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A FIXED POINT THEOREM IN INTUITIONISTIC FUZZY METRIC SPACE

SUNNY CHAUHAN [(1)], M. ALAMGIR KHAN [(2)] AND B. D. PANT [(3)]

Abstract. In this paper, we prove a common fixed point theorem for occasionally weakly compatible mappings in intuitionistic fuzzy metric space. An example is furnished to support our main result. Our results improve the results of Sharma and Deshpande [Common fixed point theorems for finite number of mappings without continuity and compatibility on intuitionistic fuzzy metric spaces, Chaos Solit. Fract. 40(2009), 2242–2256] and generalize several known results existing in the literature.

1. Introduction

The notion of fuzzy sets was initially investigated by Zadeh [38] in 1965. Since then, to use this concept in topology and analysis, many authors have expansively developed the theory of fuzzy sets and applications. As a generalization of fuzzy sets, Atanassov [8] introduced the idea of intuitionistic fuzzy set. Further, Coker et al. [10, 11] introduced the idea of the topology of intuitionistic fuzzy sets. Samanta and Mondal [31, 32] introduced the definition of the intuitionistic gradation of openness. In 2004, Park [26] introduced and discussed a notion of intuitionistic fuzzy metric spaces (briefly, IFM-spaces), which is based both on the idea of intuitionistic fuzzy sets and the concept of a fuzzy metric space given by George and Veeramani [13].

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Using the idea of intuitionistic fuzzy sets, Alaca, Turkoglu and Yildiz [5] defined the notion of intuitionistic fuzzy metric spaces (briefly, IFM-spaces) as Park [26] with the help of continuous t-norms and continuous t-conorms as a generalization of fuzzy metric space due to Kramosil and Michalek [17]. Further, they [5] proved Intuitionistic fuzzy Banach and Intuitionistic fuzzy Edelstein contraction theorems, with the different definition of Cauchy sequences and completeness than the ones given in [26]. In 2006, Turkoglu, Alaca and Yildiz [36] extended the definition of compatible mappings and compatible mappings of types (α) and (β) to IFM-spaces, which is equivalent to the concept of compatible mappings under some conditions and proved common fixed point theorems in IFM-spaces. Alaca, Altun and Turkoglu [4] introduced the concept of compatible mappings type (I) and (II) and proved common fixed point theorems for four mappings in IFM-spaces. Since then, Alaca [2] weakened the notion of compatibility by using the notion of weakly compatible mappings in IFM-spaces and showed that every pair of compatible mappings is weakly compatible but reverse is not true. Many authors proved several fixed point theorems in IFM-spaces using different contractive type conditions (see [1, 3, 6, 9, 12, 14, 18, 19, 20, 21, 22, 24, 25, 27, 28, 29, 30, 35, 37]).

In 2008, Al-Thagafi and Shahzad [7] introduced the notion of occasionally weakly compatible mappings in metric spaces and showed that the notion of occasionally weakly compatible mappings is more general among all commutativity concepts.

In the present paper, we prove a common fixed point theorem for two pairs of occasionally weakly compatible mappings in IFM-space and give an example to support our main result. Our results improve the results of Sharma and Deshpande [34].

Our results generalize several fixed point theorems in following respects:

(1) The conditions on completeness of the whole space (or underlying subspaces), containment of ranges and continuity of the involved mappings are relaxed.

(2) The conditions $\lim_{t\to\infty} M(x,y,t) = 1$ and $\lim_{t\to\infty} N(x,y,t) = 0$, for all $x,y \in X$ are not used in our result.

(3) Using the notion of occasionally weakly compatible mappings which is more general than all the commutativity concepts.

2. Preliminaries

Definition 2.1. [33] A binary operation $* : [0,1]\times[0,1] \to [0,1]$ is a continuous t-norm if it satisfies the following conditions:

(1) $*$ is commutative and associative;

(2) $*$ is continuous;

(3) $a * 1 = a$ for all $a \in [0,1]$;

(4) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$, and $a,b,c,d \in [0,1]$.

Examples of t-norm are $a * b = \min\{a,b\}$ and $a * b = ab$.

Definition 2.2. [33] A binary operation $\diamond : [0,1]\times[0,1] \to [0,1]$ is a continuous t-conorm if it satisfies the following conditions:

(1) $\diamond$ is commutative and associative;

(2) $\diamond$ is continuous;

(3) $a \diamond 0 = a$ for all $a \in [0,1]$;

(4) $a \diamond b \leq c \diamond d$ whenever $a \leq c$ and $b \leq d$, and $a,b,c,d \in [0,1]$.

Examples of t-conorm are $a \diamond b = \max\{a,b\}$ and $a \diamond b = \min\{1, a+b\}$.

Remark 1. The concepts of t-norms and t-conorms are known as the axiomatic skeletons that we use for characterizing fuzzy intersections and unions respectively. These concepts were originally introduced by Menger [23] in his study of statistical metric spaces.

Atanassov [8], Kramosil and Michalek [17] and Alaca, Turkoglu and Yildiz [5] have given the next definition on IFM-spaces:

Definition 2.3. [5] A 5-tuple $(X, M, N, *, \diamond)$ is said to be an IFM-space if X is an arbitrary set, $*$ is a continuous t-norm, $\diamond$ is a continuous t-conorm and M, N are fuzzy sets on $X^2 \times (0, \infty)$ satisfying the following conditions: for all $x, y, z \in X$ and $s, t > 0$,

(1) $M(x, y, t) + N(x, y, t) \leq 1$;
(2) $M(x, y, 0) = 0$;
(3) $M(x, y, t) = 1$ if and only if $x = y$;
(4) $M(x, y, t) = M(y, x, t)$;
(5) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$;
(6) $M(x, y, \cdot) : (0, \infty) \to [0, 1]$ is left continuous;
(7) $\lim_{t\to\infty} M(x, y, t) = 1$;
(8) $N(x, y, 0) = 1$;
(9) $N(x, y, t) = 0$ if and only if $x = y$;
(10) $N(x, y, t) = N(y, x, t)$;
(11) $N(x, y, t) \diamond N(y, z, s) \geq N(x, z, t + s)$;
(12) $N(x, y, \cdot) : (0, \infty) \to [0, 1]$ is right continuous;
(13) $\lim_{t\to\infty} N(x, y, t) = 0$.

Then (M, N) is called an intuitionistic fuzzy metric on X. The functions $M(x, y, t)$ and $N(x, y, t)$ denote the degree of nearness and the degree of non-nearness between x and y with respect to t, respectively.

Remark 2. Every fuzzy metric space $(X, M, *)$ is an IFM-space of the form $(X, M, 1-M, *, \diamond)$ such that t-norm $*$ and t-conorm $\diamond$ are associated, that is, $x \diamond y = 1 - ((1 - x) * (1 - y))$ for all $x, y \in X$.

Example 2.1. [26] *(Induced intuitionistic fuzzy metric) Let* (X, d) *be a metric space. Denote* $a * b = ab$ *and* $a \diamond b = \min\{1, a + b\}$ *for all* $a, b \in [0, 1]$ *and let* M_d *and* N_d *be fuzzy sets on* $X^2 \times (0, \infty)$ *defined by*

$$M_d(x, y, t) = \frac{ht^n}{ht^n + md(x,y)},\ N_d(x, y, t) = \frac{d(x,y)}{kt^n + md(x,y)}$$

for all $h, k, m, n \in \mathbb{N}$*. Then* $(X, M_d, N_d, *, \diamond)$ *is an intuitionistic fuzzy metric space.*

Remark 3. [26] Note the above example holds even with the t-norm $a * b = \min\{a, b\}$ and the t-conorm $a \diamond b = \max\{a, b\}$ and hence (M, N) is an intuitionistic fuzzy metric with respect to any continuous t-norms and continuous t-conorms. Putting $h = k = m = n = 1$ in Example 2.1, we get

$$M_d(x, y, t) = \frac{t}{t + d(x,y)},\ N_d(x, y, t) = \frac{d(x,y)}{t + d(x,y)}$$

Then $(X, M, N, *, \diamond)$ is an IFM-space induced by the metric d. It is obvious that $N(x, y, t) = 1 - M(x, y, t)$.

In IFM-space $(X, M, N, *, \diamond)$, $M(x, y, \cdot)$ is non-decreasing and $N(x, y, \cdot)$ is non-increasing for all $x, y \in X$.

Lemma 2.1. [4] *Let* $(X, M, N, *, \diamond)$ *be an IFM-space and for all* $x, y \in X$*,* $t > 0$ *and if for a number* $k \in (0, 1)$

$$M(x, y, kt) \geq M(x, y, t) \text{ and } N(x, y, kt) \leq N(x, y, t),$$

then $x = y$*.*

Definition 2.4. [2] Two self mappings A and S of an IFM-space $(X, M, N, *, \diamond)$ are said to be weakly compatible if they commute at coincidence point, that is, if $Ax = Sx$ for some $x \in X$, then $ASx = SAx$.

The following concept due to [7] is a proper generalization of nontrivial weakly compatible maps which do have a coincidence point. The counterpart of the concept of occasionally weakly compatible mappings in IFM-space is as follows:

Definition 2.5. Two self mappings A and S of an IFM-space $(X, M, N, *, \diamond)$ are occasionally weakly compatible if and only if there is a point $x \in X$ which is a coincidence point of A and S at which A and S commute.

From the following example, it is observe that the notion of occasionally weakly compatible mappings is more general than weakly compatible mappings.

Example 2.2. *Let $(X, M, N, *, \diamond)$ be an IFM-space, where $X = [0, \infty)$ and*

$$M(x, y, t) = \tfrac{t}{t+|x-y|} \text{ and } N(x, y, t) = \tfrac{|x-y|}{t+|x-y|},$$

$M(x, y, 0) = 0$ and $N(x, y, 0) = 1$ for all $t > 0$ and $x, y \in X$. Define the self mappings A and S by

$$A(x) = \begin{cases} 1, & \text{if } 0 \leq x \leq 1; \\ \frac{x^2+1}{2}, & \text{if } x > 1. \end{cases} \quad S(x) = \begin{cases} 1, & \text{if } 0 \leq x \leq 1; \\ 2x - 1, & \text{if } x > 1. \end{cases}$$

Here, 1 and 3 are two coincidence points of A and S. It can easily see that $AS(1) = 1 = SA(1)$ but $AS(3) \neq SA(3)$. Thus A and S are occasionally weakly compatible mappings but not weakly compatible.

Lemma 2.2. [16] *Let X be a non-empty set, A and S are occasionally weakly compatible self mappings of X. If A and S have a unique point of coincidence, $w = Ax = Sx$, then w is the unique common fixed point of A and S.*

Proof. Since A and S are occasionally weakly compatible mappings, there exists a point x in X such that $Ax = Sx = w$ and $ASx = SAx$. Thus, $AAx = ASx = SAx$, which implies Ax is also a point of coincidence of A and S. Since the point of coincidence $w = Ax$ is unique by hypothesis, $SAx = AAx = Ax$, and $w = Ax$ is a common fixed point of A and S.

Moreover, if z is any common fixed point of A and S, then $z = Az = Sz = w$ by the uniqueness of the point of coincidence. $\square$

3. Results

Theorem 3.1. *Let A, B, S and T be self mappings on an IFM-space $(X, M, N, *, \diamond)$ with $t * t \geq t$ and $(1-t) \diamond (1-t) \leq 1-t$ for all $t \in [0,1]$. Further, let (A, S) and (B, T) be each occasionally weakly compatible and satisfy the following condition: there exists $k \in (0,1)$ such that*

$$\begin{aligned}[1 + aM(Sx, Ty, kt)] * M(Ax, By, kt) \geq\ & a[M(Ax, Sx, kt) * M(By, Ty, kt) \\ & + M(Ax, Ty, kt) * M(By, Sx, kt)] \\ & + \{M(Sx, Ty, t) * M(Ax, Sx, t) \\ & * M(By, Ty, t) * M(Ax, Ty, \alpha t) \\ & * M(By, Sx, 2t - \alpha t)\}\end{aligned} \tag{3.1}$$

and

$$\begin{aligned}[1 + aN(Sx, Ty, kt)] \diamond N(Ax, By, kt) \leq\ & a[N(Ax, Sx, kt) \diamond N(By, Ty, kt) \\ & + N(Ax, Ty, kt) \diamond N(By, Sx, kt)] \\ & + \{N(Sx, Ty, t) \diamond N(Ax, Sx, t) \\ & \diamond N(By, Ty, t) \diamond N(Ax, Ty, \alpha t) \\ & \diamond N(By, Sx, 2t - \alpha t)\}\end{aligned} \tag{3.2}$$

for all $x, y \in X, a \geq 0, t > 0$ and $\alpha \in (0,2)$. Then there exists a unique point $w \in X$ such that $Aw = Sw = w$ and a unique point $z \in X$ such that $Bz = Tz = z$. Moreover, $z = w$, so that there is a unique common fixed point A, B, S and T.

Proof. Since the pairs (A, S) and (B, T) are each occasionally weakly compatible, there exist points $u, v \in X$ such that $Au = Su, ASu = SAu$ and $Bv = Tv, BTv = TBv$. Now we show that $Au = Bv$.

Putting $x = u, y = v$ with $\alpha = 1$ in inequalities (3.1) and (3.2), we get

$$\begin{aligned}
[1 + aM(Su, Tv, kt)] * M(Au, Bv, kt) \geq\ & a[M(Au, Su, kt) * M(Bv, Tv, kt) \\
& + M(Au, Tv, kt) * M(Bv, Su, kt)] \\
& + \{M(Su, Tv, t) * M(Au, Su, t) \\
& * M(Bv, Tv, t) * M(Au, Tv, t) \\
& * M(Bv, Su, t)\}
\end{aligned}$$

and

$$\begin{aligned}
[1 + aN(Su, Tv, kt)] \diamond N(Au, Bv, kt) \leq\ & a[N(Au, Su, kt) \diamond N(Bv, Tv, kt) \\
& + N(Au, Tv, kt) \diamond N(Bv, Su, kt)] \\
& + \{N(Su, Tv, t) \diamond N(Au, Su, t) \\
& \diamond N(Bv, Tv, t) \diamond N(Au, Tv, t) \\
& \diamond N(Bv, Su, t)\},
\end{aligned}$$

$$\begin{aligned}
[1+aM(Au,Bv,kt)]*M(Au,Bv,kt) &\geq a[M(Au,Au,kt)*M(Bv,Bv,kt) \\
&\quad +M(Au,Bv,kt)*M(Bv,Au,kt)] \\
&\quad +\{M(Au,Bv,t)*M(Au,Au,t) \\
&\quad *M(Bv,Bv,t)*M(Au,Bv,t) \\
&\quad *M(Bv,Au,t)\} \\
&= a[1*1+M(Au,Bv,kt)*M(Bv,Au,kt)] \\
&\quad +\{M(Au,Bv,t)*1*1*M(Au,Bv,t) \\
&\quad *M(Bv,Au,t)\}
\end{aligned}$$

and

$$\begin{aligned}
[1+aN(Au,Bv,kt)]\diamond N(Au,Bv,kt) &\leq a[N(Au,Au,kt)\diamond N(Bv,Bv,kt) \\
&\quad +N(Au,Bv,kt)\diamond N(Bv,Au,kt)] \\
&\quad +\{N(Au,Bv,t)\diamond N(Au,Au,t) \\
&\quad \diamond N(Bv,Bv,t)\diamond N(Au,Bv,t) \\
&\quad \diamond N(Bv,Au,t)\} \\
&= a[0\diamond 0+N(Au,Bv,kt)\diamond N(Bv,Au,kt)] \\
&\quad +\{N(Au,Bv,t)\diamond 0\diamond 0\diamond N(Au,Bv,t) \\
&\quad \diamond N(Bv,Au,t)\},
\end{aligned}$$

then on simplification, we have

$$M(Au,Bv,kt)+aM(Au,Bv,kt) \geq aM(Au,Bv,kt)+M(Au,Bv,t)$$

and

$$N(Au, Bv, kt) + aN(Au, Bv, kt) \leq aN(Au, Bv, kt) + N(Au, Bv, t),$$

$$M(Au, Bv, kt) \geq M(Au, Bv, t)$$

and

$$N(Au, Bv, kt) \leq N(Au, Bv, t).$$

From Lemma 2.1, we get $Au = Bv$. Therefore, $Au = Su = Bv = Tv$. Moreover, if there is another point z such that $Az = Sz$. Then using the inequalities (3.1) and (3.2) it follows that $Az = Sz = Bv = Tv$, or $Au = Az$. Hence $w = Au = Su$ is the unique point of coincidence of A and S. By Lemma 2.2, w is the unique common fixed point of A and S. Similarly, there is a unique point $z \in X$ such that $z = Bz = Tz$. Suppose that $w \neq z$ then by taking $x = w, y = z$ with $\alpha = 1$ in inequalities (3.1) and (3.2), we get

$$\begin{aligned}[1 + aM(Sw, Tz, kt)] * M(Aw, Bz, kt) \geq\ & a[M(Aw, Sw, kt) * M(Bz, Tz, kt) \\ & +M(Aw, Tz, kt) * M(Bz, Sw, kt)] \\ & +\{M(Sw, Tz, t) * M(Aw, Sw, t) \\ & *M(Bz, Tz, t) * M(Aw, Tz, t) \\ & *M(Bz, Sw, t)\}\end{aligned}$$

and

$$\begin{aligned}
[1 + aN(Sw, Tz, kt)] \diamond N(Aw, Bz, kt) \quad \leq \quad & a[N(Aw, Sw, kt) \diamond N(Bz, Tz, kt) \\
& + N(Aw, Tz, kt) \diamond N(Bz, Sw, kt)] \\
& + \{N(Sw, Tz, t) \diamond N(Aw, Sw, t) \\
& \diamond N(Bz, Tz, t) \diamond N(Aw, Tz, t) \\
& \diamond N(Bz, Sw, t)\},
\end{aligned}$$

$$\begin{aligned}
[1 + aM(w, z, kt)] * M(w, z, kt) \quad \geq \quad & a[M(w, w, kt) * M(z, z, kt) + M(w, z, kt) \\
& * M(z, w, kt)] + \{M(w, z, t) * M(w, w, t) \\
& * M(z, z, t) * M(z, w, t) * M(w, z, t)\} \\
= \quad & a[1 * 1 + M(w, z, kt) * M(z, w, kt)] \\
& + \{M(w, z, t) * 1 * 1 * M(z, w, t) * M(w, z, t)\}
\end{aligned}$$

and

$$\begin{aligned}
[1 + aN(w, z, kt)] \diamond N(w, z, kt) \quad \leq \quad & a[N(w, w, kt) \diamond N(z, z, kt) + N(w, z, kt) \\
& \diamond N(z, w, kt)] + \{N(w, z, t) \diamond N(w, w, t) \\
& \diamond N(z, z, t) \diamond N(z, w, t) \diamond N(w, z, t)\} \\
= \quad & a[0 \diamond 0 + N(w, z, kt) \diamond N(z, w, kt)] \\
& + \{N(w, z, t) \diamond 0 \diamond 0 \diamond N(z, w, t) \diamond N(w, z, t)\},
\end{aligned}$$

then on simplification, we get

$$M(w, z, kt) + aM(w, z, kt) \quad \geq \quad aM(w, z, kt) + M(w, z, t)$$

and

$$N(w,z,kt)+aN(w,z,kt) \leq a[N(w,z,kt)+N(w,z,t),$$

$$M(w,z,kt) \geq M(w,z,t)$$

and

$$N(w,z,kt) \leq N(w,z,t).$$

From Lemma 2.1 we get $w = z$. Therefore, w is the unique common fixed point of A, B, S and T in X. $\square$

From Theorem 3.1 with $a = 0$, we have the following result:

Corollary 3.1. *Let A, B, S and T be self mappings on intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ with $t * t \geq t$ and $(1-t) \diamond (1-t) \leq 1-t$ for all $t \in [0,1]$. Further, let (A, S) and (B, T) be each occasionally weakly compatible and satisfy the following condition: there exists $k \in (0,1)$ such that*

$$\begin{aligned} M(Ax,By,kt) &\geq \{M(Sx,Ty,t)*M(Ax,Sx,t)*M(By,Ty,t) \\ &\quad *M(Ax,Ty,\alpha t)*M(By,Sx,2t-\alpha t)\} \end{aligned} \tag{3.3}$$

and

$$\begin{aligned} N(Ax,By,kt) &\leq \{N(Sx,Ty,t)\diamond N(Ax,Sx,t)\diamond N(By,Ty,t) \\ &\quad \diamond N(Ax,Ty,\alpha t)\diamond N(By,Sx,2t-\alpha t)\} \end{aligned} \tag{3.4}$$

for all $x, y \in X, t > 0$ and $\alpha \in (0,2)$. Then there exists a unique point $w \in X$ such that $Aw = Sw = w$ and a unique point $z \in X$ such that $Bz = Tz = z$. Moreover, $z = w$, so that there is a unique common fixed point A, B, S and T.

Now, we give an example which illustrates Theorem 3.1 as well as Corollary 3.1.

Example 3.1. *Let $X = [0, 2]$ with the metric d defined by $d(x, y) = |x - y|$ and for each $t \in [0, 1]$ define*

$$M(x,y,t) = \begin{cases} \frac{t}{t+|x-y|}, & \text{if } t > 0; \\ 0, & \text{if } t = 0. \end{cases} \quad N(x,y,t) = \begin{cases} \frac{|x-y|}{t+|x-y|}, & \text{if } t > 0; \\ 1, & \text{if } t = 0, \end{cases}$$

for all $x, y \in X$. Clearly $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space, where $*$ and $\diamond$ are continuous t-norm and continuous t-conorm respectively. Also, we define the self maps A, B, S and T by

$$A(x) = \begin{cases} \frac{x+1}{2}, & \text{if } 0 \le x \le 1; \\ 2, & \text{if } 1 < x \le 2. \end{cases} \quad S(x) = \begin{cases} 1, & \text{if } 0 \le x \le 1; \\ \frac{1}{2}, & \text{if } 1 < x \le 2. \end{cases}$$

$$B(x) = \begin{cases} 1, & \text{if } 0 \le x \le 1; \\ 0, & \text{if } 1 < x \le 2. \end{cases} \quad T(x) = \begin{cases} 1, & \text{if } 0 \le x \le 1; \\ \frac{x}{3}, & \text{if } 1 < x \le 2. \end{cases}$$

Then A, B, S and T satisfy all the conditions of Theorem 3.1 and Corollary 3.1.

First, we have

$A(1) = 1 = S(1)$ *and* $AS(1) = 1 = SA(1)$

and

$B(1) = 1 = T(1)$ *and* $BT(1) = 1 = TB(1)$,

that is, the pairs (A, S) and (B, T) are each occasionally weakly compatible. Here 1 is the unique common fixed point of A, B, S and T. On the other hand,

$$A(X) = [-\tfrac{1}{2}, 1] \cup \{2\} \nsubseteq (\tfrac{1}{3}, \tfrac{2}{3}] \cup \{1\} = T(X)$$

and

$$B(X) = \{0, 1\} \nsubseteq \{\tfrac{1}{2}, 1\} = S(X).$$

Also, all the involved mappings A, B, S and T are discontinuous at 1.

On taking $A = B$ and $S = T$ in Theorem 3.1, we get the following result:

Corollary 3.2. *Let A and S be self mappings on intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ with $t * t \ge t$ and $(1 - t) \diamond (1 - t) \le 1 - t$ for all $t \in [0, 1]$. Further, let* *the pair (A,S) is occasionally weakly compatible and satisfy the following condition: there exists $k \in (0,1)$ such that*

$$\begin{aligned}[1+aM(Sx,Sy,kt)]*M(Ax,Ay,kt) \geq\ & a[M(Ax,Sx,kt)*M(Ay,Sy,kt)\\ & +M(Ax,Sy,kt)*M(Ay,Sx,kt)]\\ & +\{M(Sx,Sy,t)*M(Ax,Sx,t)\\ & *M(Ay,Sy,t)*M(Ax,Sy,\alpha t)\\ & *M(Ay,Sx,2t-\alpha t)\}\end{aligned} \tag{3.5}$$

and

$$\begin{aligned}[1+aN(Sx,Sy,kt)]\diamond N(Ax,Ay,kt) \leq\ & a[N(Ax,Sx,kt)\diamond N(Ay,Sy,kt)\\ & +N(Ax,Ty,kt)\diamond N(By,Sx,kt)]\\ & +\{N(Sx,Ty,t)\diamond N(Ax,Sx,t)\\ & \diamond N(By,Ty,t)\diamond N(Ax,Ty,\alpha t)\\ & \diamond N(By,Sx,2t-\alpha t)\}\end{aligned} \tag{3.6}$$

for all $x,y \in X, a \geq 0, t > 0$ and $\alpha \in (0,2)$. Then A and S have a unique common fixed point in X.

From Corollary 3.2 with $a = 0$, we have the following result:

Corollary 3.3. *Let A and S be self mappings on intuitionistic fuzzy metric space $(X,M,N,*,\diamond)$ with $t*t \geq t$ and $(1-t)\diamond(1-t) \leq 1-t$ for all $t \in [0,1]$. Further, let the pair (A,S) is occasionally weakly compatible and satisfy the following condition:*

there exists $k \in (0,1)$ *such that*

$$M(Ax, Ay, kt) \geq \{M(Sx, Sy, t) * M(Ax, Sx, t) * M(Ay, Sy, t) * M(Ax, Sy, \alpha t) * M(Ay, Sx, 2t - \alpha t)\} \tag{3.7}$$

and

$$N(Ax, Ay, kt) \leq \{N(Sx, Ty, t) \diamond N(Ax, Sx, t) \diamond N(By, Ty, t) \diamond N(Ax, Ty, \alpha t) \diamond N(By, Sx, 2t - \alpha t)\} \tag{3.8}$$

for all $x, y \in X, t > 0$ *and* $\alpha \in (0,2)$*. Then* A *and* S *have a unique common fixed point in* X*.*

Remark 4. Theorem 3.1 and Corollary 3.2 improve the results of Sharma and Deshpande [34] whereas Corollary 3.1 generalizes the results of Park [30], Alaca [2], Alaca, Turkoglu and Yildiz [5], Turkoglu, Alaca, Cho and Yildiz [35], Turkoglu, Alaca and Yildiz [36], Park and Kwun [29], Alaca, Altun and Turkoglu [4], Alaca, Turkoglu and Yildiz [6], Abu-Donia and Nase [1], Alaca [3], Kumar [18], Kumar and Kutukcu [19], Pant, Kumar and Chauhan [25], Dimri and Gariya [12] and Huang, Zhu and Wen [14].

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(1) Near Nehru Training Centre, H. No. 274, Nai Basti B-14, Bijnor-246 701, Uttar Pradesh, India.

E-mail address: sun.gkv@gmail.com

(2) Department of Natural Resources Engineering and Management, University of Kurdistan, Hewler, Iraq.

E-mail address: alam3333@gmail.com

(3) Government Degree College, Champawat-262 523, Uttarakhand, India.

E-mail address: badridatt.pant@gmail.com