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# ON RELATIONS FOR THE MOMENT GENERATING FUNCTIONS FROM THE EXTENDED TYPE II GENERALIZED LOGISTIC DISTRIBUTION BASED ON K-TH UPPER RECORD VALUES AND A CHARACTERIZATION

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Abstract. In this study we, give some new explicit expressions and recurrence relations satisfied by marginal and joint moment generating functions of $k-$th upper record values from extended type II generalized logistic distribution. Next we show that results for upper record values of this distribution can be derived from our result as special cases. Further, a characterizing result of this distribution on using the conditional expectation of upper record values is discussed.

## 1. Introduction

The logistic distribution plays an importance as a growth curve have made it one of the many important statistical distributions. The shape of the logistic distribution that is similar to that of the normal distribution makes it simpler and also profitable on suitable occasions to replace the normal by the logistic distribution with negligible errors in the respective theories.

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A random variable $X$ is said to have extended type II generalized logistic distribution if its probability density function ($pdf$) is of the form

$$f(x) = \frac{\alpha e^{-\alpha x}}{(1+e^{-x})^{\alpha+1}}, -\infty < x < \infty, \alpha > 0 \tag{1.1}$$

and the corresponding cumulative distribution function ($cdf$) is

$$F(x) = 1 - \left(\frac{e^{-x}}{1+e^{-x}}\right)^{\alpha}, -\infty < x < \infty, \alpha > 0. \tag{1.2}$$

For more details on this distribution and its applications one may refers to Balakrishnan and Leung [16].

Record values are found in many situations of daily life as well as in many statistical applications. Often we are interested in observing new records and in recording them: for example, Olympic records or world records in sport. Record values are used in reliability theory. Moreover, these statistics are closely connected with the occurrences times of some corresponding non homogeneous Poisson process used in shock models. The statistical study of record values started with Chandler [9], he formulated the theory of record values as a model for successive extremes in a sequence of independently and identically random variables. Feller [25] gave some examples of record values with respect to gambling problems. Resnick [20] discussed the asymptotic theory of records. Theory of record values and its distributional properties have been extensively studied in the literature, for example, see, Ahsanullah [11], Arnold *et al*. [2], [3], Nevzorov [23] and Kamps [21] for reviews on various developments in the area of records.

We shall now consider the situations in which the record values (e.g. successive largest insurance claims in non-life insurance, highest water-levels or highest temperatures) themselves are viewed as "outliers" and hence the second or third largest values are of special interest. Insurance claims in some non life insurance can be used as one of the examples. Observing successive $k-$th largest values in a sequence, Dziubdziela and Kopocinski [24] proposed the following model of $k-$th record values, where $k$ is some positive integer.

Let $X_1, X_2, \ldots$ be a sequence of identically independently distributed (*i.i.d.*) random variables with *pdf* $f(x)$ and *cdf* $F(x)$. Let $Xj : n$ denote the $j-$th order statistic of a sample $(X_1, X_2, \ldots, X_n)$. For a fixed $k \geq 1$ we define the sequence $U_1^{(k)}, U_2^{(k)}, \ldots$ of $k-$th upper record times of $X_1, X_2, \ldots$ as follows:

$$U_1^{(k)} = 1$$

$$U_{n+1}^{(k)} = min\{j > U_n^{(k)} : X_{j:j+k-1} > X_{U_n^{(k)}:U_n^{(k)}+k-1}\}, n = 1, 2, \ldots.$$

The sequence $\{Y_n^{(k)}, n \geq 1\}$, where $Y_n^{(k)} = X_{U_n^{(k)}}$ is called the sequences of $k-$th upper record values of the sequence $\{X_n, n \geq 1\}$. For convenience, we define $Y_0^{(k)} = 0$. Note for $k = 1$ we have $Y_n^{(1)} = X_{U_n}$, $n \geq 1$, which are record values of $\{X_n, n \geq 1\}$ (Ahsanullah [11]) .

Let $\{Y_n^{(k)}, n \geq 1\}$ be the sequence of $k-$th upper record values from (1.1). Then the *pdf* of $Y_n^k, n \geq 1$, is given by Dziubdziela and Kopocinski [24] is as follows:

$$f_{Y_n^{(k)}}(x) = \frac{k^n}{(n-1)!}[-ln(\bar{F}(x))]^{n-1}[\bar{F}(x)]^{k-1}f(x). \tag{1.3}$$

Also the joint density function of $Y_m^{(k)}$ and $Y_n^{(k)}$, $(1 \leq m \leq n)$, $n = 2, 3, \ldots$ as discussed by Grudzien [26] is given by

$$f_{Y_m^{(k)},Y_n^{(k)}}(x, y) = \frac{k^n}{(m-1)!(n-m-1)!}[-ln(\bar{F}(x))]^{m-1} \tag{1.4}$$

$$\times[-ln\bar{F}(y) + ln\bar{F}(x)]^{n-m-1}[\bar{F}(y)]^{k-1}\frac{f(x)}{\bar{F}(x)}f(y),\ x < y,$$

where $\bar{F}(x) = 1 - F(x)$.

Saran and Pandey [7], Kumar [5] have established recurrence relations for moment generating functions of $k-$th record value from extreme value and generalized logistic

distribution. Recurrence relations for moment generating functions of record value from Pareto, Gumble, power function and extreme value distributions are derived by Ahsanullah and Raqab [12], Raqab and Ahsanullah [13], [14] respectively. Recurrence relations for single and product moments of record values from Weibull, Pareto, generalized Pareto, Burr, exponential and Gumble distribution are derived by Pawalas and Szynal [17], [18], and [19]. Sultan [10], Saran and Singh [8] are established recurrence relations for moments of $k-$th record values from modified Weibull and linear exponential distribution respectively. Kumar [4], Kumar and Khan [6] have established recurrence relations for moments of $k-$th record values from exponentiated log-logistic and generalized beta II distributions respectively. Kamps [22] investigated the importance of recurrence relations of order statistics in characterization.
In this paper, we established some new explicit expressions and recurrence relations satisfied by the marginal and joint moment generating functions of $k-$th upper record values from extended type II generalized logistic distribution and also show that results for upper record values from this distribution can be derived from our result as special cases. Finally a characterization of this distribution is obtained by using the conditional expectation of upper record values.

## 2. Relations for marginal moment generating functions

In this section, we have derived the explicit expressions and recurrence relations for marginal moment generating functions of the $k-$th upper record values from the extended type II generalized logistic distribution.
Note that for type II generalized logistic distribution defined in (1.1)

$$\alpha \bar{F}(x) = (1 + e^{-x}) f(x). \tag{2.1}$$

The relation in (2.1), will be exploited in this paper to derive recurrence relations for the moment generating functions of $k-$th upper record values from the extended type II generalized logistic distribution.

Let us denote the marginal moment generating functions of $X_{U_{(n):k}}$ by $M_{U_{(n):k}}(t)$ and its $j-$th derivative by $M^{(j)}_{U_{(n):k}}(t)$. Similarly, let $M_{U_{(m,n):k}}(t_1,t_2)$ and $M^{(i,j)}_{U_{(n):k}}(t_1,t_2)$ denote the joint moment generating functions of $X_{U_{(m):k}}$ and $X_{U_{(n):k}}$ and it $(i,j)-$th partial derivatives with respect to $t_1$ and $t_2$, respectively.

We shall first established the explicit expressions for marginal moment generating functions of $k-$th upper record values $M^{(j)}_{U_{(n):k}}(t)$ by the following Theorem:

**Theorem 2.1.** *For extended type II generalized logistic distribution as given in (1.1) and $k \geq 1$, $n \geq 1$, $r = 1, 2, \dots$*

$$M_{X_{U(n):k}}(t) = (\alpha k)^n(-1)^{n-1}\frac{\partial^{n-1}}{\partial(\alpha k - t)^{n-1}}B(\alpha k - t, t+1). \tag{2.2}$$

*Proof.* From (1.3), We have

$$M_{X_{U(n):k}}(t) = \frac{k^n}{(n-1)!}\int_{-\infty}^{\infty} e^{tx}[\bar{F}(x)]^{k-1}[-ln(\bar{F}(x))]^{n-1}f(x)dx. \tag{2.3}$$

By substitution $z = [\bar{F}(x)]^{1/\alpha}$, (2.3), we get

$$M_{X_{U(n):k}}(t) = \frac{(\alpha k)^n}{(n-1)!}\int_0^1 (1-z)^t z^{\alpha k - t - 1}[-lnz]^{n-1}dz$$

$$= \frac{(\alpha k)^n(-1)^{n-1}}{(n-1)!}\int_0^1 (1-z)^t z^{\alpha k - t - 1}[lnz]^{n-1}dz. \tag{2.4}$$

Using the following results

$$a)\int_0^p (p^\delta - x^\delta)^{\beta-1}x^{\alpha-1}[lnz]^n dx = \frac{p^{\delta(\beta-1)}}{\delta}\frac{\partial^n}{\partial\alpha^n}\left[p^\alpha B\left(\beta,\frac{\alpha}{\delta}\right)\right], p,\delta,\alpha,\beta > 0$$

$$b)\frac{\partial^r B(a,b)}{\partial b^r} = \sum_{k=0}^{r-1}\begin{pmatrix} r-1 \\ k \end{pmatrix}\left[\psi^{(r-k+1)}(b) - \psi^{(r-k+1)}(a+b)\right]\frac{\partial^k B(a,b)}{\partial b^k},$$

where $B(a,b)$, $a,b>0$ is the beta function $\psi^k(x)$ is the $k-$th derivative of $\psi(x) = \frac{dln\Gamma(x)}{dx} = \frac{\Gamma\prime(x)}{\Gamma(x)}$, $x \neq 0,-1,-2,\ldots$ which is a digamma function. The result given in (2.2) is proved, in view of result (a) and (2.4). □

**Remark 2.1** Setting $k=1$ in (2.2) we deduce the explicit expression of marginal moment generating functions of upper record values from the extended type II generalized logistic distribution.

Recurrence relations for marginal moment generating functions of $k-$th upper record values from *cdf* can be derived in the following theorem.

**Theorem 2.2.** *For a positive integer $k \geq 1$ and for $n \geq 1$, $r = 0,1,2,\ldots$*

$$\left(1-\frac{t}{\alpha k}\right) M^{(j)}_{X_{U(n):k}}(t) = M^{(j)}_{X_{U(n-1):k}}(t) + \frac{j}{\alpha k} M^{(j-1)}_{X_{U(n):k}}(t) \tag{2.5}$$

$$+\frac{1}{\alpha k}[tM^{(j)}_{X_{U(n):k}}(t-1) + jM^{(j-1)}_{X_{U(n-1):k}}(t-1)].$$

*Proof.* From (1.3), we have

$$M_{X_{U(n):k}}(t) = \frac{k^n}{(n-1)!}\int_{-\infty}^{\infty} e^{tx}[\bar{F}(x)]^{k-1}[-ln(\bar{F}(x))]^{n-1} f(x)dx. \tag{2.6}$$

Integrating by parts taking $[\bar{F}(x)]^{k-1}f(x)$ as the part to be integrated and the rest of the integrand for differentiation, we get

$$M_{X_{U(n):k}}(t) = M_{X_{U(n-1):k}}(t) + \frac{tk^n}{k(n-1)!}\int_{-\infty}^{\infty} e^{tx}[\bar{F}(x)]^{k}[-ln(\bar{F}(x))]^{n-1}dx$$

the constant of integration vanishes since the integral considered in (2.6) is a definite integral. On using (2.1), we obtain

$$M_{X_{U(n):k}}(t) = M_{X_{U(n-1):k}}(t) + \frac{tk^n}{\alpha k(n-1)!}\Big(\int_{-\infty}^{\infty} e^{tx}[\bar{F}(x)]^{k-1}[-ln(\bar{F}(x))]^{n-1}f(x)dx$$

$$+\int_{-\infty}^{\infty} e^{(t-1)x}[\bar{F}(x)]^{k-1}[-ln(\bar{F}(x))]^{n-1}f(x)dx\Big)$$

$$M_{X_{U(n):k}}(t) = M_{X_{U(n-1):k}}(t) + \frac{t}{\alpha k} M_{X_{U(n):k}}(t) + \frac{t}{\alpha k} M_{X_{U(n):k}}(t-1). \tag{2.7}$$

Differentiating both the sides of (2.7) $j$ times with respect to $t$, we get

$$M^{(j)}_{X_{U(n):k}}(t) = M^{(j)}_{X_{U(n-1):k}}(t) + \frac{t}{\alpha k} M^{(j)}_{X_{U(n):k}}(t) + \frac{j}{\alpha k} M^{(j-1)}_{X_{U(n):k}}(t)$$

$$+ \frac{t}{\alpha k} M^{(j)}_{X_{U(n):k}}(t-1) - \frac{j}{\alpha k} M^{(j-1)}_{X_{U(n):k}}(t-1).$$

The recurrence relation in equation (2.5) is derived simply by rewriting the above equation.

By differentiating both sides of (2.5) with respect to $t$ and then setting $t = 0$, we obtain the recurrence relations for single moment of $k-$th upper record values from extended type II generalized logistic distribution in the form

$$E(X^{(j)}_{U(n):k}) = E(X^{(j)}_{U(n-1):k}) + \frac{j}{\alpha k}\{E(X^{(j-1)}_{U(n):k}) + E(\phi(X_{U(n):k}))\}. \tag{2.8}$$

where

$$\phi(x) = x^{j-1} e^{-x}.$$

□

**Remark 2.2** Setting $k = 1$ in (2.5) and (2.8), we deduce the recurrence relation for marginal moment generating functions and single moments of upper record values from the extended type II generalized logistic distribution.

## 3. Relations for joint moment generating functions

In this section, we have derived the explicit expressions and recurrence relations for joint moment generating functions of the $k-$th upper record values from the extended type II generalized logistic distribution. We shall first establish the explicit expression for the joint moment generating functions of $k-$th upper record values by the following Theorem:

**Theorem 3.1.** *For extended type II generalized logistic distribution as given in (1.1) and* $1 \le m \le n-2,\ r,s = 1,2,\ldots$

(3.1)
$$M_{X_{U(m,n):k}}(t_1,t_2) = \frac{(\alpha k)^n}{(m-1)!(n-m-1)!}\sum_{p=0}^{\infty}\sum_{q=0}^{\infty}\sum_{u=0}^{n-m-1}(-1)^{m-1}\begin{pmatrix} n-m-1 \\ u \end{pmatrix}$$
$$\times\frac{a_p(u)(1-\phi)_q}{[\alpha k+q-t_2]}\frac{\partial^{n-2-u}}{\partial v^{n-2-u}}B(\alpha k+q-t_2-t_1,t_1+1).$$

*Proof.* From (1.4), We have

(3.2)
$$M_{X_{U(m,n):k}}(t_1,t_2) = \frac{k^n}{(m-1)!(n-m-1)!}\int_{-\infty}^{\infty} e^{t_1 x}[-ln(\bar{F}(x))]^{m-1}\frac{f(x)}{[\bar{F}(x)]}G(x)dx,$$

where

$$G(x) = \int_x^{\infty} e^{t_2 y}[ln(\bar{F}(x)) - ln(\bar{F}(y))]^{n-m-1}[\bar{F}(x)]^{k-1}f(y)dy$$
$$= \sum_{u=0}^{n-m-1}\begin{pmatrix} n-m-1 \\ u \end{pmatrix}[ln(\bar{F}(x))]^{n-m-1-u}$$
$$\times\int_x^{\infty} e^{t_2 y}[-ln(\bar{F}(y))]^u[\bar{F}(x)]^{k-1}f(y)dy.$$

By setting $z = [\bar{F}(y)]^{1/\alpha}$ we get

$$G(x) = \sum_{u=0}^{n-m-1}\begin{pmatrix} n-m-1 \\ u \end{pmatrix}\alpha^{u+1}[ln(\bar{F}(x))]^{n-m-1-u}$$
$$\times\int_0^{[\bar{F}(y)]^{1/\alpha}}(1-z)^{t_2}[-lnz]^u z^{\alpha k-t_2-1}dz.$$

On using the logarithmic expansion

$$[-ln(1-t)]^i = \left(\sum_{p=1}^{\infty}\frac{t^p}{p}\right)^i = \sum_{p=0}^{\infty}\alpha_p(i)t^{i+p},\ |t|<1,$$

where $\alpha_p(i)$ is the coefficient of $t^{i+p}$ in the expansion of $\left(\sum_{p=1}^{\infty}\frac{t^p}{p}\right)^i$ (Balakrishnan and Cohen [15], Shawky and Bakoban [1]), integrating the resulting expression we get

$$G(x)=\sum_{p=0}^{\infty}\sum_{q=0}^{\infty}\sum_{u=0}^{n-m-1}\begin{pmatrix} n-m-1 \\ u \end{pmatrix}\frac{\alpha^u a_p(u)(1-\phi)_{(q)}}{q![k+(q-t_2)/\alpha]}$$

$$\times[\bar{F}(x)]^{k+(q-t_2)/\alpha}[ln(\bar{F}(x))]^{n-m-1-u}.$$

On substituting the above expression of $G(x)$ in (3.1), we find that

$$M_{X_{U(m,n):k}}(t_1,t_2)=\sum_{p=0}^{\infty}\sum_{q=0}^{\infty}\sum_{u=0}^{n-m-1}\begin{pmatrix} n-m-1 \\ u \end{pmatrix}\frac{\alpha^u a_p(u)(1-\phi)_{(q)}}{q![k+(q-t_2)/\alpha]} \tag{3.3}$$

$$\times\int_{-\infty}^{\infty}e^{t_1x}[-ln(\bar{F}(y))]^{n-2-u}[\bar{F}(x)]^{k+((q-t_2)/\alpha)-1}f(x)dx.$$

Again by setting $w=[\bar{F}(x)]^{1/\alpha}$ in (3.3) and simplifying the resulting expression, we get the result given in (3.1). □

**Remark 3.1** Setting $k=1$ in (3.1) we deduce the explicit expression for joint moment generating functions of $k-$th upper record value for the extended type II generalized logistic distribution.

Making use of (1.2), we can derive recurrence relations for joint moment generating functions of $k-$th upper record values.

**Theorem 3.2.** *For* $1\leq m\leq n-2$ *and* $m,n=0,1,2,\ldots,$

$$\left(1-\frac{t_2}{\alpha k}\right)M^{(i,j)}_{X_{U(m,n):k}}(t_1,t_2)=M^{(i,j)}_{X_{U(m,n-1):k}}(t_1,t_2)+\frac{j}{\alpha k}M^{(i,j-1)}_{X_{U(m,n):k}}(t_1,t_2) \tag{3.4}$$

$$+\frac{1}{\alpha k}[t_2M^{(i,j)}_{X_{U(m,n):k}}(t_1,t_2-1)+jM^{(i,j-1)}_{X_{U(m,n):k}}(t_1,t_2-1)].$$

*Proof.* From (1.4), we have

$$M_{X_{U(m,n):k}}(t_1,t_2)=\frac{k^n}{(m-1)!(n-m-1)!}\int_{-\infty}^{\infty}[-ln(\bar F(x))]^{m-1}\frac{f(x)}{[\bar F(x)]}I(x)dx, \tag{3.5}$$

where

$$I(x)=\int_x^{\infty}e^{t_1x+t_2y}[ln(\bar F(x))-ln(\bar F(y))]^{n-m-1}[\bar F(x)]^{k-1}f(y)dy.$$

Integrating $I(x)$ by parts treating $[\bar F(x)]^{k-1}f(y)$ for integration and the rest of the integrand for differentiation, and substituting the resulting expression in (3.5), we get

$$M_{X_{U(m,n):k}}(t_1,t_2)=M_{X_{U(m,n-1):k}}(t_1,t_2)+\frac{t_2k^n}{k(m-1)!(n-m-1)!}\int_{-\infty}^{\infty}\int_x^{\infty}e^{t_1x+t_2y}$$

$$\times[-ln(\bar F(x))]^{m-1}[ln(\bar F(x))-ln(\bar F(y))]^{n-m-1}[\bar F(y)]^k\frac{f(x)}{[\bar F(x)]}dydx,$$

the constant of integration vanishes since the integral in $I(x)$ is a definite integral. On using the relation (2.1), we obtain

$$M_{X_{U(m,n):k}}(t_1,t_2)=M_{X_{U(m,n-1):k}}(t_1,t_2)+\frac{t_2k^n}{\alpha k(m-1)!(n-m-1)!}$$

$$\times\Big\{\int_{-\infty}^{\infty}\int_x^{\infty}e^{t_1x+t_2y}[-ln(\bar F(x))]^{m-1}f(x)$$

$$\times[ln(\bar F(x))-ln(\bar F(y))]^{n-m-1}[\bar F(y)]^{k-1}\frac{f(x)}{[\bar F(x)]}dydx$$

$$\times\int_{-\infty}^{\infty}\int_x^{\infty}e^{t_1x+(t_2-1)y}[-ln(\bar F(x))]^{m-1}f(x)$$

$$\times[ln(\bar F(x))-ln(\bar F(y))]^{n-m-1}[\bar F(y)]^{k-1}\frac{f(x)}{[\bar F(x)]}dydx\Big\}$$

$$M_{X_{U(m,n):k}}(t_1,t_2)=M_{X_{U(m,n-1):k}}(t_1,t_2)+\frac{t_2}{\alpha k}M_{X_{U(m,n):k}}(t_1,t_2) \tag{3.6}$$

$$+\frac{t_2}{\alpha k}M_{X_{U(m,n):k}}(t_1,t_2-1).$$

Differentiating both the sides of (3.6), $i$ times with respect to $t_1$ and then $j$ times with respect to $t_2$ , we get

$$M^{(i,j)}_{X_{U(m,n):k}}(t_1,t_2) = M^{(i,j)}_{X_{U(m,n-1):k}}(t_1,t_2) + \frac{1}{\alpha k}[t_2 M^{(i,j)}_{X_{U(m,n):k}}(t_1,t_2) + jM^{(i,j-1)}_{X_{U(m,n):k}}(t_1,t_2)]$$

$$+\frac{1}{\alpha k}[t_2 M^{(i,j)}_{X_{U(m,n):k}}(t_1,t_2-1) + jM^{(i,j-1)}_{X_{U(m,n):k}}(t_1,t_2-1)],$$

which, when rewritten gives the recurrence relation in (3.4).

By differentiating both sides of equation (3.6) with respect to $t_1, t_2$ and then setting $t_1 = t_2 = 0$, we obtain the recurrence relations for product moments of $k-$th upper record values from extended type II generalized logistic distribution in the form

$$E(X^{(i,j)}_{U(m,n):k}) = E(X^{(i,j)}_{U(m,n-1):k}) + \frac{j}{\alpha k}\{E(X^{(i,j-1)}_{U(m,n):k}) + E(\phi(X_{U(m,n):k}))\}, \tag{3.7}$$

where

$$\phi(x) = x^i y^{j-1} e^{-y}.$$

□

**Remark 3.2** Setting $k = 1$ in (3.4) and (3.7), we deduce the recurrence relation for joint moment generating functions and product moments of upper record value for the extended type II generalized logistic distribution.

## 4. Characterization

This Section contains characterizations of extended type II generalized logistic distribution by conditional expectation of upper record values.

Let $\{X_n, n \geq 1\}$ be a sequence of *i.i.d.* continuous random variables with *cdf* $F(x)$ and *pdf* $f(x)$. Let $X_{U(n)}$ be the $n-$th upper record values, then the conditional *pdf* of $X_{U(n)}$ given $X_{U(m)} = x$, $1 \leq m < n$ in view of (1.2) and (1.3) for $k = 1$ is

$$f(Y_{U(n)}|Y_{U(m)} = x) = \frac{1}{(n-m-1)!}[-ln\bar{F}(y) + ln\bar{F}(x)]^{n-m-1}\frac{f(y)}{\bar{F}(x)}, x > y. \tag{4.1}$$

**Theorem 4.1.** *Let $X$ be an absolutely continuous random variable with cdf $F(x)$ and pdf $f(x)$ on the support $(-\infty, \infty)$, then for $m < n$,*

$$E[e^{tX_{U(n)}}|X_{U(m)} = x] = \sum_{p=0}^{t}(-1)^{p+t}\begin{pmatrix} t \\ p \end{pmatrix}(1+e^x)^p\left(\frac{\alpha}{\alpha - p}\right)^{n-m}, \tag{4.2}$$

if and only if

$$F(x) = 1 - \left(\frac{e^{-x}}{(1+e^{-x})}\right)^{\alpha}, -\infty < x < \infty, \alpha > 0.$$

*Proof.* From (4.1), we have

$$E[e^{tX_{U(n)}}|X_{U(m)} = x] = \frac{1}{(n-m-1)!}\int_x^{\infty} e^{ty}\Big[ln\Big(\frac{\bar{F}(x)}{\bar{F}(y)}\Big)\Big]^{n-m-1}\frac{f(y)}{\bar{F}(x)}dy. \tag{4.3}$$

By setting $u = ln\Big(\frac{\bar{F}(x)}{\bar{F}(y)}\Big)$ from (1.2) in (4.3), we obtain

$$E[e^{tX_{U(n)}}|X_{U(m)} = x] = \frac{1}{(n-m-1)!}\sum_{p=0}^{t}(-1)^{p+t}\begin{pmatrix} t \\ p \end{pmatrix}(1+e^x)^p.$$

$$\times \int_0^{\infty} u^{n-m-1}e^{-[1-(p/\alpha)]u}du.$$

Simplifying the above expression, we derive the relation given in (4.2).

To prove sufficient part, we have from (4.1) and (4.2)

$$\frac{1}{(n-m-1)!}\int_x^{\infty} e^{ty}[-ln\bar{F}(y) + ln\bar{F}(x)]^{n-m-1}f(y)dy = \bar{F}(x)H_r(x), \tag{4.4}$$

where

$$H_r(x) = \sum_{p=0}^{t}(-1)^{p+t}\begin{pmatrix} t \\ p \end{pmatrix}(1+e^x)^p\left(\frac{\alpha}{\alpha - p}\right)^{n-m}.$$

Differentiating (4.4) both sides with respect to $x$, we get

$$-\frac{1}{(n-m-2)!}\int_x^{\infty} e^{ty}[-ln\bar{F}(y) + ln\bar{F}(x)]^{n-m-2}\frac{f(x)}{\bar{F}(x)}f(y)dy$$

$$= -f(x)H_r(x) + \bar{F}(x)H_r'(x)$$

or

$$\frac{f(x)}{\bar{F}(x)} = -\frac{H_r'(x)}{[H_{r+1}(x) - H_r(x)]} = \left(\frac{\alpha e^x}{1+e^x}\right)$$

which proves that

$$F(x) = 1 - \left(\frac{e^{-x}}{(1+e^{-x})}\right)^{\alpha}, -\infty < x < \infty, \alpha > 0.$$

□

## 5. Conclusions

In this study, some new explicit expressions and recurrence relations for marginal and joint moment generating functions of $k-$th upper record values from the extended type II generalized logistic distribution have been established. Further, characterization of this distribution has also been obtained on using the conditional expectation of upper record values is discussed.

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