

## TRIPLED COINCIDENCE POINT THEOREM IN FUZZY METRIC SPACES

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ABSTRACT. In this present paper we prove some common tripled fixed point theorems for contractive mappings in fuzzy metric spaces under geometrically convergent  $t$ -norms. Our aim of this paper is to improve the result of A. Gupta, R. Narayan and R. N. Yadava, Tripled Fixed Point For Compatible Mappings In Partially Ordered Fuzzy Metric Spaces, The Journal Of Fuzzy Mathematics 22(3), 565-580, 2014. Our technique for the proof of the theorem is different. We also give an example in support of our theorem.

### 1. INTRODUCTION

The fixed point theorems in metric spaces are playing a major role to construct methods in mathematics to solve problems in applied mathematics and sciences. So the attraction of metric spaces to a large numbers of mathematicians is understandable. Some generalizations of the notion of a metric space have been proposed by some authors.

The concept of fuzzy sets was introduced initially by Zadeh [17] in 1965. After that, to use this concept in topology and analysis, many authors have expansively developed the theory of fuzzy sets and application [5], [6].

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Recently, Vasile Berinde and Marin Borcut [2] extended and generalized the results of [14] to the case of contractive operator  $F : X \times X \times X \rightarrow X$ , where  $X$  is a complete ordered metric space. They introduced the concept of a tripled fixed point and the mixed monotone property of a mapping  $F : X \times X \times X \rightarrow X$ . For more detail of tripled fixed point results we refer the reader to ([2] and [1]).

In the present paper, we prove a tripled coincidence point result in fuzzy metric spaces as defined by George and Veeramani under geometrically convergent  $t$ -norms. We use a new fuzzy contractive inequality and improve the result of A.Gupta et al [8]. Our technique for the proof is different from the other existing results on the same topic. We assume that the associated  $t$ -norm is a Hadžić type  $t$ -norm. The result is illustrated with an example and is an extension of some known results.

## 2. PRELIMINARIES

A triangular norm ( $t$ -norm for short) is a binary operation on the unit interval  $[0, 1]$ , i.e., a function  $T : [0, 1]^2 \rightarrow [0, 1]$  which is commutative, associative, monotone and  $T(x, 1) = x$ .  $t$ -conorm  $S$  is defined by  $S(x, y) = 1T(1x, 1y)$

If  $T$  is a  $t$ -norm,  $x \in [0, 1]$  and  $n \in N \cup \{0\}$  then we shall write

$$x_T^{(n)} = \begin{cases} 1, & \text{if } n = 0 \\ T\left(x_T^{(n-1)}, x\right), & \text{if } otherwise. \end{cases}$$

We assume that the reader is familiar with the basic concepts and terminology of the theory of fuzzy metric spaces. We only recall that a  $t$ -norm  $T$  is said to be of Hadžić-type ( denoted  $T \in H$  and  $T^\infty$  means infinite sequence of Hadžić-type) if the family  $\{T^n(t)\}_{n=1}^\infty$  defined by

$$T^1(t) = t; \quad T^{n+1}(t) = T(t, T^n(t)) \quad (n = 1, 2, \dots, t \in [0, 1])$$

is equicontinuous at  $t = 1$ , and that a  $t$ -norm  $T$  is called geometrically convergent (or  $g$ -convergent) [10] if, for all  $q \in (0, 1)$ ,

$$\lim_{n \rightarrow \infty} T_{i=n}^{\infty}(1 - q^i) = 1.$$

It is worth noting that if for a  $t$ -norm there exists  $q_0 \in (0, 1)$  such that

$$\lim_{n \rightarrow \infty} T_{i=n}^{\infty}(1 - q_0^i) = 1,$$

then

$$\lim_{n \rightarrow \infty} T_{i=n}^{\infty}(1 - q^i) = 1$$

for all  $q \in (0, 1)$ .

The well-known  $t$ -norms  $T_P(x, y) = xy$ ,  $T_L(x, y) = \max\{x+y-1, 0\}$  and  $T_M(x, y) = \max\{x, y\}$  are  $g$ -convergent. Also, every member of the *Dombi family*  $(T_{\lambda}^D)_{\lambda \in (0,1)}$ , *Aczel-Alsina family*  $(T_{\lambda}^{AA})_{\lambda \in (0,1)}$  and *Sugeno-Weber family*  $(T_{\lambda}^{SW})_{\lambda \in (0,1)}$  are  $g$ -convergent [10]. A large class of  $g$ -convergent  $t$ -norms, in terms of the generators of strict  $t$ -norms is described in [10].

We shall need some families of  $t$ -norms given in the following example.

**Example 1.** The Dombi family of  $t$ -norms  $(T_{\lambda}^D)_{\lambda \in [0, \infty]}$  is defined by

$$T_{\lambda}^D(x, y) = \begin{cases} T_D(x, y), & \text{if } \lambda = 0 \\ T_M(x, y), & \text{if } \lambda = \infty \\ \left( 1 + \left( \left( \frac{1-x}{x} \right)^{\lambda} + \left( \frac{1-y}{y} \right)^{\lambda} \right)^{\frac{1}{\lambda}} \right)^{-1} & \text{if } \lambda \in (0, \infty). \end{cases}$$

**Example 2.** The Schweizer-Sklar family of  $t$ -norms  $(T_\lambda^{SS})_{\lambda \in [-\infty, \infty]}$  is defined by

$$T_\lambda^{SS}(x, y) = \begin{cases} T_D(x, y), & \text{if } \lambda = -\infty \\ (x^\lambda + y^\lambda - 1)^{\frac{1}{\lambda}}, & \text{if } \lambda \in (-\infty, 0), \\ T_P(x, y), & \text{if } \lambda = 0, \\ (\max\{x^\lambda + y^\lambda - 1, 0\})^{\frac{1}{\lambda}}, & \text{if } \lambda \in (0, \infty), \\ T_D(x, y), & \text{if } \lambda = \infty. \end{cases}$$

**Example 3.** The Aczél-Alsina family of  $t$ -norms  $(T_\lambda^{AA})_{\lambda \in [0, \infty]}$  is defined by

$$T_\lambda^{AA}(x, y) = \begin{cases} T_D(x, y), & \text{if } \lambda = 0 \\ T_M(x, y), & \text{if } \lambda = \infty \\ e^{-(|\log x|^\lambda + |\log y|^\lambda)^{\frac{1}{\lambda}}}, & \text{if } \lambda \in (0, \infty). \end{cases}$$

**Example 4.** The Sugeno-Weber family of  $t$ -norms  $(T_\lambda^{SW})_{\lambda \in [-1, +\infty]}$  is defined by

$$T_\lambda^{SW}(x, y) = \begin{cases} T_D(x, y), & \text{if } \lambda = -1 \\ T_P(x, y), & \text{if } \lambda = +\infty \\ \max\left(0, \frac{x+y-1+\lambda xy}{1+\lambda}\right) & \text{if otherwise.} \end{cases}$$

In the following we consider  $M$ -complete fuzzy metric spaces in the sense of Kramosil and Michalek [13], satisfying the condition (FM-6):

$$(2.1) \quad \lim_{t \rightarrow \infty} M(x, y, t) = 1$$

for all  $x, y \in X$ .

**Definition 5.** [2] An element  $(x, y, z) \in X \times X \times X$  is called a tripled fixed point of the mapping  $F : X \times X \times X \rightarrow X$  if

$$F(x, y, z) = x, \quad F(y, x, y) = y \quad \text{and} \quad F(z, y, x) = z.$$

**Definition 6.** [2] Let  $X$  be a nonempty set. The mappings  $F : X \times X \times X \rightarrow X$  and  $g : X \rightarrow X$  are said to commute if

$$g(F(x, y, z)) = F(gx, gy, gz)$$

for all  $x, y, z \in X$ .

**Definition 7.** [2] An element  $(x, y, z) \in X \times X \times X$  is called a *tripled coincidence point* of the mappings  $F : X \times X \times X \rightarrow X$  and  $g : X \rightarrow X$  if

$$F(x, y, z) = gx, \quad F(y, x, y) = gy \quad \text{and} \quad F(z, y, x) = gz.$$

**Definition 8.** [2] An element  $(x, y, z) \in X \times X \times X$  is called a *tripled common fixed point* of the mappings  $F : X \times X \times X \rightarrow X$  and  $g : X \rightarrow X$  if

$$F(x, y, z) = gx = x, \quad F(y, x, y) = gy = y \quad \text{and} \quad F(z, y, x) = gz = z.$$

**Definition 9.** [2] An element  $x \in X$  is called a *common fixed point* of the mappings  $F : X \times X \times X \rightarrow X$  and  $g : X \rightarrow X$  if

$$F(x, x, x) = gx = x.$$

### 3. MAIN RESULTS

**Theorem 10.** Let  $(X, M, T)$  be a complete Fuzzy metric space, satisfying 2.1, with  $T$  is a  $g$ -convergent  $t$ -norm. Let  $F : X \times X \times X \rightarrow X$  and  $g : X \rightarrow X$  be two mappings and there exists  $k \in (0, 1)$  such that,

$$(3.1) \quad M(F(x, y, z), F(u, v, w), kt) \geq \min\{M(gx, gu, t), M(gy, gv, t), M(gz, gw, t)\}$$

for all  $x, y, z, u, v, w \in X, t > 0$ .

Suppose that  $F(X \times X \times X) \subset g(X)$  and  $g$  is continuous,  $F$  and  $g$  are commuting. If there exist  $a > 0$  and  $x_0, y_0, z_0 \in X$  such that

$$\sup_{t>0} t^a (1 - M(gx_0, F(x_0, y_0, z_0), t)) < \infty,$$

$$\sup_{t>0} t^a (1 - M(gy_0, F(y_0, x_0, y_0), t)) < \infty,$$

$$\sup_{t>0} t^a (1 - M(gz_0, F(z_0, y_0, x_0), t)) < \infty,$$

then there exists a unique  $x \in X$  such that  $x = g(x) = F(x, x, x)$ , which means that,  $F$  and  $g$  have a unique common fixed point in  $X$ .

It should be noted that  $(x_0, y_0, z_0)$  is a tripled coincidence point of  $F$  and  $g$ , then the conditions

$$\sup_{t>0} t^a (1 - M(gx_0, F(x_0, y_0, z_0), t)) < \infty,$$

$$\sup_{t>0} t^a (1 - M(gy_0, F(y_0, x_0, y_0), t)) < \infty,$$

$$\sup_{t>0} t^a (1 - M(gz_0, F(z_0, y_0, x_0), t)) < \infty,$$

are satisfied.

*Proof.* Let  $x_0, y_0, z_0 \in X$  be three arbitrary points in  $X$ . Since  $F(X \times X \times X) \subseteq g(X)$ , we can choose  $x_1, y_1, z_1 \in X$  such that  $g(x_1) = F(x_0, y_0, z_0)$ ,  $g(y_1) = F(y_0, z_0, x_0)$  and  $g(z_1) = F(z_0, x_0, y_0)$  using same arguments, we can construct three sequences  $\{x_n\}$ ,  $\{y_n\}$ ,  $\{z_n\}$  in  $X$  such that

$$g(x_{n+1}) = F(x_n, y_n, z_n), \quad g(y_{n+1}) = F(y_n, x_n, y_n), \quad g(z_{n+1}) = F(z_n, x_n, y_n).$$

The proof of the Theorem is divided into five steps,

**Step - 1.** Prove that  $\{x_n\}$ ,  $\{y_n\}$ ,  $\{z_n\}$  are Cauchy sequences.

Let  $\alpha > 0$  be such that

$$t^a (1 - M(gx_0, F(x_0, y_0, z_0), t)) \leq \alpha,$$

$$t^a (1 - M(gy_0, F(y_0, x_0, y_0), t)) \leq \alpha,$$

$$t^a (1 - M(gz_0, F(z_0, y_0, x_0), t)) \leq \alpha,$$

for all  $t > 0$ . Then

$$\begin{aligned} M\left(gx_0, gx_1, \frac{1}{t^n}\right) &\geq 1 - \alpha(t^a)^\alpha, \\ M\left(gy_0, gy_1, \frac{1}{t^n}\right) &\geq 1 - \alpha(t^a)^\alpha, \\ M\left(gz_0, gz_1, \frac{1}{t^n}\right) &\geq 1 - \alpha(t^a)^\alpha, \end{aligned}$$

for every  $t > 0$  and  $n \in N$ .

If  $t > 0$  and  $\epsilon \in (0, 1)$  are given, we choose  $\mu$  in the interval  $(k, 1)$  such that

$$T_{i=n+1}^\infty(1 - (\mu^a)^i) > 1 - \epsilon$$

and  $\delta = \frac{k}{\mu}$ . As  $\delta \in (0, 1)$ , we can find  $n_1 = n_1(t)$  such that  $\sum_{n=n_1}^\infty \delta^n < t$ .

Condition 3.1 implies that, for all  $s > 0$ ,

$$\begin{aligned} M(gx_1, gx_2, ks) &= M(F(x_0, y_0, z_0), F(x_1, y_1, z_1), ks) \\ &\geq \min\{M(gx_0, gx_1, s), M(gy_0, gy_1, s), M(gz_0, gz_1, s)\}, \end{aligned}$$

$$\begin{aligned} M(gy_1, gy_2, ks) &= M(F(y_0, x_0, y_0), F(y_1, x_1, y_1), ks) \\ &\geq \min\{M(gy_0, gy_1, s), M(gz_0, gz_1, s), M(gy_0, gy_1, s)\}, \end{aligned}$$

and

$$\begin{aligned} M(gz_1, gz_2, ks) &= M(F(z_0, y_0, x_0), F(z_1, y_1, x_1), ks) \\ &\geq \min\{M(gz_0, gz_1, s), M(gy_0, gy_1, s), M(gx_0, gx_1, s)\}. \end{aligned}$$

It follows by induction that

$$M(gx_1, gx_2, k^n s) \geq \min\{M(gx_0, gx_1, s), M(gy_0, gy_1, s), M(gz_0, gz_1, s)\},$$

$$M(gy_1, gy_2, k^n s) \geq \min\{M(gy_0, gy_1, s), M(gz_0, gz_1, s), M(gy_0, gy_1, s)\},$$

and

$$M(gz_1, gz_2, k^n s) \geq \min\{M(gz_0, gz_1, s), M(gy_0, gy_1, s), M(gx_0, gx_1, s)\},$$

for all  $n \in N$ . Then for all  $n \geq n_1$  and all  $m \in N$ , we obtain

$$\begin{aligned} & M(gx_n, gx_{n+m}, t) \\ & \geq M(gx_n, gx_{n+m}, \Sigma_{i=n_1}^{\infty} \delta^i) \\ & \geq M(gx_n, gx_{n+m}, \Sigma_{i=n}^{n+m-1} \delta^i) \\ & \geq T_{i=n}^{n+m-1} M(gx_i, gx_{i+1}, \delta^i) \\ & \geq T_{i=n}^{n+m-1} \left( \min \left\{ M\left(gx_0, gx_1, \frac{1}{\mu^i}\right), M\left(gy_0, gy_1, \frac{1}{\mu^i}\right), M\left(gz_0, gz_1, \frac{1}{\mu^i}\right) \right\} \right) \\ & \geq T_{i=n}^{n+m-1} (1 - \alpha \mu^{\alpha i}). \end{aligned}$$

If we choose  $l_0 \in N$  such that  $\alpha \mu^{al_0} \leq \mu^a$ , then

$$1 - \alpha(\mu^a)^{n+l_0} \geq 1 - (\mu^a)^{n+1},$$

for all  $n$ . Thus,

$$M(gx_{n+l_0}, gx_{n+l_0+m}, t) \geq T_{i=n+1}^{\infty} (1 - (\mu^a)^i) > 1 - \epsilon,$$

for every  $n \geq n_1$  and  $m \in N$ , hence  $\{gx_n\}$  is a Cauchy sequence.

Similarly, one can show that  $\{gy_n\}$  and  $\{gz_n\}$  are Cauchy sequences.

**Step 2.** Prove that  $g$  and  $F$  have common coincidence point.

Since  $X$  is complete space, there exist  $x, y, z \in X$  such that,



$$\lim_{n \rightarrow \infty} F(x_n, y_n, z_n) = \lim_{n \rightarrow \infty} g(x_n) = x,$$

$$\lim_{n \rightarrow \infty} F(y_n, z_n, x_n) = \lim_{n \rightarrow \infty} g(y_n) = y,$$

$$\lim_{n \rightarrow \infty} F(z_n, x_n, y_n) = \lim_{n \rightarrow \infty} g(z_n) = z.$$

Next, we prove that  $g(x) = F(x, y, z)$ ,  $g(y) = F(y, x, y)$  and  $g(z) = F(z, y, x)$ .

From the continuity of  $g$ , it follows that

$$\lim_{n \rightarrow \infty} g g x_n = g x,$$

$$\lim_{n \rightarrow \infty} g g y_n = g y,$$

$$\lim_{n \rightarrow \infty} g g z_n = g z.$$

As  $F$  and  $g$  commuting,

$$g g x_{n+1} = g F(x_n, y_n, z_n) = F(g x_n, g y_n, g z_n),$$

$$g g y_{n+1} = g F(y_n, x_n, y_n) = F(g y_n, g x_n, g y_n),$$

and

$$g g z_{n+1} = g F(z_n, y_n, x_n) = F(g z_n, g y_n, g x_n).$$

Consequently, for all  $t > 0$  and  $n \in N$ ,

$$\begin{aligned} M(g g x_{n+1}, F(x, y, z), kt) &= M(F(x_n, y_n, z_n), F(x, y, z), kt) \\ &= M(F(g x_n, g y_n, g z_n), F(x, y, z), kt) \\ &\geq \min\{M(g g x_n, g x, t), M(g g y_n, g y, t), M(g g z_n, g z, t)\}. \end{aligned}$$

Letting  $n \rightarrow \infty$  yields  $M(g x, F(x, y, z), kt) \geq 1$  for all  $t > 0$ , hence  $g x = F(x, y, z)$ .

Similarly one can deduce that  $g(y) = F(y, x, y)$  and  $g(z) = F(z, y, x)$ .

**Step 3.** We show that  $gx = y$ ,  $gy = z$  and  $gz = x$ .

Indeed letting  $n \rightarrow \infty$  in the inequality

$$\begin{aligned} M(gx, gy_{n+1}, kt) &= M(F(x, y, z), F(y_n, x_n, y_n), kt) \\ &\leq \min\{M(gx, gy_n, t), M(gy, gx_n, t), M(gz, gy_n, t)\} \\ M(gx, y, kt) &\leq \min\{M(gx, y, t), M(gy, x, t), M(gz, y, t)\}. \end{aligned}$$

Similarly

$$\begin{aligned} M(gy, gz_{n+1}, kt) &= M(F(y, x, y), F(z_n, y_n, x_n), kt) \\ &\leq \min\{M(gy, gz_n, t), M(gx, gy_n, t), M(gy, gx_n, t)\} \\ M(gy, z, kt) &\leq \min\{M(gy, z, t), M(gx, z, t), M(gy, x, t)\} \end{aligned}$$

$$\begin{aligned} M(gx, gz_{n+1}, kt) &= M(F(x, y, z), F(z_n, y_n, x_n), kt) \\ &\leq \min\{M(gx, gz_n, t), M(gy, gy_n, t), M(gz, gx_n, t)\} \\ M(gx, z, kt) &\leq \min\{M(gx, z, t), M(gy, y, t), M(gz, x, t)\}. \end{aligned}$$

Thus

$$\begin{aligned} &\min\{M(gx, y, t), M(gy, z, t), M(gx, z, t)\} \\ &\geq \min\left\{M\left(gx, y, \frac{t}{k^n}\right), M\left(gy, z, \frac{t}{k^n}\right), M\left(gx, z, \frac{t}{k^n}\right)\right\} \end{aligned}$$

for all  $n \in N$ , implying

$$\min\{M(gx, y, t), M(gy, z, t), M(gx, z, t)\} = 1$$

for all  $t > 0$ . It follows that

$$M(gx, y, t) = 1,$$

$$M(gy, z, t) = 1,$$

$$M(gx, z, t) = 1,$$

for all  $t > 0$ , whence  $gx = y$ ,  $gy = z$  and  $gz = x$ .

**Step 4.** We prove that  $x = y = z$ .

Indeed, letting  $n \rightarrow \infty$  in the inequality

$$\begin{aligned} M(gx_{n+1}, gy_{n+1}, kt) &= M(F(x_n, y_n, z_n)F(y_n, x_n, y_n), kt) \\ &\geq \min\{M(gx_n, gy_n, t), M(gy_n, gx_n, t), M(gz_n, gy_n, t)\} \\ M(x, y, kt) &\geq \min\{M(x, y, t), M(y, x, t), M(z, y, t)\}. \end{aligned}$$

Similarly we have

$$\begin{aligned} M(gz_{n+1}, gy_{n+1}, kt) &= M(F(z_n, y_n, x_n)F(y_n, x_n, y_n), kt) \\ &\geq \min\{M(gz_n, gy_n, t), M(gy_n, gx_n, t), M(gx_n, gy_n, t)\} \\ M(z, y, kt) &\geq \min\{M(z, y, t), M(y, x, t), M(x, y, t)\} \end{aligned}$$

$$\begin{aligned} M(gx_{n+1}, gz_{n+1}, kt) &= M(F(x_n, y_n, z_n)F(z_n, y_n, x_n), kt) \\ &\geq \min\{M(gx_n, gz_n, t), M(gy_n, gy_n, t), M(gz_n, gx_n, t)\} \\ M(x, z, kt) &\geq \min\{M(x, z, t), M(y, y, t), M(z, x, t)\}. \end{aligned}$$

It follows that

$$\min\{M(x, y, kt), M(z, y, kt), M(x, z, kt)\} \geq \min\{M(x, y, t), M(y, x, t), M(z, y, t)\}$$

for all  $t > 0$  and so  $x = y = z$ .

**Step 5.** We show that the fixed point is unique.

Let  $p, q$  be common fixed points for  $F$  and  $g$ . Then from 3.1 we obtain

$$\begin{aligned} M(gp, gq, kt) &= M(F(p, p, p), F(q, q, q), kt) \\ &\geq \min\{M(gp, gq, t), M(gp, gq, t), M(gp, gq, t)\} \\ &\geq \min\{M(p, q, t), M(p, q, t), M(p, q, t)\} \\ M(p, q, kt) &\geq M(p, q, t) \end{aligned}$$

for all  $t > 0$ , implying  $p = q$ .

Hence the proof is completed. □

Our next theorem shows that, if the  $t$ -norm  $T$  is of Hadžić -type, then the conditions

$$\begin{aligned} \sup_{t>0} t^a (1 - M(gx_0, F(x_0, y_0, z_0), t)) &< 1 \\ \sup_{t>0} t^a (1 - M(gy_0, F(y_0, x_0, y_0), t)) &< 1 \\ \sup_{t>0} t^a (1 - M(gz_0, F(z_0, y_0, x_0), t)) &< 1 \end{aligned}$$

can be dropped.

**Theorem 11.** *Let  $(X, M, T)$  be a complete Fuzzy metric space, be a complete fuzzy metric space satisfying 2.1, with  $T \in H$ . Let  $F : X \times X \times X \rightarrow X$  and  $g : X \rightarrow X$  be two mappings such that, for some  $k \in (0, 1)$ ,*

$$(3.2) \quad M(F(x, y, z), F(u, v, w), kt) \geq \min\{M(gx, gu, t), M(gy, gv, t), M(gz, gw, t)\}$$

for all  $x, y, z, u, v, w \in X$ ,  $t > 0$ .

Suppose that  $F(X \times X \times X) \subset g(X)$  and  $g$  is continuous,  $F$  and  $g$  are commuting. Then  $F$  and  $g$  have a unique common fixed point in  $X$ .

*Proof.* We only verify Step -1 of Theorem 10, so we need to prove that  $\{x_n\}$ ,  $\{y_n\}$  and  $\{z_n\}$  are Cauchy sequences.

Let  $t > 0$  and  $\epsilon \in (0, 1)$  be given. Since  $T$  is a  $t$ -norm of Hadžić type, then there exists  $\mu > 0$  such that

$$T^k(1 - \mu) > 1 - \epsilon,$$

for all  $k \in \mathbb{N}$ .

By 2.1, we can find  $s > 0$  such that

$$M(gx_0, gx_1, s) > 1 - \mu,$$

$$M(gy_0, gy_1, s) > 1 - \mu$$

and

$$M(gz_0, gz_1, s) > 1 - \mu.$$

Let  $n_0 \in \mathbb{N}$  be such that  $t > \sum_{i=n_0}^{\infty} k^i s$ .

As in Step 1 of the proof of Theorem 10 it can be proved that

$$M(gx_n, gx_{n+1}, k^n s) \geq \min\{M(gx_0, gx_1, s), M(gy_0, gy_1, s), M(gz_0, gz_1, s)\} > 1 - \mu,$$

$$M(gy_n, gy_{n+1}, k^n s) \geq \min\{M(gy_0, gy_1, s), M(gz_0, gz_1, s), M(gx_0, gx_1, s)\} > 1 - \mu,$$

and

$$M(gz_n, gz_{n+1}, k^n s) \geq \min\{M(gz_0, gz_1, s), M(gy_0, gy_1, s), M(gx_0, gx_1, s)\} > 1 - \mu$$

for all  $n \in N$ . Therefore, for all  $n \geq n_0$  and all  $m \in N$  the following inequalities hold:

$$\begin{aligned}
 M(gx_n, gx_{n+m}, t) &\geq M(gx_n, gx_{n+m}, \Sigma_{i=n_1}^{\infty} k^i s) \\
 &\geq M(gx_n, gx_{n+m}, \Sigma_{i=n}^{n+m-1} k^i s) \\
 &\geq T_{i=n}^{n+m-1} M(gx_i, gx_{i+1}, k^n s) \\
 &\geq T_{i=n}^{n+m-1} (1 - \mu) \\
 M(gx_n, gx_{n+m}, t) &> 1 - \epsilon.
 \end{aligned}$$

This complete the proof. □

Next we give an example to illustrate Theorem 3.1.

**Example 12.** Let  $X = [-3, 3]$  and  $M(x, y, t) = \left(\frac{t}{t+1}\right)^{|x-y|}$ . It is easy to verify that  $(X, M, T_P)$  is a complete fuzzy metric spaces.

Let  $F : X \times X \times X \rightarrow X$ , be defined by

$$F(x, y, z) = \frac{x^2}{18} + \frac{y^2}{18} + \frac{z^2}{18} - 3$$

and  $g : X \rightarrow X$  be such that

$$g(x) = x.$$

Then  $F(X \times X \times X) = [-3, -\frac{3}{2}]$  and 3.1 is verified with  $k = \frac{1}{3}$ .

Indeed, since

$$\frac{t/3}{t/3+1} \geq \left(\frac{t}{t+1}\right)^3$$

for all  $t \geq 0$ , then

$$\begin{aligned}
 M\left(F(x, y, z), F(u, v, w), \frac{t}{3}\right) &= \left(\frac{\frac{t}{3}}{\frac{t}{3} + 1}\right)^{\frac{|x^2 - u^2 + y^2 - v^2 + z^2 - w^2|}{18}} \\
 &\geq \left(\frac{t}{t + 1}\right)^{\frac{|x^2 - u^2 + y^2 - v^2 + z^2 - w^2|}{6}} \\
 &\geq \left(\frac{t}{t + 1}\right)^{\frac{|x - u| + |y - v| + |z - w|}{3}} \\
 &\geq \min \left\{ \left(\frac{t}{t + 1}\right)^{|x - u|}, \left(\frac{t}{t + 1}\right)^{|y - v|}, \left(\frac{t}{t + 1}\right)^{|z - w|} \right\} \\
 M\left(F(x, y, z), F(u, v, w), \frac{t}{3}\right) &\geq \min\{M(gx, gu, t), M(gy, gv, t), M(gz, gw, t)\}
 \end{aligned}$$

for all  $x, y, z, u, v, w \in X$ ,  $t > 0$ . The point  $x = 4(1 - \sqrt{3}) \in X$  and it is the unique common fixed point of  $F$  and  $g$ .

#### 4. CONCLUSION

In this paper we have proved tripled coincidence point results in ordered fuzzy metric spaces by assuming an inequality, certain conditions on the  $t$ -norm and commutativity condition between the mappings. Here we also remove the monotone property and prove a common tripled fixed point for fuzzy metric spaces.

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