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RECURRENCE RELATIONS FOR MOMENTS AND MOMENT GENERATING FUNCTIONS FROM THE EXTENDED TYPE I GENERALIZED LOGISTIC DISTRIBUTION BASED ON $k-th$ LOWER RECORD VALUES

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Abstract. In this study we give explicit expressions and some recurrence relations for marginal and joint moment generating functions of $k-$th lower record values from extended type I generalized logistic distribution. Further a characterization of this distribution by considering recurrence relations for marginal moment generating functions of the $k-$th lower record values is presented.

1. Introduction

Record values are used in many statistical applications, statistical modeling and inference involving data pertaining to weather, athletic events, economics, life testing studies and so on. For example, Guinness World Records, fastest time taken to recite the periodic table of the elements or shortest ever tennis matches both in terms of number of games and duration of time or fastest indoor marathon, etc. People make several attempts to make records but records are made only when the attempt is a success. Usually, we don't get the data on all of the attempts made to break the

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records around the world. The data that we have are the records. Because of importance of record values in many fields of application, these kind of ordered data have been extensively studied in the literature. There are hundreds of papers and several books published on record-breaking data and its distributional properties (see, for instance, Chandler [10], Resnick [23], Shorrock [22], Glick [19], Nevzorov [25], Ahsanullah [11], Balakrishnan and Ahsanullah [15, 16], Grunzien and Szynal [26] and Arnold *et al*. [3, 4]). Hence, it is pertinent that one has to study the properties based on records.

In the context of order statistics model and reliability theory, the life length of the $r-$out-of$-n$ system is the $(n-r+1)$th order statistic in a sample of size n. Another related model is the model of record statistics defined by Chandler [10] as a model for successive extremes in a sequence of independent and identically distributed (iid) random variables. The study of record values and associated statistics are of great significance in many real life situations such as meteorology, seismology, athletic events, economics, and life testing.

Let $\{X_n, n \geq 1\}$ be a sequence of iid random variables with cumulative distribution function (*cdf*) $F(x)$ and probability density function (*pdf*) $f(x)$. The $j-$th order statistic of a sample $(X_1, X_2, \ldots, X_n)$ is denoted by $X_{j:n}$. For a fixed $k \geq 1$ we define the sequence $\{L^{(k)}(n), n \geq 1\}$ of $k-$th lower record times of $X_1, X_1 \ldots$ as follows:

$$L^{(k)}(1) = 1,$$

$$L^{(k)}(n+1) = min\{j > L^{(k)}(n) : X_{k:L^{(k)}(n)+k-1} > X_{k:j+k-1}\}.$$

The sequences $\{Z_n^{(k)}, n \geq 1\}$ with $Z_n^{(k)} = X_{k:L^{(k)}(n)+k-1}$, $n = 1, 2, \ldots,$ are called the sequences of $k-$th lower record values of $\{X_n, n \geq 1\}$. For convenience, we shall also take $Z_0^{(k)} = 0$. Note that $k = 1$ we have $Y_n^{(1)} = X_{L(n)},\ n \geq 1$, *i.e.* record values of $\{X_n,\ n \geq 1\}$.

The joint *pdf* of $k-$th lower record values $Z_1^{(k)}, Z_2^{(k)}, \ldots, Z_n^{(k)}$ can be given as the joint *pdf* of $k-$th upper record values of $\{-X_n, n \geq 1\}$, Pawlas and Szynal [20]

$$f_{z_1^{(k)},\ldots,z_n^{(k)}}(z_1,\ldots,z_n) = k^n \left(\prod_{i=1}^{n-1} \frac{f(z_i)}{F(z_i)}\right) [F(z_n)]^{k-1} f(z_n),\ z_1 > z_2 > \ldots > z_n.$$

In view of above equation, the marginal *pdf* of $X_{L(n)}^{(k)}$, $n \geq 1$ is given by

$$f_{X_{L(n)}^{(k)}}(x) = \frac{k^n}{\Gamma(n)} [-ln(F(x))]^{n-1} [F(x)]^{k-1} f(x),\ n \geq 1, \tag{1.1}$$

and the joint *pdf* of $X_{L(m)}^{(k)}$ and $X_{L(n)}^{(k)}$, $1 \leq m < n$, $n > 2$ is given by

$$f_{X_{L(m)}^{(k)}, X_{L(n)}^{(k)}}(x,y) = \frac{k^n}{\Gamma(m)\Gamma(n-m)} [-ln(F(x))]^{m-1}$$

$$\times [-ln(F(y)) + ln(F(x))]^{n-m-1} [F(y)]^{k-1} \frac{f(x)}{F(x)} f(y),\ x > y, \tag{1.2}$$

where $\Gamma(x)$ is a gamma function. When x is a positive integer, $\Gamma(x) = (x-1)!$.

Let $X_1, X_2, \ldots, X_n$ be a random sample of the extened type I generalized logistic distribution with *pdf* and *cdf* as in (1.3) and (1.4) respectively, and let $X_{L(1)}, X_{L(2)}$, $\ldots, X_{L(n)}$ be the first n lower record values obtained from this sample. Let us denote the marginal moment generating functions of $X_{L_{(n):k}}$ by $M_{L_{(n):k}}(t)$ and its $j-$th derivative by $M^{(j)}_{L_{(n):k}}(t)$. Similarly, let $M_{L_{(m,n):k}}(t_1,t_2)$ and $M^{(i,j)}_{L_{(n):k}}(t_1,t_2)$ denote the joint moment generating functions of $X_{L_{(m):k}}$ and $X_{L_{(n):k}}$ and it $(i,j)-$th partial derivatives with respect to t_1 and t_2, respectively.

Ahsanullah and Raqab [12], Raqab and Ahsanullah [13, 14] and Kumar [6] have established recurrence relations for moment generating functions of record values from Pareto and Gumble, power function, extreme value and generalized logistic distributions respectively.

Recurrence relations for marginal and joint moment generating functions of generalized order statistics from power function distribution are derived by Saran and Pandey [7]. Recurrence relations for single and product moments of record values from exponential and generalized extreme value distribution are derived by Balakrishnan and Ahsanullah [17] and Balakrishnan *et al*. [18]. Pawlas and Szynal [20, 21] and Saran and Singh [8] have established recurrence relations for single and product moments of k-th record values from Weibull, Gumbel and linear exponential distribution. Kumar [5] have established explicit expression and recurrence relations for single and product moments of k-th lower record values from exponentiated Log-logistic distribution. Kamps [24] investigated the importance of recurrence relations of order statistics in characterization.

In the present study, we established some explicit expressions and recurrence relations for marginal and joint moment generating functions of k-th lower record values from extended type I generalized logistic distribution. A characterization of this distribution has also been obtained on using a recurrence relation for marginal moment generating function. A random variable X is said to have extended type I generalized logistic distribution if its *pdf* is of the form

$$f(x) = \frac{\alpha\lambda^{\alpha} e^{-x}}{(\lambda + e^{-x})^{\alpha+1}}, -\infty < x < \infty, \alpha > 0 \tag{1.3}$$

and the corresponding (*cdf*) is

$$F(x) = \left(\frac{\lambda}{\lambda + e^{-x}}\right), -\infty < x < \infty, \alpha > 0. \tag{1.4}$$

When $\alpha = \lambda = 1$, we have the ordinary logistic distribution and when $\lambda = 1$, we have the type I generalized logistic distribution. For more details on this distribution and its applications one may refer to Olapade [1, 2].

2. Relations for marginal moment generating functions

In this section, we have derived the explicit expressions and recurrence relations for marginal moment generating functions of the $k-$th lower record values from the extended type I generalized logistic distribution.
Note that for extended type I generalized logistic defined in (1.3)

$$\alpha\bar{F}(x) = (1+\lambda e^{-x})f(x). \tag{2.1}$$

The relation in (2.1), will be exploited in this paper to derive recurrence relations for the moment generating functions of $k-$th lower record values from the extended type I generalized logistic distribution.

We shall first established the explicit expressions for marginal moment generating functions of $k-$th lower record values $M^{(j)}_{L_{(n):k}}(t)$. Using (1.1)

$$M_{X_{L(n):k}}(t) = \frac{k^n}{(n-1)!}\int_{-\infty}^{\infty} e^{tx}[F(x)]^{k-1}[-ln(F(x))]^{n-1}f(x)dx. \tag{2.2}$$

By substitution $z = [\bar{F}(x)]^{1/\alpha}$, (2.2), we get

$$M_{X_{L(n):k}}(t) = \frac{(\alpha k)^n}{\lambda^t(n-1)!}\int_0^1 (1-z)^{-t}z^{\alpha k+t-1}[-lnz]^{n-1}dz.$$

On using the Maclaurine series expansion $(1-t)^{-t} = \sum_{p=0}^{\infty}\frac{(t)_{(p)}z^p}{p!}$, where

$$t_(p) = \begin{cases} t(t+1)...(t+p-1), & p=1,2... \\ 1, & p=0 \end{cases}$$

and integrating the resulting expression, we obtain

$$M_{X_{L(n):k}}(t) = \frac{(\alpha k)^n}{\lambda^t}\sum_{p=0}^{\infty}\frac{(t)_{(p)}}{p!(\alpha k+p+t)^n},\ t \neq 0. \tag{2.3}$$

Remark 2.1 Setting $k = 1$ in (2.3) we deduce the explicit expression of marginal moment generating functions of lower record values from the extended type I generalized logistic distribution.

Recurrence relations for marginal moment generating functions of $k-$th lower record values from *cdf* can be derived in the following theorem.

Theorem 2.1. *For a positive integer $k \geq 1$ and for $n \geq 1$,*

$$\left(1+\frac{t}{\alpha k}\right) M^{(j)}_{X_{L(n):k}}(t) = M^{(j)}_{X_{L(n-1):k}}(t) - \frac{j}{\alpha k} M^{(j-1)}_{X_{L(n):k}}(t) - \frac{\lambda}{\alpha k}\left[t M^{(j)}_{X_{L(n):k}}(t+1) + j M^{(j-1)}_{X_{L(n-1):k}}(t+1)\right]. \tag{2.4}$$

Proof. From (1.1), we have

$$M_{X_{L(n):k}}(t) = \frac{k^n}{(n-1)!}\int_{-\infty}^{\infty} e^{tx}[F(x)]^{k-1}[-ln(F(x))]^{n-1} f(x)dx. \tag{2.5}$$

Integrating by parts taking $[F(x)]^{k-1}f(x)$ as the part to be integrated and the rest of the integrand for differentiation, we get

$$M_{X_{L(n):k}}(t) = M_{X_{L(n-1):k}}(t) - \frac{tk^n}{k(n-1)!}\int_{-\infty}^{\infty} e^{tx}[F(x)]^{k}[-ln(F(x))]^{n-1}dx$$

the constant of integration vanishes since the integral considered in (2.5) is a definite integral. On using (2.1), we obtain

$$M_{X_{L(n):k}}(t) = M_{X_{L(n-1):k}}(t) - \frac{tk^n}{\alpha k(n-1)!}\Big\{\int_{-\infty}^{\infty} e^{tx}[F(x)]^{k-1}[-ln(F(x))]^{n-1} f(x)dx + \lambda\int_{-\infty}^{\infty} e^{(t+1)x}[F(x)]^{k-1}[-ln(F(x))]^{n-1} f(x)dx\Big\}$$

$$M_{X_{L(n):k}}(t) = M_{X_{L(n-1):k}}(t) - \frac{t}{\alpha k}M_{X_{L(n):k}}(t) - \frac{\lambda t}{\alpha k}M_{X_{L(n):k}}(t+1). \tag{2.6}$$

Differentiating both the sides of (2.6) j times with respect to t, we get

$$M^{(j)}_{X_{L(n):k}}(t) = M^{(j)}_{X_{L(n-1):k}}(t) - \frac{t}{\alpha k}M^{(j)}_{X_{L(n):k}}(t) - \frac{j}{\alpha k}M^{(j-1)}_{X_{L(n):k}}(t) - \frac{\lambda t}{\alpha k}M^{(j)}_{X_{L(n):k}}(t+1) - \frac{j\lambda}{\alpha k}M^{(j-1)}_{X_{L(n):k}}(t+1).$$

The recurrence relation in equation (2.4) is derived simply by rewriting the above equation.

By differentiating both sides of (2.4) with respect to t and then setting $t = 0$, we obtain the recurrence relations for single moment of $k-$th lower record values from extended type I generalized logistic distribution in the form

$$E(X_{L(n):k}^{(j)}) = E(X_{L(n-1):k}^{(j)}) - \frac{j}{\alpha k}\{E(X_{L(n):k}^{(j-1)}) + \lambda E(\phi(X_{L(n):k}))\}. \tag{2.7}$$

where

$$\phi(x) = x^{j-1}e^{x}.$$

Remark 2.2 Setting $k = 1$ in (2.4) and (2.7), we deduce the recurrence relation for marginal moment generating functions and single moments of lower record values from the extended type I generalized logistic distribution.

3. Relations for joint moment generating functions

In this section, we have derived the explicit expressions and recurrence relations for joint moment generating functions of the $k-$th lower record values from the extended type I generalized logistic distribution. We shall first establish the explicit expression for the joint moment generating functions of $k-$th lower record values. On using (1.2), the explicit expression for the joint moment generating of k-th lower record values $M_{X_{L(m,n):k}}(t_1, t_2)$ can be obtained

$$M_{X_{L(m,n):k}}(t_1, t_2) = \frac{k^n}{(m-1)!(n-m-1)!}\int_{-\infty}^{\infty} e^{t_1 x}[-ln(F(x))]^{m-1}\frac{f(x)}{[F(x)]}G(x)dx, \tag{3.1}$$

where

$$G(x) = \int_{-\infty}^{x} e^{t_2 y}\ [ln(F(x)) - ln(F(y))]^{n-m-1}[F(x)]^{k-1}f(y)dy. \tag{3.2}$$

By setting $w = ln(F(x)) - ln(F(y))$ in (3.2), we obtain

$$G(x) = \frac{\alpha^{n-m}}{\lambda^{t_2+p}} \sum_{p=0}^{\infty} \frac{(t_2)_{(p)}[F(x)]^{k+(t_2+p)/\alpha}\ \Gamma(n-m)}{p![\alpha k + p + t_2]^{n-m}}.$$

On substituting the above expression of $G(x)$ in (3.1) and simplifying the resulting equation, we obtain

$$M_{X_{L(m,n):k}}(t_1, t_2) = \frac{(\alpha k)^n}{\lambda^{t_1+t_2+p+q}} \sum_{p=0}^{\infty}\sum_{q=0}^{\infty} \frac{(t_2)_{(p)}(t_1)_{(q)}}{p!q![\alpha k + p + t_2]^{n-m}} \times \frac{1}{[\alpha k + p + q + t_1 + t_2]^m}. \tag{3.3}$$

Remark 3.1 Setting $k = 1$ in (3.3) we deduce the explicit expression for joint moment generating functions of $k-$th lower record values for the extended type I generalized logistic distribution.

Making use of (1.1), we can derive recurrence relations for joint moment generating functions of $k-$th lower record values.

Theorem 3.1. *For* $1 \leq m \leq n-2$ *and* $m, n = 0, 1, 2, \ldots,$

$$\left(1 + \frac{t_2}{\alpha k}\right) M^{(i,j)}_{X_{L(m,n):k}}(t_1, t_2) = M^{(i,j)}_{X_{L(m,n-1):k}}(t_1, t_2) - \frac{j}{\alpha k} M^{(i,j-1)}_{X_{L(m,n):k}}(t_1, t_2) - \frac{\lambda}{\alpha k}\left[t_2 M^{(i,j)}_{X_{L(m,n):k}}(t_1, t_2+1) + j M^{(i,j-1)}_{X_{L(m,n):k}}(t_1, t_2+1)\right]. \tag{3.4}$$

Proof. From (1.2), for $1 \leq m \leq n-1$, $m, n = 0, 1, 2, \ldots$

$$M_{X_{L(m,n):k}}(t_1, t_2) = \frac{k^n}{(m-1)!(n-m-1)!} \int_{-\infty}^{\infty} [-ln(F(x))]^{m-1} \frac{f(x)}{[F(x)]} I(x) dx, \tag{3.5}$$

where

$$I(x) = \int_{-\infty}^{x} e^{t_1 x + t_2 y}\ [ln(F(x)) - ln(F(y))]^{n-m-1} [F(x)]^{k-1} f(y) dy.$$

Integrating $I(x)$ by parts treating $[F(x)]^{k-1}f(y)$ for integration and the rest of the integrand for differentiation, and substituting the resulting expression in (3.5), we get

$$M_{X_{L(m,n):k}}(t_1,t_2) = M_{X_{L(m,n-1):k}}(t_1,t_2) - \frac{t_2 k^n}{k(m-1)!(n-m-1)!}\int_{-\infty}^{\infty}\int_{-\infty}^{x} e^{t_1x+t_2y}$$

$$\times[-ln(F(x))]^{m-1}[ln(F(x))-ln(F(y))]^{n-m-1}[F(y)]^k\frac{f(x)}{[F(x)]}dydx,$$

the constant of integration vanishes since the integral in $I(x)$ is a definite integral. On using the relation (2.1), we obtain

$$M_{X_{L(m,n):k}}(t_1,t_2) = M_{X_{L(m,n-1):k}}(t_1,t_2) - \frac{t_2 k^n}{\alpha k(m-1)!(n-m-1)!}$$

$$\times\Big\{\int_{-\infty}^{\infty}\int_{-\infty}^{x} e^{t_1x+t_2y}[-ln(F(x))]^{m-1}f(x)$$

$$\times[ln(F(x))-ln(F(y))]^{n-m-1}[F(y)]^{k-1}\frac{f(y)}{[F(x)]}dydx$$

$$+\lambda\int_{-\infty}^{\infty}\int_{-\infty}^{x} e^{t_1x+(t_2-1)y}[-ln(F(x))]^{m-1}f(x)$$

$$\times[ln(F(x))-ln(F(y))]^{n-m-1}[F(y)]^{k-1}\frac{f(y)}{[F(x)]}dydx\Big\}$$

$$M_{X_{L(m,n):k}}(t_1,t_2) = M_{X_{L(m,n-1):k}}(t_1,t_2) - \frac{t_2}{\alpha k}M_{X_{L(m,n):k}}(t_1,t_2) - \frac{\lambda t_2}{\alpha k}M_{X_{L(m,n):k}}(t_1,t_2-1). \tag{3.6}$$

Differentiating both the sides of (3.6), i times with respect to t_1 and then j times with respect to t_2 , we get

$$M^{(i,j)}_{X_{L(m,n):k}}(t_1,t_2) = M^{(i,j)}_{X_{L(m,n-1):k}}(t_1,t_2) - \frac{1}{\alpha k}\left[t_2 M^{(i,j)}_{X_{L(m,n):k}}(t_1,t_2) + jM^{(i,j-1)}_{X_{L(m,n):k}}(t_1,t_2)\right]$$

$$-\frac{\lambda}{\alpha k}\left[t_2 M^{(i,j)}_{X_{L(m,n):k}}(t_1,t_2+1) + jM^{(i,j-1)}_{X_{L(m,n):k}}(t_1,t_2+1)\right],$$

which, when rewritten gives the recurrence relation in (3.4).

By differentiating both sides of equation (3.6) with respect to t_1, t_2 and then setting

$t_1 = t_2 = 0$, we obtain the recurrence relations for product moments of $k-$th lower record values from extended type I generalized logistic distribution in the form

$$E(X^{(i,j)}_{L(m,n):k}) = E(X^{(i,j)}_{L(m,n-1):k}) - \frac{j}{\alpha k}\{E(X^{(i,j-1)}_{L(m,n):k}) + \lambda E(\phi(X_{L(m,n):k}))\}, \tag{3.7}$$

where

$$\phi(x) = x^i y^{j-1} e^y.$$

Remark 3.2 Setting $k = 1$ in (3.4) and (3.7), we deduce the recurrence relation for joint moment generating functions and product moments of loer record value for the extended type I generalized logistic distribution.

4. Characterization

This Section contains characterizations of extended type I generalized logistic distribution by using recurrence relations for marginal moment generating functions of k-th lower record values.

Theorem 4.1. *Let X be a non-negative random variable having an absolutely continuous cumulative distribution function $F(x)$ with $F(0) = 0$ and $0 < F(x) < 1$ for all $x > 0$,*

$$M_{X_{L(n):k}}(t) = M_{X_{L(n-1):k}}(t) - \frac{t}{\alpha k} M_{X_{L(n):k}}(t) - \frac{\lambda t}{\alpha k} M_{X_{L(n):k}}(t+1). \tag{4.1}$$

Proof. The necessity part follows immediately from equation (2.4). On the other hand if the recurrence relation in equation (4.1) is satisfied, then on using equation (1.1), we have

$$\frac{k^n}{(n-1)!}\int_{-\infty}^{\infty} e^{tx}[-ln(F(x))]^{n-1}[F(x)]^{k-1} f(x)dx \tag{4.2}$$

$$= \frac{(n-1)k^n}{k(n-1)!}\int_{-\infty}^{\infty} e^{tx}[-ln(F(x))]^{n-2}[F(x)]^{k-1}f(x)dx$$

$$-\frac{tk^n}{\alpha k(n-1)!}\int_{-\infty}^{\infty} e^{tx}[-ln(F(x))]^{n-1}[F(x)]^{k-1}f(x)dx$$

$$-\frac{\lambda tk^n}{\alpha k(n-1)!}\int_{-\infty}^{\infty} e^{(t+1)x}[-ln(F(x))]^{n-1}[F(x)]^{k-1}f(x)dx$$

Integrating the first integral on the right hand side of equation (4.2), by parts we get

$$\frac{k^n}{(n-1)!}\int_{-\infty}^{\infty} e^{tx}[-ln(F(x))]^{n-1}[F(x)]^{k-1}f(x)dx$$

$$= \frac{tk^n}{k(n-1)!}\int_{-\infty}^{\infty} e^{tx}[-ln(F(x))]^{n-1}[F(x)]^{k}dx$$

$$+\frac{k^n}{(n-1)!}\int_{-\infty}^{\infty} e^{tx}[-ln(F(x))]^{n-1}[F(x)]^{k-1}f(x)dx$$

$$-\frac{tk^n}{\alpha k(n-1)!}\int_{-\infty}^{\infty} e^{tx}[-ln(F(x))]^{n-1}[F(x)]^{k-1}f(x)dx$$

$$-\frac{\lambda tk^n}{\alpha k(n-1)!}\int_{-\infty}^{\infty} e^{(t+1)x}[-ln(F(x))]^{n-1}[F(x)]^{k-1}f(x)dx$$

which reduces to

$$(4.3) \qquad \frac{tk^n}{k(n-1)!}\int_{-\infty}^{\infty} e^{tx}[-ln(F(x))]^{n-1}[F(x)]^{k-1}$$

$$\times\left\{F(x) - \frac{f(x)}{\alpha} - \frac{\lambda e^x}{\alpha}f(x)\right\}dx = 0.$$

Now applying a generalization of the Müntz-Szász Theorem (Hwang and Lin, [9] to equation (4.3), we get

$$\frac{f(x)}{F(x)} = \frac{\alpha}{1+\lambda e^x}$$

which proves that

$$F(x) = \left(\frac{\lambda}{\lambda + e^{-x}}\right), \ -\infty < x < \infty, \ \alpha, \ \lambda > 0.$$

5. Conclusions

In this study we give exact expressions and some recurrence relations for marginal and joint moment generating functions of k-th lower record values from extended type I generalized logistic distribution. Further, characterization of this distribution has also been obtained on using a recurrence relation for marginal moment generating functions of k-th lower record values.

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