ON D - CONTINUOUS FUNCTIONS AND ITS SOME PROPERTIES

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ABSTRACT. In this paper, we introduce a new class of continuous functions called D-continuous functions by utilizing D-closed sets. We study their properties in topological space. It turns out, among others, the D-continuous is weaker than perfect continuity and stronger than both gp-continuity and πgp-continuity.

1. INTRODUCTION

Continuous functions in topology found a valuable place in the applications of mathematics as it has applications to engineering especially to digital signal processing and neural networks. Topologist studied weaker and stronger forms of continuous functions in topology using the sets stronger and weaker than open and closed sets. Balachandran et.al [7], Levine [19], Mashour et.al [21], Rajesh et.al [27], Ghanambal et.al [15], Park et.al [25], J. K. Kohli et al.[18], E. Ekici [13], I. L. Reilly et al.[29], K. Dass et al.[8], M. Akdağ [1] have introduced g-continuity, semi-continuity, pre-continuity, $\widetilde{g}$ continuity, gpr-continuity and πgp-continuity, D-super continuity, perfectly continuity, $\delta-$continuity, contra D-continuity, lower and upper multi D-continuity respectively. As generalization of closed sets , D-closed sets were introduced and studied by the same author [3]. The aim of this paper is to introduce new

classes of functions called D-continuous functions. Moreover, the relationships and properties of D-continuous functions are obtained.

2. Preliminaries

Throughout this paper (X, τ), (Y, σ), (Z, η) represent non-empty topological spaces on which no separation axioms are assumed unless otherwise mentioned. We recall the following definitions in the sequel.

Definition 2.1. Let (X, τ) be a topological space. A subset A of the space X is said to be

  (1) Pre-open [21] if $A \subseteq Int(cl(A))$ and pre-closed if $cl(Int(A)) \subseteq A$.
  (2) Semi-open [19] if $A \subseteq cl(Int(A))$ and semi-closed if $Int(cl(A)) \subseteq A$.
  (3) Semi-preopen [2] if $A \subseteq cl(int(cl(A)))$ and semi-preclosed if $int(cl(int(A))) \subseteq A$.
  (4) Regular open [30] if $A = Int(cl(A))$ and regular closed if $A = cl(Int(A))$.
  (5) π-open [39] if it is a finite union of regular open sets.

Recall that the intersection of all semi-closed (resp. pre-closed, semi-preclosed) sets containing A is called the semi-closure of A and is denoted by $scl(A)$ (resp. $pcl(A)$, $spcl(A)$).

Definition 2.2. Let (X, τ) be a topological space. A subset $A \subseteq X$ is said to be

  (1) generalized closed (briefly g-closed) [20] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.
  (2) generalized pre-closed (briefly gp-closed) [23] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.
  (3) generalized pre-regular closed (briefly gpr-closed) [14] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X.

(4) pre-generalized closed (briefly pg-closed) [23] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is preopen in X.

(5) g^*-preclosed (briefly g^*p-closed) [37] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open in X.

(6) generalized semi-preclosed (briefly gsp-closed) [10] if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

(7) pre semi-closed [38] $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open in X.

(8) ω-closed [32] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in X.

(9) πgp-closed [24] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is π-open in X.

(10) $\widehat{g}$-closed [34] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in X.

(11) $^*g$-closed [36] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\widehat{g}$-open in X.

(12) #g-semi closed (briefly #gs-closed)[35] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is $^*g$-open in X.

(13) $\widetilde{g}$-closed [17] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is #gs-open in X.

(14) ρ-closed [9] if $pcl(A) \subseteq Int(U)$ whenever $A \subseteq U$ and U is $\widetilde{g}$-open in X.

(15) D-closed [3] if $pcl(A) \subseteq Int(U)$ whenever $A \subseteq U$ and U is ω-open in X.

The complements of the above mentioned sets are called their respective open sets

Definition 2.3. A function $f : (X, \tau) \to (Y, \sigma)$ is called

(1) semi-continuous [19] if $f^{-1}(V)$ is semi-open in (X, τ) for every open set V in (Y, σ).

(2) pre-continuous [21] if $f^{-1}(V)$ is pre-closed in (X, τ) for every closed set V in (Y, σ).

(3) g-continuous [7] if $f^{-1}(V)$ is g-closed in (X, τ) for every closed set V in (Y, σ).

(4) ω-continuous [31] if $f^{-1}(V)$ is ω-closed in (X, τ) for every closed set V in (Y, σ).

(5) gsp-continuous [10] if $f^{-1}(V)$ is gsp-closed in (X, τ) for every closed set V in (Y, σ).

(6) gp-continuous [5] if $f^{-1}(V)$ is gp-closed in (X, τ) for every closed set V in (Y, σ).

(7) gpr-continuous [15] if $f^{-1}(V)$ is gpr-closed in (X, τ) for every closed set V in (Y, σ).

(8) semi-pre-continuous [2] if $f^{-1}(V)$ is semi-preopen in (X, τ) for every open set V in (Y, σ).

(9) pre-semi-continuous [38] if $f^{-1}(V)$ is pre-semiclosed in (X, τ) for every closed set V in (Y, σ).

(10) πgp-continuous [25] if $f^{-1}(V)$ is πgp-closed in (X, τ) for every closed set V in (Y, σ)).

(11) pg-continuous [23] if $f^{-1}(V)$ is pg-closed for every closed set V in (Y, σ).

(12) g^*p-continuous [37] if $f^{-1}(V)$ is g^*p-closed for every closed set V in (Y, σ)).

(13) $\#g$-semi-continuous [35] if $f^{-1}(V)$ is $\#gs$-closed in (X, τ) for every closed set V in (Y, σ).

(14) $\widetilde{g}$-continuous [27] if $f^{-1}(V)$ is $\widetilde{g}$-closed in (X, τ) for every closed set V in (Y, σ).

(15) contra-continuous [11] if $f^{-1}(V)$ is closed in (X, τ) for every open set V in (Y, σ).

(16) perfectly continuous [6] if $f^{-1}(V)$ is clopen in (X, τ) for every open set V in (Y, σ).

(17) contra-pre-continuous [16] if $f^{-1}(V)$ is pre-closed in (X, τ) for every open set V in (Y, σ).

(18) $\widetilde{g}$-irresolute [28] if $f^{-1}(V)$ is $\widetilde{g}$-closed in (X, τ) for every $\widetilde{g}$-closed set V in (Y, σ).

(19) M-pre closed [22] if $f(V)$ is pre closed in (Y, σ) for every pre closed set V in (X, τ).

(20) RC-continuous [12] if $f^{-1}(V)$ is regular closed in (X, τ) for every open set V in (Y, σ).

(21) D-irresolute [4] if $f^{-1}(V)$ is D-closed in (X, τ) for every D-closed set V in (Y, σ).

Definition 2.4. A space (X, τ) is called

(1) a $T_{\frac{1}{2}}$-space [20] if every g-closed set is closed.

(2) a T_ω-space [31] if every ω-closed set is closed.

(3) a $g_s T_{\frac{1}{2}}^{\#}$-space [35] if every $\#g$-semi closed set is closed.

(4) a $T\widetilde{g}$-space [37] if every $\widetilde{g}$-closed set is closed.

Theorem 2.5. [3]

(1) *Every open and preclosed subset of (X, τ) is D-closed.*

(2) *Every D-closed set is ρ-closed (resp.gp-closed ,gpr-closed,gsp-closed). Converse need not be true.*

(3) *If $D[A] \subseteq D_p[A]$ for each subset A of a space (X, τ), then the union of two D-closed set is D-closed.*

(4) *A subset A of (X, τ) is regular open iff A is both open and D-closed.*

3. D-CONTINUOUS FUNCTIONS

Definition 3.1. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be D-continuous if $f^{-1}(V)$ is D-closed in (X, τ) for every closed set V in (Y, σ).

Example 3.2. Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{c\}, \{a, c\}, X\}$ and $\sigma = \{\emptyset, \{a, c\}, Y\}$. Then the identity function $f : (X, \tau) \to (Y, \sigma)$ is D-continuous.

Theorem 3.3. *Every continuous is D-continuous.*

Proof. Let V be closed in (Y, σ). Since f is continuous, $f^{-1}(V)$ is closed in (Y, σ). By Theorem 2.8 [4], $f^{-1}(V)$ is D-closed in (X, τ). Hence f is D-continuous. $\square$

Remark 3.4. The converse of the above theorem need not be true as seen from the following example.

Example 3.5. Let $X = \{a, b, c\} = Y$, $\tau = \{\emptyset, \{a\}, \{b, c\}, X\}$ and $\sigma = \{\emptyset, \{a, c\}, Y\}$. Define $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = a$; $f(b) = b = f(c)$. Then f is D-continuous but not continuous. Since for the closed set $V = \{b\}$, $f^{-1}(V) = \{b, c\}$ is D-closed but not closed.

Proposition 3.6. *Every contra-continuous and Pre-continuous is D-continuous*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be Contra-continuous and pre-continuous. Let V be closed in (Y, σ). Then $f^{-1}(V)$ is pre-closed also open in (X, τ). Hence by theorem 2.5(1), $f^{-1}(V)$ is D-closed in (X, τ). Hence f is D-continuous. $\square$

Remark 3.7. The converse of the above proposition need not be true as seen from the following example.

Example 3.8. Let $X = \{a, b, c\} = Y$, $\tau = \{\emptyset, \{b\}, X\}$ and $\sigma = \{\emptyset, \{a, b\}, Y\}$. Define $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = f(b) = c$ and $f(c) = b$. Then f is D-continuous but neither contra-continuous nor pre-continuous. Since for the closed set $V = \{c\}$ in (Y, σ), $f^{-1}(V) = \{a, b\}$ is D-closed and it is neither pre-closed nor open in (X, τ).

Proposition 3.9. *Every D-continuous is gp-continuous.*

Proof. By theorem 2.5(2), every D-closed set is gp-closed, the proof follows. $\square$

Remark 3.10. The converse of the above proposition need not be true as seen from the following example.

Example 3.11. Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{b, c\}, X\}$ and $\sigma = \{\emptyset, \{a, b\}, Y\}$. Define $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = a$; $f(b) = c$ and $f(c) = b$. Then f is gp-continuous but not D-continuous. Since for the closed set $V = \{c\}$ in (Y, σ), $f^{-1}(V) = \{b\}$ is gp-closed but not D-closed in (X, τ).

Proposition 3.12. *Every D-continuous is gpr-continuous.*

Proof. By theorem 2.5(2), every D-closed set is gpr-closed, the proof follows. □

Remark 3.13. The converse of the above proposition need not be true as seen from the following example.

Example 3.14. By Example 3.11, f is gpr-continuous but not D-continuous. Since for the closed set $V = \{c\}$ in (Y, σ), $f^{-1}(V) = \{b\}$ is gpr-closed but not D-closed in (X, τ).

Proposition 3.15. *Every D-continuous is gsp-continuous.*

Proof. By theorem 2.5(2), every D-closed set is gsp-closed, the proof follows. □

Remark 3.16. The converse of the above proposition need not be true as seen from the following example.

Example 3.17. By example 3.14, f is gsp-continuous but not D-continuous. Since for the closed set $V = \{c\}$ in (Y, σ), $f^{-1}(V) = \{b\}$ is gsp-closed but not D-closed in (X, τ).

Proposition 3.18. *Every D-continuous is πgp-continuous.*

Proof. From the definitions 2.2 (9) and (15), every D-closed is πgp-closed, the proof follows. □

Remark 3.19. The converse of the above proposition need not be true as seen from the following example.

Example 3.20. Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{b, c\}, X\}$ and $\sigma = \{\emptyset, \{a, c\}, Y\}$. Define $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = a$; $f(b) = c$ and $f(c) = b$. Then f is πgp-continuous but not D-continuous. Since for the closed set $V = \{b\}$ in (Y, σ), $f^{-1}(V) = \{c\}$ is πgp-closed but not D-closed in (X, τ).

Remark 3.21. D-continuous and pre-continuous are independent. It is shown by the following examples.

Example 3.22. By example 3.20, f is pre-continuous but not D-continuous. Since for the closed set $V = \{b\}$, $f^{-1}(V) = \{c\}$ is pre-closed but not D-closed.

Example 3.23. Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{c\}, \{a, c\}, X\}$ and $\sigma = \{\emptyset, \{a, c\}, Y\}$. Define $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = a$; $f(b) = b = f(c)$. Then f is D-continuous but not pre-continuous. Since for the closed set $V = \{b\}$, $f^{-1}(V) = \{b, c\}$ is D-closed but not pre-closed.

Remark 3.24. D-continuous is independent of semi-continuous and semi-pre continuous. It is shown by the following examples.

Example 3.25. By example 3.23, f is D-continuous but neither semi-continuous nor semi-pre continuous. Since for the closed set $V = \{b\}$ in (Y, σ), $f^{-1}(V) = \{b, c\}$ is D-closed but neither semi-closed nor semi-pre closed.

Example 3.26. Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$ and $\sigma = \{\emptyset, \{a, c\}, Y\}$. Then the identity function $f : (X, \tau) \to (Y, \sigma)$ is semi-continuous and semi-pre continuous but not D-continuous. Since for the closed set $V = \{b\}$, $f^{-1}(V) = \{b\}$ semi-closed and semi-pre-closed but not D-closed .

Remark 3.27. D-continuous and pre-semi-continuous are independent. It is shown by the following examples.

Example 3.28. Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{c\}, X\}$ and $\sigma = \{\emptyset, \{b, c\}, Y\}$. Define $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = f(c) = a$ and $f(b) = b$. Then f is D-continuous but not pre-semi-continuous. Since for the closed set $V = \{a\}$, $f^{-1}(V) = \{a, c\}$ is D-closed but not pre-semi-closed.

Example 3.29. By example 3.20, f is pre-semi-continuous but not D-continuous. Since for the closed set $V = \{b\}$ in (Y, σ), $f^{-1}(V) = \{c\}$ is pre-semi-closed but not D-closed in (X, τ).

Remark 3.30. D-continuous and pg-continuous are independent. It is shown by the following examples.

Example 3.31. By example 3.23, f is D-continuous but not pg-continuous. Since for the closed set $V = \{b\}$ in (Y, σ), $f^{-1}(V) = \{b, c\}$ is D-closed but not pg-closed in (X, τ).

Example 3.32. By example 3.20, f is pg-continuous but not D-continuous. Since for the closed set $V = \{b\}$ in (Y, σ), $f^{-1}(V) = \{c\}$ is pg-closed but not D-closed in (X, τ).

Remark 3.33. D-continuous and g*p-continuous are independent. It is shown by the following examples.

Example 3.34. By example 3.28, f is D-continuous but not g^*p-continuous. Since for the closed set $V = \{a\}$ in (Y, σ), $f^{-1}(V) = \{a, c\}$ is D-closed but not g^*p-closed.

Example 3.35. By example 3.20, f is g^*p-continuous but not D-continuous. Since for the closed set $V = \{b\}$ in (Y, σ), $f^{-1}(V) = \{c\}$ is g^*p-closed but not D-closed.

Remark 3.36. D-continuous and g-continuous are independent. It is shown by the following examples.

Example 3.37. Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{b\}, \{a, b\}, X\}$ and $\sigma = \{\emptyset, \{b, c\}, Y\}$. Then the identity function $f : (X, \tau) \to (Y, \sigma)$ is D-continuous but not g-continuous. Since for the closed set $V = \{a\}$, $f^{-1}(V) = \{a\}$ is D-closed but not g-closed.

Example 3.38. Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{b, c\}, X\}$ and $\sigma = \{\emptyset, \{a, c\}, Y\}$. Then the identity function $f : (X, \tau) \to (Y, \sigma)$ is g-continuous but not D-continuous. Since for the closed set $V = \{b\}$, $f^{-1}(V) = \{b\}$ is g-closed but not D-closed.

Proposition 3.39. *Every D-continuous is ρ-continuous.*

Proof. By theorem 2.5(2), every D-closed set is ρ-closed, the proof follows.   □

Remark 3.40. The converse of the above theorem need not be true as seen from the following example.

Example 3.41. Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a, b\}, X\}$ and $\sigma = \{\emptyset, \{a, c\}, Y\}$. Then the identity function $f : (X, \tau) \to (Y, \sigma)$ is ρ-continuous but not D-continuous. Since for the closed set $V = \{b\}$, $f^{-1}(V) = \{b\}$ is ρ-closed but not D-closed.

Remark 3.42. We have the following relationship between D-continuous and other related generalized continuous. $A \to B$ ($A \not\longleftrightarrow B$) represent A implies B but not conversely (A and B are independent of each other).

4. Characterization of D-continuous functions

Now we shall obtain characterization of D-continuous functions in the sense of definition 3.1

Theorem 4.1. *A function $f : (X, \tau) \to (Y, \sigma)$ is D-continuous iff $f^{-1}(U)$ is D-open in (X, τ) for every open set U in (Y, σ).*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be D-continuous and U be open set in (X, τ). Then $f^{-1}(U^c)$ is D-closed in (X, τ). But $f^{-1}(U^c) = (f^{-1}(U))^c$ and so $f^{-1}(U)$ is D-open in (X, τ). Conversely, let U be an open set in (Y, σ). Then U^c is a closed set in (Y, σ). Since $f^{-1}(U)$ is D-open in (X, τ), $(f^{-1}(U))^c$ is D-closed in (X, τ). Therefore $f^{-1}(U^c) = (f^{-1}(U))^c$ is D-closed in (X, τ). $\square$

Remark 4.2. The composition of two D-continuous functions need not be D-continuous. It is shown by the following example.

Example 4.3. Let $X = Y = Z = \{a, b, c\}$, $\tau = \{\emptyset, \{a, b\}, X\}$, $\sigma = \{\emptyset, \{a\}, X\}$ and $\eta = \{\emptyset, \{a, c\}, X\}$. Define $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = b$; $f(b) = a$; $f(c) = c$ and define $g : (Y, \sigma) \to (Z, \eta)$ by $g(x) = x$. Then f and g are D-continuous but gof is not D-continuous. Since $\{b\}$ is closed in (Z, η), $(gof)^{-1}(\{b\}) = f^{-1}(g^{-1}\{b\}) = f^{-1}(\{b\}) = \{a\}$ is not D-closed in (X, τ).

Definition 4.4.   (1) A space (X, τ) is said to be $D - T_s$ space if every D-closed set is closed.

  (2) A space (X, τ) is said to be $D - T_{\frac{1}{2}}$ space if every D-closed set is pre-closed.

Theorem 4.5. *Let (X, τ) and (Z, η) be topological spaces and (Y, σ) be $D - T_s$ space. Then the composition $gof : (X, \tau) \to (Z, \eta)$ of D-continuous (resp.continuous) function $f : (X, \tau) \to (Y, \sigma)$ and the D-continuous function $g : (Y, \sigma) \to (Z, \eta)$ is D-continuous (resp. continuous).*

Proof. Let G be any closed set of (Z, η). Then by assumption $g^{-1}(G)$ is closed in (Y, σ). Since f is D-continuous (resp. continuous), then $f^{-1}(g^{-1}(G)) = (gof)^{-1}(G)$ is D-closed (resp. closed) in (X, τ). Thus gof is D-continuous (resp. continuous). $\square$

Theorem 4.6. *Let (X, τ) and (Z, η) be topological spaces and (Y, σ) be $T_{\frac{1}{2}}$-space (resp $T\omega$-space, $T\widetilde{g}$-space, $gsT_{\frac{1}{2}}^{\#}$-space). Then the composition $gof : (X, \tau) \to (Z, \eta)$ of D-continuous function $f : (X, \tau) \to (Y, \sigma)$ and the g-continuous (resp ω-continuous, $\widetilde{g}$-continuous, #gs-continuous) function $g : (Y, \sigma) \to (Z, \eta)$ is D-contiounus.*

Proof. Let G be any closed set of (Z, η). Then $g^{-1}(G)$ is g-closed (resp ω-closed, $\widetilde{g}$-closed, #gs-closed) in (Y, σ) and by assumption, $g^{-1}(G)$ is closed in (Y, σ). Since f is D-continuous, $f^{-1}(g^{-1}(G)) = (gof)^{-1}(G)$ is D-closed in (X, τ). Thus gof is D-continuous. $\square$

Theorem 4.7. *Let $f : (X, \tau) \to (Y, \sigma)$ be D-continuous and $g : (Y, \sigma) \to (Z, \eta)$ be continuous. Then their composition $gof : (X, \tau) \to (Z, \eta)$ is D-continuous.*

Proof. Let G be any closed set of (Z, η). Then $g^{-1}(G)$ is closed in (Y, σ). Since f is D-continuous, $f^{-1}(g^{-1}(G)) = (gof)^{-1}(G)$ is D-closed in (X, τ). Thus gof is D-continuous. $\square$

Theorem 4.8. *Let $f : (X, \tau) \to (Y, \sigma)$ be contra-continuous and $g : (Y, \sigma) \to (Z, \eta)$ be contra-continuous. Then their composition $gof : (X, \tau) \to (Z, \eta)$ is D-continuous.*

Proof. Let G be any closed set of (Z, η). Since g is contra- continuous, then $g^{-1}(G)$ is open in (Y, σ). Since f is contra-continuous, $f^{-1}(g^{-1}(G)) = (gof)^{-1}(G)$ is closed in (X, τ). Then by theorem 2.8[4] , $(gof)^{-1}(G)$ is D-closed in (X, τ). Hence gof is D-continuous. $\square$

Theorem 4.9. *Let $f : (X, \tau) \to (Y, \sigma)$ be D-irresolute and $g : (Y, \sigma) \to (Z, \eta)$ be D-continuous. Then their composition $gof : (X, \tau) \to (Z, \eta)$ is D-continuous.*

Proof. Let G be any closed set of (Z, η). Since g is D-continuous, $g^{-1}(G)$ is D-closed in (Y, σ). Since f is D-irresolute, $f^{-1}(g^{-1}(G)) = (gof)^{-1}(G)$ is D-closed in (X, τ). Thus gof is D-continuous. $\square$

Theorem 4.10. *Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a D-continuous then f is continuous if (X, τ) is $D - T_s$.*

Proof. Let G be any closed set of (Y, σ). Since f is D-continuous and by assumption $f^{-1}(G)$ is closed in (X, τ), f is continuous. $\square$

Definition 4.11.   (1) Let x be a point of (X, τ) and V be a subset of X. Then V is called a D-neighborhood of x in (X, τ) if there exists a D-open set U of (X, τ) such that $x \in U \subseteq V$.

  2 The intersection of all D-closed sets containing a set A in a topological space X is called a D-closure of A and in denoted by $D - cl(A)$.

Theorem 4.12. *Let A be a subset of (X, τ). Then $x \in D - cl(A)$ if and only if for any D-neighborhood N_x of x in (X, τ) such that $A \cap N_x \neq \emptyset$.*

Proof. Necessity : Assume that $x \in D - cl(A)$. Suppose that there exists a D-neighborhood N_x of x such that $A \cap N_x = \emptyset$. Since N_x is a D-Neighborhood of x in (X, τ), by definition 4.11, there exists a D-open set V_x such that $x \in V_x \subseteq N_x$. Therefore, we have $A \cap V_x = \emptyset$ and so $A \subseteq (V_x)^c$. Since $(V_x)^c$ is a D-closed set containing A, we have $D - cl(A) \subseteq (V_x)^c$ and therefore $x \notin D - cl(A)$. Which is a contradiction.

  Sufficiency : Assume that for each D-neighborhood N_x of x in (X, τ) such that $A \cap N_x \neq \emptyset$. Suppose $x \notin D - cl(A)$. Then there exists a D-closed set $Vof(X, \tau)$ such that $A \subseteq V$ and $x \notin V$. Thus $x \in V^c$ is D-open in (X, τ). But $A \cap V^c = \emptyset$. Which is a contradiction. $\square$

Theorem 4.13. *Let $f : (X, \tau) \to (Y, \sigma)$ be a function. Then the following statements are equivalent:*

(1) *The function f is D-continuous.*

(2) *The inverse of each open set in (Y, σ) is D-open in (X, τ)*

(3) *The inverse of each closed set in (Y, σ) is D-closed in (X, τ)*

(4) *For each x in (X, τ) the inverse of every neighborhood of $f(x)$ is a D-neighborhood of x.*

(5) *For each x in (X, τ) and each neighborhood N of $f(x)$, there is a D-neighborhood W of x such that $f(W) \subseteq N$*

(6) *For each subset A of (X, τ), $f(D - cl(A)) \subseteq cl(f(A))$.*

(7) *For each subset B of (Y, σ), $D - cl(f^{-1}(B)) \subseteq f^{-1}(cl(B))$*

Proof. $1 \Leftrightarrow 2$ This follows from theorem 4.1

$2 \Leftrightarrow 3$ The proof is clear from the result $f^{-1}(A^c) = (f^{-1}(A))^c$

$2 \Leftrightarrow 4$ *Let* $x \in X$ and let N be a neighborhood of $f(x)$. Then there exists an open set V in (Y, σ) such that $f(x) \in V \subseteq N$. Consequently $f^{-1}(V)$ is D-open in (X, τ) and $x \in f^{-1}(V) \subseteq f^{-1}(N)$. Thus $f^{-1}(N)$ is a D-neighborhood of x.

$4 \Leftrightarrow 5$ Let $x \in X$ and let N be a neighborhood of $f(x)$. Then by assumption, $W = f^{-1}(N)$ is a D-neighborhood of x and $f(W) = f(f^{-1}(N)) \subseteq N$.

$5 \Leftrightarrow 6$ Suppose that (5) holds. Let $y \in f(D - cl(A))$ and let N be any neighborhood of y. Then there exists $x \in X$ and a D-neighborhood W of x such that $f(x) = y$, $x \in W$. Hence $x \in D - cl(A)$ and $f(W) \subseteq N$. By theorem 4.12, $W \cap A \neq \emptyset$ and hence $f(A) \cap N \neq \emptyset$. Hence $y = f(x) \in cl(f(A))$. Therefore $f(D - cl(A)) \subseteq cl(f(A))$.

Conversely,suppose that (6) holds. Let $x \in X$ and N be any neighborhood of $f(x)$. Let $A = f^{-1}(N^c)$. Since $f(D - cl(A)) \subseteq cl(f(A)) \subseteq N^c$, $D - cl(A) \subseteq A$. Hence $D - cl(A) = A$. Since $x \notin D - cl(A)$, there exists a D-neighborhood W of x such that $W \cap A = \emptyset$. Hence $f(W) \subseteq f(A^c) \subseteq N$.

$6 \Leftrightarrow 7$ Suppose that (6) holds. Let B be any subset of (Y, σ). Then replacing A by $f^{-1}(B)$ in (6), we obtain $f(D - cl(f^{-1}(B)) \subseteq cl(f(f^{-1}(B))) \subseteq cl(B)$. That is $D - cl(f^{-1}(B)) \subseteq f^{-1}(cl(B))$.

Conversely,suppose (7) holds. Let $B = f(A)$, where A is a subset of (X, τ). Then $D - cl(A) \subseteq D - cl(f^{-1}(B)) \subseteq f^{-1}(cl(f(A))$ and so $f(D - cl(A)) \subseteq cl(f(A))$.   □

Definition 4.14. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called ω-irresolute if $f^{-1}(V)$ is a ω-closed set of (X, τ) for every ω-closed set V of (Y, σ).

Proposition 4.15. *If $f : (X, \tau) \rightarrow (Y, \sigma)$ is ω-irresolute and M-pre closed function then $f(A)$ is D-closed in (Y, σ) for every D-closed set A of (X, τ).*

Proof. Let U be any ω-open set in (Y, σ) such that $f(A) \subseteq U$. Then $A \subseteq f^{-1}(U)$. Since f is ω-irresolute then $f^{-1}(U)$ is ω-open. Since A is D-closed in (X, τ) we have $pcl(A) \subseteq Int(f^{-1}(U))$. Hence $f(pcl(A)) \subseteq Int(U)$. Since f is M-pre closed, $f(pcl(A))$ is a preclosed in (Y, σ). Now $pcl(f(A)) \subseteq pcl(f(Pcl(A))) = f(pcl(A)) \subseteq Int(U)$. Hence $f(A)$ is D-closed in (Y, σ).   □

Theorem 4.16. *If the bijective function $f : (X, \tau) \rightarrow (Y, \sigma)$ is pre-irresolute and ω-open then f is D-irresoulte.*

Proof. Let A be D-closed in (X, τ) and let U be any ω-open set in (X, τ) such that $f^{-1}(A) \subseteq U$. Then $A \subseteq f(U)$. Since f is ω-open, $f(U)$ is ω-open in (Y, σ). Since A is D-closed in (Y, σ), we have $pcl(A) \subseteq Int(f(U))$. Thus $f^{-1}(pcl(A)) \subseteq f^{-1}(Int(f(U))) \subseteq Int(f^{-1}(f(U))) = Int(U)$. Since f is pre-irresolute, we have

$f^{-1}(pcl(A))$ is a pre-closed in (X, τ). Now, $pcl(f^{-1}(A)) \subseteq pcl(f^{-1}(pcl(A))) = f^{-1}(pcl(A)) \subseteq Int(U)$. Hence $f^{-1}(A)$ is D-closed in (X, τ) and so f is D-irresolute. $\qquad \square$

Theorem 4.17.   (1) *If $f : (X, \tau) \to (Y, \sigma)$ is gp-continuous and contra-continuous then f is D-continuous.*

(2) *If $f : (X, \tau) \to (Y, \sigma)$ is gpr-continuous and RC-continuous then f is D-continuous.*

Proof.   (1) Let V be any closed set in (Y, σ). Since f is gp-continuous and contra-continuous, $f^{-1}(V)$ is gp-closed and open in (X, τ). By Theorem 4.17 [3], $f^{-1}(V)$ is D-closed in (X, τ). Hence f is D-continuous.

(2) Let V be any closed set in (Y, σ). Since f is gpr-continuous and RC-continuous, $f^{-1}(V)$ is gpr-closed and regular open. By theorem 4.15 [3], $f^{-1}(V)$ is D-closed in (X, τ). Hence f is D-continuous.

$\qquad \square$

Theorem 4.18. *If $f : (X, \tau) \to (Y, \sigma)$ is D-irresolute and $g : (Y, \sigma) \to (Z, \eta)$ is D-irrusolute then $gof : (X, \tau) \to (Z, \eta)$ is D-irresolute.*

Proof. Let G be any D-closed set in (Z, η). Since g is D-irresolute, $g^{-1}(G)$ is D-closed in (Y, σ). Since f is D-irresolute, $f^{-1}(g^{-1}(G)) = (gof)^{-1}(G)$ is D-closed in (X, τ). Thus gof is D-irresolute. $\qquad \square$

Regarding the restriction of a D-continuous function, we have the following.

Lemma 4.19. [17]

(1) *Let A be ω-closed in (X, τ). If A is regular closed, then $pcl(A)$ is also ω-closed.*

(2) *If $A \subseteq Y \subseteq X$ where A is ω-open in Y and Y is ω-open in X then A is ω-open in X.*

(3) *Let $A \subseteq Y \subseteq X$ and suppose that A is ω-closed in X then A is ω-closed in Y.*

Theorem 4.20. *Let $f : (X, \tau) \to (Y, \sigma)$ be a D-continuous function and H be a open D-closed subset of X. Assume that $DC(X, \tau)$ (the class of all D-closed sets of (X, τ)) is D-closed under finite intersections. Then the restriction $f|H : (H, \tau|H) \to (Y, \sigma)$ is D-continuous.*

Proof. Let F be a closed subset of Y. By hypothesis and assumption, $f^{-1}(F) \cap H = H_1$ (say) is D-closed in X. Since $(f|H)^{-1}(F) = H_1$, it is sufficient to show that H_1 is D-closed in H. Let G_1 be a ω-open set in H such that $H_1 \subseteq G_1$. Then by hypothesis and by lemma 4.19(2), G_1 is ω-open in X. Since H_1 is D-closed in X, $pclX(H_1) \subseteq Int(G_1)$. Since H is open and by lemma 2.10[15], $pclH(H_1) = pclX(H_1) \cap H \subseteq Int(G_1) \cap H = Int(G_1) \cap Int(H) = Int(G_1 \cap H) \subseteq Int(G_1)$. Hence $H_1 = (f|H)^{-1}(F)$ is D-closed in H. Thus $f|H$ is D-continuous. $\square$

Theorem 4.21. *Let A and Y be subsets of (X, τ) such that $A \subseteq Y \subseteq X$. Let A be ω-closed and regular closed in (X, τ). If A is D-closed in (Y, σ) and Y is open and D-closed in (X, τ) then A is D-closed in (X, τ).*

Proof. Let U be a ω-open set of (X, τ) such that $A \subseteq U$. Since Y is open in (X, τ) and A is D-closed in (Y, σ), we have $pcl_Y(A) \subseteq Int_Y(U \cap Y)$. Thus $pcl(A) \cap Y \subseteq pcl_Y(A) \subseteq Int_Y(U \cap Y) = Int(U \cap Y)$. By lemma 4.19 (1), $(pcl(A))^c$ is ω-open in (X, τ). Hence $Int(U \cap Y) \cup (pcl(A))^c$ is ω-open in (X, τ) and it contains Y. Since Y is D-closed in (X, τ), we have $pcl(A) \subseteq pcl(Y) \subseteq Int[Int(U \cap Y) \cup (pcl(A))^c] \subseteq Int(U) \cup (pcl(A))^c$. Thus $pcl(A) \subseteq Int(U)$. Hence A is D-closed in (X, τ). $\square$

Theorem 4.22. *Let $X = G \cup H$ be a topological space with topology τ and Y be a topological space with topology σ. Let $f : (G, \tau|G) \to (Y, \sigma)$ and $g : (H, \tau|H) \to (Y, \sigma)$ be D-continuous functions such that $f(x) = g(x)$ for every $x \in G \cap H$. Assume that $D[E] \subseteq D_p[E]$, for any $E \subseteq X$. Suppose that both G and H are open and D-closed in (X, τ). Then their combination $f \triangle g : (X, \tau) \to (Y, \sigma)$ defined by $(f \triangle g)(x) = f(x)$ if $x \in G$ and $(f \triangle g)(x) = g(x)$ if $x \in H$ is D-continuous.*

Proof. Let F be a closed subset of (Y, σ). Then $f^{-1}(F)$ is D-closed in $(G, \tau|G)$ and $g^{-1}(F)$ is D-closed in $(H, \tau|H)$. Since G and H are both open and D-closed subsets of (X, τ), by Theorem 4.20, $f^{-1}(F)$ and $g^{-1}(F)$ are both D-closed sets in (X, τ). By theorem 2.5(3), $f^{-1}(F) \cup g^{-1}(F)$ is D-closed in (X, τ). By definition, $(f \triangle g)^{-1}(F) = f^{-1}(F) \cup g - 1(F)$ is D-closed in (X, τ). Hence $f \triangle g$ is D-continuous. $\square$

5. STRONGLY D-CONTINUOUS AND PERFECTLY D-CONTINUOUS FUNCTIONS

Definition 5.1. A function $f : (X, \tau) \to (Y, \sigma)$ is called

(1) Strongly gp-continuous [26] if $f^{-1}(V)$ is closed (resp.open) in (X, τ) for every gp-closed set (resp. gp-open set) V of (X, τ).

(2) Strongly πgp-continuous [25] if $f^{-1}(V)$ is closed (resp.open) in (X, τ) for every πgp-closed set (resp. πgp-open set) V of (Y, σ).

(3) Perfectly D-continuous [4] if $f^{-1}(V)$ is clopen in (X, τ) for every D-closed set (resp. D-open set) V of (Y, σ).

(4) Strongly D-continuous if $f^{-1}(V)$ is closed (resp.open) in (X, τ) for every D-closed set (resp. D-open set) V of (Y, σ).

(5) Pre-D-continuous if $f^{-1}(V)$ is D-closed in (X, τ) for every pr-closed set V of (Y, σ).

Remark 5.2. From the above definition and closed $\longrightarrow$ D-closed, D-closed $\longrightarrow$ gp-closed (resp. πgp-closed) we have the following
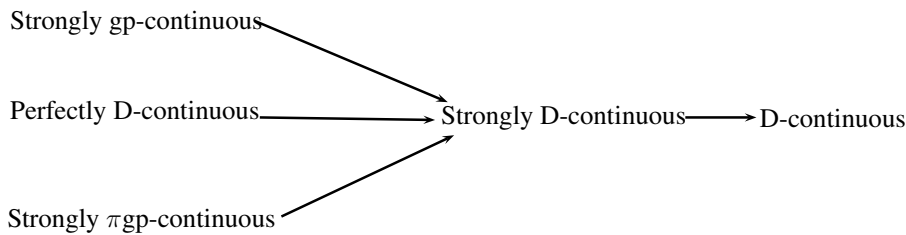
Theorem 5.3. (1) *If $f : (X, \tau) \to (Y, \sigma)$ is perfectly D-continuous then f is strongly D-continuous and also D-irresoulte.*

(2) *If $f : (X, \tau) \to (Y, \sigma)$ is pre-D-continuous then f is D-continuous.*

(3) *If $f : (X, \tau) \to (Y, \sigma)$ is strongly D-continuous and $g : (Y, \sigma) \to (Z, \eta)$ is D-continuous then $gof : (X, \tau) \to (Z, \eta)$ is continuous.*

(4) *If $f : (X, \tau) \to (Y, \sigma)$ is strongly D-continuous and $g : (Y, \sigma) \to (Z, \eta)$ is perfectly D-continuous then $gof : (X, \tau) \to (Z, \eta)$ is strongly D-continuous.*

(5) *If $f : (X, \tau) \to (Y, \sigma)$ is perfectly D-continuous and $g : (Y, \sigma) \to (Z, \eta)$ is pre D-continuous then $gof : (X, \tau) \to (Z, \eta)$ is D-continuous.*

Proof. (1) Let V be D-closed in (Y, σ). Then $f^{-1}(V)$ is clopen in (X, τ) and hence $f^{-1}(V)$ is closed in (X, τ) and so f is strongly D-continuous. By Theorem 2.8 [4], closed set implies D-closed, $f^{-1}(V)$ is D-closed in (X, τ). Thus f is D-irresolute.

(2) Closed set implies pre-closed set and the proof is obvious.

(3) Let V be closed in (Z, η). Then $g^{-1}(V)$ is D-closed in (Y, σ) and $f^{-1}(g^{-1}(V)) = (gof)^{-1}(V)$ is closed in (X, τ). Then gof is continuous.

(4) Let V be D-closed in (Z, η). Then $g^{-1}(V)$ is clopen in (Y, σ). Since closed set implies D-closed and by theorem 2.8 [4], $g^{-1}(V)$ is D-closed in (Y, σ). Hence $f^{-1}(g^{-1}(V)) = (gof)^{-1}(V)$ is closed in (X, τ). Then gof is strongly D-continuous.

(5) Let V be closed in (Z, η). Since closed implies pre-closed , $g^{-1}(V)$ is D-closed in (Y, σ). Hence $f^{-1}(g^{-1}(V)) = (gof)^{-1}(V)$ is clopen in (X, τ). By Theorem 2.8 [4], $(gof)^{-1}(V)$ is D-closed in (X, τ). Hence gof is D-continuous.

$\square$

Theorem 5.4. *Let $f : (X, \tau) \to (Y, \sigma)$ be a bijective, D-irresolute and M-pre closed. If (X, τ) is a D-$T_{\frac{1}{2}}$ space ,then (Y, σ) is also D-$T_{\frac{1}{2}}$ space.*

Proof. Let A be D-closed in (Y, σ). Since f is D-irresolute, $f^{-1}(A)$ is D-closed in (X, τ). Since (X, τ) is a D-$T_{\frac{1}{2}}$ space, $f^{-1}(A)$ is pre-closed in (X, τ). Since f is M-pre closed then $f(f^{-1}(A)) = A$ is pre-closed in (Y, σ). Hence (Y, σ) is a D-$T_{\frac{1}{2}}$ space. $\square$

6. D-COMPACT AND D-CONNECTED

Definition 6.1.   (1) A topological space (X, τ) is S-closed [33] if every regular closed cover of X has a finite sub cover.

(2) A topological space (X, τ) is strongly S-closed [11] if every closed cover of X has a finite sub cover.

Definition 6.2. A topological space (X, τ) is D-compact if every D-open cover of X has a finite subcover.

Theorem 6.3. *Let $f : (X, \tau) \to (Y, \sigma)$ be a bijective, D-continuous function. If X is D-compact then Y is compact.*

Proof. Let $\{A_i : i \in I\}$ be an open cover of Y. Then $\{f^{-1}(A_i) : i \in I\}$ is a D-open cover of X. Since X is D-compact, it has a finite sub cover say $\{f^{-1}(A_1), \ldots, f^{-1}(A_n)\}$. Since f is surjective, $\{A_1, A_2, \ldots, A_n\}$ is a finite sub cover of Y. Hence Y is compact. $\square$

Definition 6.4. A subset A of a space X is called D-compact relative to X if every collection $\{U_i : i \in I\}$ of D-open subsets of X such that $A \subseteq \{U_i : i \in I\}$, there exists a finite subset I_0 of I such that $A \subseteq \{U_i : i \in I_0\}$.

Theorem 6.5. *Every D-closed subset of a D-compact space X is D-compact relative to X.*

Proof. Let A be a D-closed subset of a D-compact space X. Let $\{U_i : i \in I\}$ be a cover of A by D-open subsets of X. So, $A \subseteq \cup\{U_i : i \in I\}$ and then $A^c \cup (\cup\{U_i :$

$i \in I\}) = X$. Since X is D-compact, there exists a finite subset I_0 of I such that $A^c \cup (\cup\{U_i : i \in I_0\}) = X$. Then $A \subseteq \cup\{U_i : i \in I_0\}$. Hence A is D-compact relative to X. $\qquad \square$

Theorem 6.6. *If $f : (X, \tau) \to (Y, \sigma)$ is D-irresolute and a subset A of X is D-compact relative to X then its image $f(A)$ is D-compact relative to Y.*

Proof. Let $\{f(U_i) : i \in I\}$ be a cover of $f(A)$ by D-open subsets of (Y, σ). Since f is D-irresolute, $\{U_i : i \in I\}$ is a cover of A by D-open subsets of (X, τ). Since A is compact relative to X, there exists a finite sub set I_0 of I such that $A \subseteq \cup\{U_i : i \in I_0\}$. Hence $f(A) \subseteq \cup\{f(U_i) : i \in I\}$. Thus $f(A)$ is compact relative to Y. $\qquad \square$

Theorem 6.7. *If $p : X \times Y \to X$ be a projection, then p is D-irresolute.*

Proof. Let A be a D-closed subset of X. Since p is a projection, $p^{-1}(A) = A \times Y$ is a subset of $X \times Y$. Now to show that $p^{-1}(A) = A \times Y$ is D closed in $X \times Y$. Let U be ω-open subset of $X \times Y$ such that $A \times Y \subseteq U$. Then $V \times Y = U$, for some open set V of X containing A. Since A is D-closed in X, we have $pcl_X(A) \subseteq Int(V)$ and $pcl_X(A) \times Y \subseteq Int(V) \times Y$. That is $pcl_{X \times Y}(A \times Y) \subseteq Int(V \times Y) = Int(U)$. Hence $p^{-1}(A) = A \times Y$ is D-closed in $X \times Y$. $\qquad \square$

Theorem 6.8. *If the product space $X \times Y$ is D-compact then each of the spaces X and Y is D-compact.*

Proof. Let $X \times Y$ be D-compact. By theorem 6.7, the projection $p : X \times Y \to X$ is D-irresolute and then by theorem 6.6, $p(X \times Y) = X$ is D-compact. The proof for the space Y is similar to the case of X. $\qquad \square$

Lemma 6.9. *(The tube lemma)*
Consider the product space $X \times Y$, where Y is compact. If N is an open set of $X \times Y$

containing the slice $x_0 \times Y$ of $X \times Y$ then N contains some tube $W \times Y$ about $x_0 \times Y$, where W is a neighborhood of x_0 in X.

Theorem 6.10. *Let A be any subset of Y.*

(1) *If $X \times A$ is D-closed in the product space $X \times Y$ and Y is $T\widetilde{g}$-space then A is D-closed in Y.*

(2) *If X is compact and A is D-closed in Y and $X \times Y$ is $T\widetilde{g}$-space then $X \times A$ is D-closed in $X \times Y$.*

Proof.   (1) Let U be a ω-open set of Y such that $A \subseteq U$. Then $X \times A \subseteq X \times U$. Since Y is $T\widetilde{g}$-space, U is open in Y and $X \times U$ is open in $X \times Y$. Hence $X \times U$ is ω-open in $X \times Y$. Since $X \times A$ is D-closed in $X \times Y$, $pcl(X \times A) \subseteq Int(X \times U) = X \times U$. By proposition 2.8[26], $X \times pcl(A) \subseteq X \times Int(U)$. Thus $pcl(A) \subseteq Int(U)$. Hence A is D-closed in Y.

(2) Let U be a ω-open set of $X \times Y$ such that $X \times A \subseteq U$. Since X is compact and $X \times Y$ is $T\widetilde{g}$-space and by the generalization of lemma 6.9, there exists an open set V in Y containing A such that $X \times V \subseteq U$. Since A is D-closed in Y, $pcl(A) \subseteq Int(V)$. Therefore $X \times pcl(A) \subseteq X \times Int(V)$. This implies $X \times pcl(A) = Int(X) \times Int(V) \subseteq Int(X \times U)$. By proposition 2.8[26], $pcl(X \times A) \subseteq Int(X \times V) \subseteq Int(U)$. Hence $X \times A$ is D-closed in $X \times Y$.

$\square$

Definition 6.11. A topological space (X, τ) is D-connected if X can not be written as the disjoint union of two non-empty D-open sets.

Theorem 6.12. *Let $f : (X, \tau) \to (Y, \sigma)$ be a surjective, D-continuous (resp. D-irresolute) function. If X is D-connected then Y is connected (resp. D-connected)*

Proof. Suppose Y is not connected (resp. not D-connected). Then $Y = A \cup B$, where $A \cap B = \emptyset$, $A \neq \emptyset$, $B \neq \emptyset$ and A, B are open (resp.D-open) sets in Y.

Since f is surjective, $f(X) = Y$ and since f is D-continuous (resp.D-irresolute), $X = f^{-1}(A) \cup f^{-1}(B)$ is the disjoint union of two non-empty D-open sets. Which is a contradiction to X is D-connected. □

Theorem 6.13. *If the product space $X \times Y$ is D-connected then each of the spaces X and Y is D-connected.*

Proof. Let $X \times Y$ be D-connected. By theorem 6.7, the projection $p : X \times Y \to X$ is D-irresolute and then by theorem 6.12, $p(X \times Y) = X$ is D-connected. The proof for the space Y is similar to the case of X. □

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