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ANOTHER DECOMPOSITION OF $*$-CONTINUITY VIA IDEAL TOPOLOGICAL SPACES

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Abstract. In this paper, we introduce the notions of ω-$\mathcal{I}$-LC *-sets, $\mathcal{I}^*_\omega$-closed sets and $\mathcal{I}$-ω_t-sets. Also defined the notions of ω-$\mathcal{I}$-LC *-continuous maps, $\mathcal{I}^*_\omega$-continuous maps, $\mathcal{I}$-ω_t-continuous maps and obtained decompositions of $*$-continuity.

1. Introduction and Preliminaries

The concept of ideals in topological spaces is treated in the classic text by Kuratowski [11] and Vaidyanathaswamy [17]. The notion of $\mathcal{I}$-open sets in topological spaces was introduced by Jankovic and Hamlett [9]. Dontchev et al. [2] introduced and studied the notion of $\mathcal{I}_g$-closed sets. An ideal $\mathcal{I}$ on a topological space (X, τ) is a non-empty collection of subsets of X satisfying the following properties:(1) $A \in \mathcal{I}$ and $B \subseteq A$ imply $B \in \mathcal{I}$ (heredity); (2) $A \in \mathcal{I}$ and $B \in \mathcal{I}$ imply $A \cup B \in \mathcal{I}$ (finite additivity). A topological space (X, τ) with an ideal $\mathcal{I}$ on X is called an *ideal topological space* and is denoted by $(X, \tau, \mathcal{I})$. For a subset $A \subseteq X$, $A^*(\mathcal{I}) = \{x \in X : U \cap A \notin \mathcal{I}$ for every $U \in \tau(x)\}$, is called the *local function* [11] of A with respect to $\mathcal{I}$ and τ. We simply write A^* in case there is no chance for confusion. A Kuratowski closure operator $Cl^*(.)$ for a topology $\tau^*(\mathcal{I})$ called the $*$-topology finer than τ is defined by $Cl^*(A) = A \cup A^*$ [17]. Let (X, τ) denote a topological space on which no separation axioms are assumed unless explicitly stated. In a topological space (X, τ), the

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closure and the interior of any subset A of X will be denoted by $Cl(A)$ and $Int(A)$, respectively. A subset A of a topological space (X, τ) is said to be g-closed [12] (resp. ω-closed [15]) if $Cl(A) \subset U$ whenever $A \subset U$ and U is open (resp. semi-open) in X. The complement of a g-closed (resp. ω-closed) set is said to be g-open (resp.ω-open). A subset A of a topological space (X, τ) is said to be g^*-closed [18] if $Cl(A) \subset U$ whenever $A \subset U$ and U is g-open in X. A subset A of an ideal space $(X, \tau, \mathcal{I})$ is $*$-*closed* [9] (resp. $*$-perfect [5]) if $A^* \subset A$ (resp. $A = A^*$).

Definition 1.1. A subset A of a topological space (X, τ) is called

(1) locally closed [3] (briefly LC) if $A = U \cap V$, where U is open and V is closed.
(2) ω-LC *-set [15] if $A = U \cap V$, where U is ω-open and V is closed.
(3) t-set [16] if $Int(Cl(A)) = Int(A)$.
(4) ω_t-set [8] if $A = C \cap D$, where C is ω-open and D is a t-set.

Definition 1.2. A subset A of an ideal topological space $(X, \tau, \mathcal{I})$ is called

(1) t-$\mathcal{I}$-set [4] if $Int(Cl^*(A)) = Int(A)$.
(2) α^*-$\mathcal{I}$-set [4] if $Int(Cl^*(Int(A))) = Int(A)$.
(3) $\mathcal{I}$-LC set [1] if $A = C \cap D$, where $C \in \tau$ and D is $*$-perfect.
(4) weakly-$\mathcal{I}$-LC set [10] if $A = C \cap D$, where $C \in \tau$ and D is $*$-closed.
(5) $C_{\mathcal{I}}$-set [4] if $A = C \cap D$, where $C \in \tau$ and D is an α^*-$\mathcal{I}$-set.
(6) G-$\mathcal{I}$-LC *-set [7] if $A = C \cap D$, where C is g-open and D is $*$-closed.

Definition 1.3. [7] A subset A of an ideal space $(X, \tau, \mathcal{I})$ is said to be $\mathcal{I}_g^*$-closed if $A^* \subset U$ whenever $A \subset U$ and U is g-open in X.

Definition 1.4. [6] A function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is said to be $*$-continuous if $f^{-1}(A)$ is $*$-closed in $(X, \tau, \mathcal{I})$ for every closed set A in (Y, σ).

Definition 1.5. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be ω-LC *-continuous [15] (resp. g^*-continuous [18], ω_t-continuous [8]) if $f^{-1}(A)$ is ω-LC *-set (resp. g^*-closed, ω_t-set) in (X, τ) for every closed set A of (Y, σ).

Definition 1.6. [7] A function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is said to be $\mathcal{I}_g^*$-continuous (resp. G-$\mathcal{I}$-LC *-continuous) if $f^{-1}(V)$ is $\mathcal{I}_g^*$-closed(resp. G-$\mathcal{I}$-LC *-set) in $(X, \tau, \mathcal{I})$ for every closed set V in (Y, σ).

For a subset A of an ideal space $(X, \tau, \mathcal{I})$, if A is $*$-closed, then by [13] A is weakly-$\mathcal{I}$-LC. Also by Definition 1.3 it follows that if A is $*$-closed, then A is $\mathcal{I}_g^*$-closed.

2. ω-$\mathcal{I}$-LC *-SETS

Definition 2.1. A subset A of an ideal topological space $(X, \tau, \mathcal{I})$ is said to be an ω-$\mathcal{I}$-LC *-set if $A = C \cap D$, where C is ω-open and D is $*$-closed.

Proposition 2.1. *Let $(X, \tau, \mathcal{I})$ be an ideal space and $A \subset X$. Then the following hold:*

(1) *If A is ω-open, then A is an ω-$\mathcal{I}$-LC *-set;*

(2) *If A is $*$-closed, then A is an ω-$\mathcal{I}$-LC *-set;*

(3) *If A is weakly-$\mathcal{I}$-LC set, then A is an ω-$\mathcal{I}$-LC *-set;*

(4) *If A is an ω-$\mathcal{I}$-LC *-set, then A is a G-$\mathcal{I}$-LC *-set.*

The converse of Proposition 2.1 need not be true as seen from the following examples.

Example 2.1. *Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, \{b\}, \{b, c, d\}, X\}$ and $\mathcal{I} = \{\emptyset, \{b\}\}$. Then,*

(1) *$A = \{a\}$ is an ω-$\mathcal{I}$-LC *-set but not an ω-open set.*

(2) *$A = \{c, d\}$ is an ω-$\mathcal{I}$-LC *-set but not a $*$-closed set.*

(3) *$A = \{c\}$ is a G-$\mathcal{I}$-LC *-set but not an ω-$\mathcal{I}$-LC *-set.*

Example 2.2. *Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, \{b\}, \{a, d\}, \{a, b, d\}, X\}$ and $\mathcal{I} = \{\emptyset, \{b\}\}$. Then, $A = \{a\}$ is an ω-$\mathcal{I}$-LC *-set but not a weakly-$\mathcal{I}$-LC set.*

Theorem 2.1. *Let $(X, \tau, \mathcal{I})$ be an ideal space and A be an ω-$\mathcal{I}$-LC *-subset of X. Then the following hold:*

1. *If B is a $*$-closed set, then $A \cap B$ is an ω-$\mathcal{I}$-LC *-set;*
2. *If B is an ω-open set, then $A \cap B$ is an ω-$\mathcal{I}$-LC *-set;*
3. *If B is an ω-$\mathcal{I}$-LC *-set, then $A \cap B$ is an ω-$\mathcal{I}$-LC *-set.*

Proof. 1. Let B be $*$-closed and A is ω-$\mathcal{I}$-LC *-set, then $A \cap B = (C \cap D) \cap B = C \cap (D \cap B)$, where $D \cap B$ is $*$-closed. Hence $A \cap B$ is an ω-$\mathcal{I}$-LC *-set.

2. Let B be ω-open and A is ω-$\mathcal{I}$-LC *-set, then $A \cap B = (C \cap D) \cap B = (C \cap B) \cap D$, where $C \cap B$ is ω-open [15]. Hence $A \cap B$ is an ω-$\mathcal{I}$-LC *-set.

3. Let A and B be ω-$\mathcal{I}$-LC *-sets, then $A \cap B = (C \cap D) \cap (U \cap V) = (C \cap U) \cap (D \cap V)$, where $C \cap U$ is ω-open and $D \cap V$ is $*$-closed. Hence $A \cap B$ is an ω-$\mathcal{I}$-LC *-set. □

Remark 1. The union of any two ω-$\mathcal{I}$-LC *-sets need not be an ω-$\mathcal{I}$-LC *-set.

Example 2.3. *Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$ and $\mathcal{I} = \{\emptyset, \{c\}\}$. Then, $A = \{a\}$ and $B = \{c\}$ are ω-$\mathcal{I}$-LC *-sets but $A \cup B = \{a, c\}$ is not an ω-$\mathcal{I}$-LC *-set.*

Definition 2.2. A subset A of an ideal space $(X, \tau, \mathcal{I})$ is said to be $\mathcal{I}^*_\omega$-closed if $A^* \subset U$ whenever $A \subset U$ and U is ω-open in X.

Proposition 2.2. *Let $(X, \tau, \mathcal{I})$ be an ideal space and A be a subset of X. If A is $\mathcal{I}^*_g$-closed, then A is $\mathcal{I}^*_\omega$-closed.*

The converse of Proposition 2.2 need not be true as seen from the following example.

Example 2.4. *Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$ and $\mathcal{I} = \{\emptyset, \{a\}\}$. Then, $A = \{c\}$ is $\mathcal{I}^*_\omega$-closed but not $\mathcal{I}^*_g$-closed.*

Definition 2.3. A subset A of an ideal space $(X, \tau, \mathcal{I})$ is said to be an $\mathcal{I}$-ω_t-set if $A = C \cap D$, where C is ω-open and D is a t-$\mathcal{I}$-set.

Proposition 2.3. *Let $(X, \tau, \mathcal{I})$ be an ideal space and A be a subset of X. If A is an ω-$\mathcal{I}$-LC^*-set, then A is an $\mathcal{I}$-ω_t-set.*

The converse of Proposition 2.3 need not be true as seen from the following example.

Example 2.5. *Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$ and $\mathcal{I} = \{\emptyset, \{a\}\}$. Then $A = \{c\}$ is an $\mathcal{I}$-ω_t-set but not an ω-$\mathcal{I}$-LC^*-set.*

Proposition 2.4. *Let $(X, \tau, \mathcal{I})$ be an ideal space and A be a subset of X. If A is an $\mathcal{I}$-ω_t-set, then A is a $C_{\mathcal{I}}$-set.*

The converse of Proposition 2.4 need not be true as seen from the following example.

Example 2.6. *Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, \{b\}, \{a, d\}, \{a, b, d\}, X\}$ and $\mathcal{I} = \{\emptyset, \{b\}\}$. Then $A = \{a, c\}$ is a $C_{\mathcal{I}}$-set but not an $\mathcal{I}$-ω_t-set.*

Remark 2. From the above, we have the following implications:

$$
\begin{array}{c}
\text{G-}\mathcal{I}\text{-LC}^*\text{-set} \\
\Uparrow \\
*\text{-closed set} \Rightarrow \omega\text{-}\mathcal{I}\text{-LC}^*\text{-set} \Rightarrow \mathcal{I}\text{-}\omega_t\text{-set} \Rightarrow C_{\mathcal{I}}\text{-set} \\
\Uparrow \\
\mathcal{I}\text{-LC set} \Rightarrow \text{weakly-}\mathcal{I}\text{-LC set}
\end{array}
$$

Theorem 2.2. *Let $(X, \tau, \mathcal{I})$ be an ideal space and A be an $\mathcal{I}$-ω_t-subset of X. Then the following hold:*

(1) *If B is a t-$\mathcal{I}$-set, then $A \cap B$ is an $\mathcal{I}$-ω_t-set;*

(2) *If B is an ω-open set, then $A \cap B$ is an $\mathcal{I}$-ω_t-set;*

(3) *If B is an $\mathcal{I}$-ω_t-set, then $A \cap B$ is an $\mathcal{I}$-ω_t-set.*

Remark 3. The union of any two $\mathcal{I}$-ω_t-sets need not be an $\mathcal{I}$-ω_t-set.

Example 2.7. *Let* $X = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$ *and* $\mathcal{I} = \{\emptyset, \{c\}\}$. *Then,* $A = \{a\}$ *and* $B = \{c\}$ *are* $\mathcal{I}$-ω_t*-sets but* $A \cup B = \{a, c\}$ *is not an* $\mathcal{I}$-ω_t*-set.*

Theorem 2.3. *The following are equivalent for a subset* A *of an ideal topological space* $(X, \tau, \mathcal{I})$*:*

(1) *A is $*$-closed;*

(2) *A is a weakly-$\mathcal{I}$-LC set and an $\mathcal{I}_g^*$-closed set;*

(3) *A is an ω-$\mathcal{I}$-LC^*-set and an $\mathcal{I}_g^*$-closed set;*

(4) *A is an ω-$\mathcal{I}$-LC^*-set and an $\mathcal{I}_\omega^*$-closed set;*

(5) *A is an $\mathcal{I}$-ω_t-set and an $\mathcal{I}_\omega^*$-closed set.*

Proof. (1)⇒(2): This is obvious.

(2)⇒(3): Follows from Proposition 2.1.

(3)⇒(4): Follows from Proposition 2.2.

(4)⇒(5): Follows from Proposition 2.3.

(5)⇒(1): Let A be an $\mathcal{I}$-ω_t-set and an $\mathcal{I}_\omega^*$- closed set. Since A is an $\mathcal{I}$-ω_t-set, $A = C \cap D$, where C is ω-open and D is a t-$\mathcal{I}$-set. Now $A \subseteq C$ and A is $\mathcal{I}_\omega^*$-closed implies $A^* \subseteq C$. Also $A \subseteq D$ and D is a t-$\mathcal{I}$-set implies $Int(D) = Int(Cl^*(D)) = Int(D \cup D^*) \supseteq Int(D) \cup Int(D^*)$. This shows that $Int(D^*) \subseteq Int(D)$. Thus $D^* \subseteq D$ and hence $A^* \subseteq D$. Therefore $A^* \subseteq C \cap D = A$. Hence A is $*$-closed. □

Remark 4. 1. The notions of weakly-$\mathcal{I}$-LC-sets and $\mathcal{I}_g^*$-closed sets are independent.

2. The notions of ω-$\mathcal{I}$-LC *-sets and $\mathcal{I}_g^*$-closed sets are independent.

3. The notions of ω-$\mathcal{I}$-LC *-sets and $\mathcal{I}_\omega^*$-closed sets are independent.

4. The notions of $\mathcal{I}$-ω_t-sets and $\mathcal{I}_\omega^*$-closed sets are independent.

Example 2.8. *Let* $X = \{a, b, c, d, e\}$, $\tau = \{\emptyset, \{a, b\}, \{c, d\}, \{a, b, c, d\}, X\}$ *and* $\mathcal{I} = \{\emptyset, \{a\}\}$. *Then*

(1) $A = \{a, b\}$ *is a weakly-*$\mathcal{I}$*-LC-set but not an* $\mathcal{I}_g^*$*-closed set.*

(2) $A = \{b, e\}$ *is an* $\mathcal{I}_g^*$*-closed set but not a weakly-*$\mathcal{I}$*-LC-set.*

Example 2.9. *Let* $X = \{a, b, c, d, e\}$, $\tau = \{\emptyset, \{a, b\}, \{c, d\}, \{a, b, c, d\}, X\}$ *and* $\mathcal{I} = \{\emptyset, \{a\}\}$. *Then*

(1) $A = \{b\}$ *is an* ω*-*$\mathcal{I}$*-LC* **-set but not an* $\mathcal{I}_g^*$*-closed set.*

(2) $A = \{a, b\}$ *is an* $\mathcal{I}_g^*$*-closed set but not an* ω*-*$\mathcal{I}$*-LC* **-set.*

Example 2.10. *Let* $X = \{a, b, c, d\}$, $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$ *and* $\mathcal{I} = \{\emptyset, \{a\}\}$. *Then*

(1) $A = \{a, b\}$ *is an* ω*-*$\mathcal{I}$*-LC* **-set but not an* $\mathcal{I}_\omega^*$*-closed set.*

(2) $A = \{c\}$ *is an* $\mathcal{I}_\omega^*$*-closed set but not an* ω*-*$\mathcal{I}$*-LC* **-set.*

Example 2.11. *Let* $X = \{a, b, c, d\}$, $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$ *and* $\mathcal{I} = \{\emptyset, \{a\}\}$. *Then*

(1) $A = \{b\}$ *is an* $\mathcal{I}$*-*ω_t*-set but not an* $\mathcal{I}_\omega^*$*-closed set.*

(2) $A = \{a, b\}$ *is an* $\mathcal{I}_\omega^*$*-closed set but not an* $\mathcal{I}$*-*ω_t*-set.*

3. Decompositions of $*$-continuity

Definition 3.1. A function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is said to be $\mathcal{I}_\omega^*$-continuous (resp. ω-$\mathcal{I}$-LC *-continuous, $\mathcal{I}$-ω_t-continuous) if $f^{-1}(V)$ is $\mathcal{I}_\omega^*$-closed (resp. ω-$\mathcal{I}$-LC *-set, $\mathcal{I}$-ω_t-set) in $(X, \tau, \mathcal{I})$ for every closed set V in (Y, σ).

Remark 5. 1. Every $*$-continuous function is weakly $\mathcal{I}$-LC-continuous.

2. Every weakly $\mathcal{I}$-LC-continuous function is ω-$\mathcal{I}$-LC *-continuous.
3. Every $*$-continuous function is $\mathcal{I}_g^*$-continuous.
4. Every $\mathcal{I}_g^*$-continuous function is $\mathcal{I}_\omega^*$-continuous.

Definition 3.2. A subset A of a topological space (X, τ) is said to be ω^*-closed if $Cl(A) \subset U$ whenever $A \subset U$ and U is ω-open in X.

Definition 3.3. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be ω^*-continuous if $f^{-1}(A)$ is ω^*-closed set in (X, τ) for every closed set A of (Y, σ).

Remark 6. 1. Every g^*-continuous function is ω^*-continuous.

2. Every ω-LC *-continuous function is ω_t-continuous.

Example 3.1. *Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$ and $\sigma = \{\emptyset, \{a, b, d\}, X\}$. Let $f : (X, \tau) \to (X, \sigma)$ be the identity map. Then for the set $V = \{c\}$, $f^{-1}(V) = \{c\}$ is ω^*-closed (resp. ω_t-set) but not g^*-closed (resp. ω-LC *-set). Hence f is ω^*-continuous (resp. ω_t-continuous) but not g^*-continuous (resp. ω-LC *-continuous).*

Proposition 3.1. *1. Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ be $\mathcal{I}^*_\omega$-continuous and $g : (Y, \sigma, \mathcal{J}) \to (Z, \eta)$ be continuous. Then $g \circ f$:$(X, \tau, \mathcal{I}) \to (Z, \eta)$ is $\mathcal{I}^*_\omega$-continuous.*

2. Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ be $\mathcal{I}^*_\omega$-continuous and $g : (Y, \sigma, \mathcal{J}) \to (Z, \eta)$ be $*$-continuous. Then $g \circ f$:$(X, \tau, \mathcal{I}) \to (Z, \eta)$ is $\mathcal{I}^*_\omega$-continuous.

Theorem 3.1. *For a function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$, the following are equivalent:*

(1) *f is $*$-continuous;*

(2) *f is weakly-$\mathcal{I}$-LC continuous and $\mathcal{I}^*_g$-continuous;*

(3) *f is ω-$\mathcal{I}$-LC *-continuous and $\mathcal{I}^*_g$-continuous;*

(4) *f is ω-$\mathcal{I}$-LC *-continuous and $\mathcal{I}^*_\omega$-continuous;*

(5) *f is $\mathcal{I}$-ω_t-continuous and $\mathcal{I}^*_\omega$-continuous.*

Proof. Immediately follows from Theorem 2.3. □

Corollary 3.1. *Let $(X, \tau, \mathcal{I})$ be an ideal space and $\mathcal{I} = \{\emptyset\}$, for a function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$, the following are equivalent:*

(1) *f is continuous;*

(2) *f is LC-continuous and g^*-continuous;*

(3) *f is ω-LC *-continuous and g^*-continuous;*

(4) *f is ω-LC *-continuous and ω^*-continuous.*

(5) *f is ω_t-continuous and ω^*-continuous.*

4. On $\mathcal{I}_\omega^*$-normal spaces

Definition 4.1. An ideal space $(X, \tau, \mathcal{I})$ is said to be $\mathcal{I}_\omega^*$-normal, if for any two disjoint closed sets F and G in $(X, \tau, \mathcal{I})$ there exist disjoint $\mathcal{I}_\omega^*$-open sets U and V such that $F \subseteq U$ and $G \subseteq V$.

Theorem 4.1. *For an ideal space $(X, \tau, \mathcal{I})$, the following are equivalent:*

(1) *$(X, \tau, \mathcal{I})$ is $\mathcal{I}_\omega^*$-normal.*

(2) *For each closed set F and for each open set V containing F, there exists an $\mathcal{I}_\omega^*$-open set U such that $F \subseteq U \subseteq Cl^*(U) \subseteq V$.*

Proof. (1)⇒(2): Let F be a closed subset of X and D be an open set such that $F \subseteq D$. Then F and $X - D$ are disjoint closed sets in X. Therefore, by hypothesis there exist disjoint $\mathcal{I}_\omega^*$-open sets U and V such that $F \subseteq U$ and $X - D \subseteq V$. Hence $F \subseteq U \subseteq X - V \subseteq D$. Now with D being open it is also ω-open and since $X - V$ is $\mathcal{I}_\omega^*$-closed, we have $F \subseteq U \subseteq Cl^*(U) \subseteq Cl^*(X - V) \subseteq D \subseteq V$.

(2)⇒(1): Let F and G be two disjoint closed subsets of X. Then by hypothesis, there exists an $\mathcal{I}_\omega^*$-open set U such that $F \subseteq U \subseteq Cl^*(U) \subseteq X - G$. If we take $W = X - Cl^*(U)$, then U and W are the required disjoint $\mathcal{I}_\omega^*$-open sets containing F and G respectively. Hence $(X, \tau, \mathcal{I})$ is $\mathcal{I}_\omega^*$-normal. □

Theorem 4.2. *Let $(X, \tau, \mathcal{I})$ be $\mathcal{I}_\omega^*$-normal. Then the following statements are true.*

(1) *If F is closed and A is an ω-closed set such that $A \cap F = \phi$, then there exist disjoint $\mathcal{I}_\omega^*$-open sets U and V such that $A \subseteq U$ and $F \subseteq V$.*

(2) *If A is closed and B is an ω-open set containing A, then there exists $\mathcal{I}_\omega^*$-open set U such that $A \subseteq Int^*(U) \subseteq U \subseteq B$.*

(3) *If A is ω-closed and B is an open set containing A, then there exists $\mathcal{I}_\omega^*$-open set U such that $A \subseteq U \subseteq Cl^*(U) \subseteq B$.*

Proof. (1) Since $A \cap F = \phi$, $A \subseteq X - F$, where $X - F$ is open and hence semi-open. Hence by hypothesis, $Cl(A) \subseteq X - F$. Since $Cl(A) \cap F = \phi$ and X is $\mathcal{I}^*_\omega$-normal, there exist disjoint $\mathcal{I}^*_\omega$-open sets U and V such that $Cl(A) \subseteq U$ and $F \subseteq V$. The proofs of (2) and (3) are similar. □

Definition 4.2. A function $f : (X, \tau, \mathcal{I}) \rightarrow (Y, \sigma, \mathcal{J})$ is said to be $\mathcal{I}^*_\omega$-irresolute if $f^{-1}(V)$ is $\mathcal{I}^*_\omega$-open in $(X, \tau, \mathcal{I})$ for every $\mathcal{J}^*_\omega$- open set V in $(Y, \sigma, \mathcal{J})$.

Theorem 4.3. *If $f : (X, \tau, \mathcal{I}) \rightarrow (Y, \sigma, \mathcal{I})$ is an $\mathcal{I}^*_\omega$-irresolute closed injection and Y is an $\mathcal{I}^*_\omega$-normal space, then X is $\mathcal{I}^*_\omega$-normal.*

Proof. Let F and G be disjoint closed sets of X. Since f is a closed injection, $f(F)$ and $f(G)$ are disjoint closed sets of Y. Now from the $\mathcal{I}^*_\omega$-normality of Y, there exist disjoint $\mathcal{I}^*_\omega$-open sets U and V such that $f(F) \subseteq U$ and $f(G) \subseteq V$. Also since, f is $\mathcal{I}^*_\omega$-irresolute, $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint $\mathcal{I}^*_\omega$-open sets containing F and G respectively. Hence by Defention 4.1, it follows that X is $\mathcal{I}^*_\omega$-normal. □

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