_A \widetilde{\cup} G_B)^\bullet = \{(e_1, \{u_1\}), (e_1, \{u_2\}), (e_1, \{u_3\}), (e_1, \{u_4\}), (e_2, \{u_2\}), (e_2, \{u_3\}),$
$(e_2, \{u_5\}), (e_3, \{u_1\}), (e_3, \{u_5\})\}$.
Thus, $(F_A \widetilde{\cup} G_B)^\bullet = F_A^\bullet \cup G_B^\bullet$, $(F_A \widetilde{\cap} G_B)^\bullet = F_A^\bullet \cap G_B^\bullet$.
Moreover we see that $F_A \widetilde{\cup} G_B \in S_f(U)$ *but* $F_A \widetilde{\cap} G_B \notin S_f(U)$.

The conclusions of the above example will be generalized in the following proposition and its proof is being omitted as it is straightforward.

Proposition 4.1. *For each* $F_A, G_B \in S_f(U)$,
$(i)\,(F_A \widetilde{\cup} G_B)^\bullet = F_A^\bullet \cup G_B^\bullet$ $(ii)\,(F_A \widetilde{\cap} G_B)^\bullet = F_A^\bullet \cap G_B^\bullet$.

Note 4.3. *For* $F_A, G_B \in S_f(U)$, $F_A \widetilde{\cup} G_B \in S_f(U)$, *but* $F_A \widetilde{\cap} G_B$ *may not belong to* $S_f(U)$.

Theorem 4.6. *Let* $F_A^\bullet$ *and* $G_B^\bullet$ *be two soft groups over* (E, U) *with respect to binary composition* $\widetilde{*}$. *If* $F_A \widetilde{\cap} G_B \in S_f(U)$ *and* $A \cap B \neq \emptyset$, *then* $(F_A \widetilde{\cap} G_B)^\bullet = F_A^\bullet \cap G_B^\bullet$ *is a soft group over* (E, U).

Proof. Since $F_A^\bullet, G_B^\bullet$ are two soft groups over (E, U), then A, B are subgroups of E. Therefore $A \cap B$ is a subgroup of E. Let $F_A \widetilde{\cap} G_B = H_C$, then $C = A \cap B$ and $H(e) = F(e) \cap G(e), \forall\, e \in C$. Since $F_A \widetilde{\cap} G_B \in S_f(U)$, $H(e) \neq \emptyset$ for all $e \in A \cap B$. Now let, $e_i, e_j \in A \cap B$, and $u_k \in H(e_i)$, $u_l \in H(e_j)$. This implies that $e_i, e_j \in A, B$ and $u_k \in F(e_i), G(e_i)$ and $u_l \in F(e_j), G(e_j)$. Now $F_A^\bullet$ is a soft group over (E, U), and $e_i, e_j \in A, u_k \in F(e_i), u_l \in F(e_j) \Rightarrow e_i \circ e_j \in A, u_k * u_l \in F(e_i \circ e_j)$ (by

Theorem 4.2). Similarly, $G_B^\bullet$ is a soft group over (E, U), and $e_i, e_j \in B, u_k \in G(e_i), u_l \in G(e_j) \Rightarrow e_i \circ e_j \in B, u_k * u_l \in G(e_i \circ e_j)$. Hence $e_i \circ e_j \in A \cap B$ and $u_k * u_l \in F(e_i \circ e_j) \cap G(e_i \circ e_j) = H(e_i \circ e_j)$. Again by Theorem 4.2, $e_i \in A, u_k \in F(e_i) \Rightarrow u_k^{-1} \in F(e_i^{-1})$ and $e_i \in B, u_k \in G(e_i) \Rightarrow u_k^{-1} \in G(e_i^{-1})$. Therefore $u_k^{-1} \in F(e_i^{-1}) \cap G(e_i^{-1}) = H(e_i^{-1})$. Hence by the Theorem 4.2, $H_C^\bullet$ is a soft group over (E, U). Therefore $(F_A \tilde{\cap} G_B)^\bullet = F_A^\bullet \cap G_B^\bullet$ is a soft group over (E, U). □

Theorem 4.7. *Let $F_A^\bullet$ and $G_B^\bullet$ be two soft groups over (E, U) with respect to binary composition $\tilde{*}$. Then $(F_A \tilde{\cup} G_B)^\bullet = F_A^\bullet \cup G_B^\bullet$ forms a soft group over (E, U) if $F_A \tilde{\subseteq} G_B$, or $G_B \tilde{\subseteq} F_A$.*

Proof. Proof follows by Note 2.1. □

Example 4.6. *In Example 3.1, let $B = \{e_1, e_2\}$ and define a soft set $G : B \to P(U)$ by $G(e_1) = \{\rho_0, \rho_1, \rho_2\}, G(e_2) = \{\rho_3, \rho_4, \rho_5\}$. Then the soft elements of $G_B^\bullet$ are $(e_1, \{\rho_0\}), (e_1, \{\rho_1\}), (e_1, \{\rho_2\}), (e_2, \{\rho_3\}), (e_2, \{\rho_4\}), (e_2, \{\rho_5\})$. Then it is easy to verify that $(G_B^\bullet, \tilde{*})$ is a soft group over (E, U).*
Now let $F_A \tilde{\cap} G_B = H_C$. Then $C = \{e_1\}$ and $H(e_1) = F(e_1) \cap G(e_1) = \{\rho_0, \rho_1, \rho_2\}$. Therefore $(F_A \tilde{\cap} G_B)^\bullet = H_C^\bullet = \{(e_1, \{\rho_0\}), (e_1, \{\rho_1\}), (e_1, \{\rho_2\})\}$ is a soft group with respect to $\tilde{*}$.
Let $F_A \tilde{\cup} G_B = J_C$. Then $C = A \cup B = \{e_1, e_2, e_3\}$ and $J(e_1) = \{\rho_0, \rho_1, \rho_2\}, J(e_2) = \{\rho_3, \rho_4, \rho_5\}, J(e_3) = \{\rho_3, \rho_4, \rho_5\}$. Therefore $J_C^\bullet = \{(e_1, \{\rho_0\}), (e_1, \{\rho_1\}), (e_1, \{\rho_2\}), (e_2, \{\rho_3\}), (e_2, \{\rho_4\}), (e_2, \{\rho_5\}), (e_3, \{\rho_3\}), (e_3, \{\rho_4\}), (e_3, \{\rho_5\})\}$. Here $(F_A \tilde{\cup} G_B)^\bullet = J_C^\bullet$ is not a soft group with respect to $\tilde{*}$ because $(e_2, \{\rho_4\}) \tilde{*} (e_3, \{\rho_5\}) = (e_2 \circ e_3, \{\rho_4 * \rho_5\}) = (e_4, \{\rho_1\}) \notin J_C^\bullet$. Also note that neither $F_A \tilde{\subseteq} G_B$ nor $G_B \tilde{\subseteq} F_A$.
Again let $D = \{e_1\}$ and define a soft set $K : D \to P(U)$ by $K(e_1) = \{\rho_0\}$. Therefore $K_D^\bullet = \{(e_1, \{\rho_0\})\}$. Obviously, $(K_D^\bullet, \tilde{*})$ is a soft group. Moreover $K_D \tilde{\subseteq} F_A$. Therefore $(F_A \tilde{\cap} K_D)^\bullet = K_D^\bullet$ and $(F_A \tilde{\cup} K_D)^\bullet = F_A^\bullet$ are soft groups.

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