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COUPLED FIXED POINT THEOREMS FOR COMPATIBLE MAPS AND ITS VARIANTS IN FUZZY METRIC SPACES

SUMITRA

ABSTRACT: In this note , first we introduce the notion of common coupled coincidence point for pairs of mappings and secondly, we introduce the variants of compatible maps (compatible map of type (A) and type (P)) in fuzzy metric spaces for coupled maps. At the end, we prove some common fixed point theorems for pairs of maps that generalize the results of various authors present in literature.

1. INTRODUCTION

The notion of fuzzy sets introduced by Zadeh [12] proved a turning point in the development of mathematics. Fuzzy set theory has applications in applied sciences such as neural network theory, stability theory, mathematical programming, modelling theory, engineering sciences, medical sciences (medical genetics, nervous system), image processing, control theory, communication etc.

The work of various authors like Kramosil and Michalek[8], George and Veeramani[3], Kaleva and Seikkala [7], Bhaskar and Lakshmikantham [1], Lakshmikantham and Ćirić [9] and Zhu et al. [10] are of great importance in the theory of fixed points in fuzzy metric spaces. Consequently, the last three decades remained very productive for fuzzy mathematics and the recent literature has observed the fuzzification in almost every

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direction of mathematics such as airthmatics, topology, graph theory, probability theory, logic etc.

Bhaskar and Lakshmikantham[1], Lakshmikantham and $\acute{C}$iri$\acute{c}$ [9], Xin-Qi Hu [6], Fang[2] and Zhu et al. [10] proved some coupled fixed point theorems under contraction conditions in different spaces. The aim of this paper is to establish coupled coincidence fixed point theorems for quadruple of maps for compatible maps and its variants in fuzzu metric spaces.

2. Definition and Preliminaries

Definition 2.1

A fuzzy set A in X is a function with domain X and values in [0, 1].

Definition 2.2

A binary operation *: $[0,1] \times [0,1] \rightarrow [0,1]$ is a continuous t-norm if ([0,1], *) is a topological abelian monoid with unit 1 s.t. a * b $\leq$ c * d whenever a $\leq$ c and b $\leq$ d , $\forall$ a, b, c, d $\in$ [0,1].For instance, (i) *(a, b) = ab (ii) *(a, b) = min {a, b}.

Definition 2.3

Let $\underset{0<t<1}{sup.}\Delta(t, t) = 1$. A t-norm Δ is said to be of H-type if the family of functions $\{\Delta^m(t)\}_{m=1}^{\infty}$ is equicontinuous at t = 1, where $\Delta^1(t)$ = t, $\Delta^{m+1}(t)$ = t Δ ($\Delta^m(t)$), m= 1, 2,…., t $\in$ [0, 1].

i.e a t-norm Δ is a H-type t-norm iff for any $\lambda \in (0, 1)$, there exists $\delta(\lambda) \in (0, 1)$ such that $\Delta^m(t) > (1-\lambda)$ for all m $\in$ N, when t > (1-δ).

The t-norm Δ_M = min. is an example of t-norm of H-type.

Definition 2.4

The 3-tuple (X, M, *) is called a fuzzy metric space if X is an arbitrary set, * is a continuous t-norm and M is a fuzzy set on X2 × [0,∞) satisfying the following conditions:

(FM-1) M(x, y, 0) > 0,

(FM-2) M(x, y, t) = 1 iff x = y,

(FM-3) M(x, y, t) = M(y, x, t),

(FM-4) M(x, y, t) * M(y, z, s) ≤M(x, z, t + s),

(FM-5) M(x, y, .) : (0,∞) →[0,1] is continuous, for all x, y, z ∈ X and s, t > 0.

In present paper , we consider M to be fuzzy metric space with condition:

(FM-6) $\lim_{n \to \infty} M(x, y, t) = 1$, ∀x, y ∈ X and t > 0.

Definition 2.5

Let (X, M, *) be fuzzy metric space . A sequence {xn} in X is said to be convergent to a point x ∈ X, if $\lim_{n \to \infty} M(xn, x, t) = 1$, for all t > 0;

Cauchy sequence if $\lim_{n \to \infty} M(xn+p, xn, t) = 1$, for all t > 0 and p > 0.

Definition 2.6

A fuzzy metric space (X, M, *) is said to be complete if and only if every Cauchy sequence in X is convergent.

Lemma 2.1

M(x, y, .) is non-decreasing for all x, y ∈ X.

Definition 2.7

An element (x, y) ∈X × X is called a coupled fixed point of the mapping f: X × X → X if f(x, y) = x , f(y, x) = y.

Definition 2.8

An element (x, y) $\in X \times X$ is called a coupled coincidence point of the mappings f: $X \times X \to X$ and g: $X \to X$ if f(x, y) = g(x), f(y, x) = g(y).

Definition 2.9

An element (x, y) $\in X \times X$ is called a common coupled fixed point of the mappings f: $X \times X \to X$ and g: $X \to X$ if x = f(x, y) = g(x), y = f(y, x) = g(y).

Definition 2.10

An element x ∈ X is called a common fixed point of the mappings f: $X \times X \to X$ and g: $X \to X$ if x = f(x, x) = g(x).

Definition 2.11

Let A, B : $X \times X \to X$ and S , T : $X \to X$, be four mappings. Then, the pair of maps (B, S) and (A, T) are said to have Common Coupled Coincidence Point if there exist a, b in X such that B (a, b) = S(a) = T(a) = A(a, b) and B(b, a) = S(b) = T(b) = A(b, a).

Definition 2.12

The mappings f: $X \times X \to X$ and g: $X \to X$ are called commutative if gf(x, y) = f(gx, gy), for all x, y ∈ X.

Definition 2.13

The mappings f: $X \times X \to X$ and g: $X \to X$ are said to be compatible if

$$\lim_{n\to\infty} M(gf(x_n, y_n), f\big(g(x_n), g(y_n)\big), t) = 1,$$

$$\lim_{n\to\infty} M(gf(y_n, x_n), f\big(g(y_n), g(x_n)\big), t) = 1,$$

for all t > 0 whenever {xn} and {yn} are sequences in X, such that $\lim_{n\to\infty} f(x_n, y_n) = \lim_{n\to\infty} g(x_n) = x$, $\lim_{n\to\infty} f(y_n, x_n) = \lim_{n\to\infty} g(y_n) = y$, for some x, y ∈ X.

Now, we introduce the followings:

Definition 2.14

The mappings f: X × X → X and g: X → X are said to be compatible of type (A) if $\lim_{n\to\infty} M(f(gx_n, gy_n), g^2 x_n, t) = 1$, $\lim_{n\to\infty} M(f(gy_n, gx_n), g^2 y_n, t) = 1$ and $\lim_{n\to\infty} M\big(gf(x_n, y_n), f\big(f(x_n, y_n), f(y_n, x_n)\big), t\big) = 1$,

$\lim_{n\to\infty} M\big(gf(y_n, x_n), f\big(f(y_n, x_n), f(x_n, y_n)\big), t\big) = 1$, whenever {xn} and {yn} are sequences in X such that $\lim_{n\to\infty} f(x_n, y_n) = \lim_{n\to\infty} g(x_n) =$ x, $\lim_{n\to\infty} f(y_n, x_n) = \lim_{n\to\infty} g(y_n) =$ y for some x, y ∈ X and t > 0.

Definition 2.15

The mappings f: X × X → X and g: X → X are said to be compatible of type (P) if

$\lim_{n\to\infty} M(f(f(x_n, y_n),\ f(y_n, x_n)), g^2 x_n, t) = 1$ and

$\lim_{n\to\infty} M(f(f(y_n, x_n),\ f(x_n, y_n)), g^2 y_n, t) = 1$,

whenever {xn} and {yn} are sequences in X such that $\lim_{n\to\infty} f(x_n, y_n) = \lim_{n\to\infty} g(x_n) =$ x, $\lim_{n\to\infty} f(y_n, x_n) = \lim_{n\to\infty} g(y_n) =$ y for some x, y ∈ X and t > 0.

Lemma 2.2

Let (X, M, *) be a FM-Space and f: X × X → X, g: X → X be compatible maps. Let {xn} and {yn} be sequences in X such that

$$\lim_{n\to\infty} f(x_n, y_n) = \lim_{n\to\infty} g(x_n) = x,$$

$$\lim_{n\to\infty} f(y_n, x_n) = \lim_{n\to\infty} g(y_n) = y, \text{ for some x, y in X.}$$

If g is continuous, then

$$\lim_{n\to\infty} f(gx_n, gy_n) = g(x), \lim_{n\to\infty} f(gy_n, gx_n) = g(y).$$

Proof.

As f and g are compatible, it follows that

$\lim_{n\to\infty} M(gf(x_n, y_n), f(gx_n, gy_n), t)$ = 1, $\lim_{n\to\infty} M(gf(y_n, x_n), f(gy_n, gx_n), t)$ = 1,

for all t > 0.Using the definition of FM-Space and continuity of g, it is easy to see that

$\lim_{n\to\infty} f(gx_n, gy_n)$ = g(x), $\lim_{n\to\infty} f(gy_n, gx_n)$ = g(y).

3. Main results

Xin – Qi Hu[6] proved the following theorem for a pair of compatible maps.

Theorem . 3.1

Let (X, M, *) be a complete FM-Space, * being continuous t – norm of H-type. Let F: X × X → X and g: X → X be two mappings such that M(F(x, y), F(u, v), ϕ(t)) ≥M(gx, gu, t) * M(gy, gv, t), for all x, y, u, v in X and t > 0.

Suppose that F(X × X) ⊆ g(X) and g is continuous, F and g are compatible. Then there exists a unique x in X such that x = g(x) = F(x, x) , where ϕ : R+ → R+ is non decreasing , upper-semi-continuous from right and $\sum_{n=0}^{\infty} \phi^n(t) < +\infty$ for all t > 0.

Now, we prove the following result for quadruple maps.

Theorem 3.2

Let (X, M, *) be a Complete Fuzzy Metric Space, * being continuous t-norm of H-type with $a*b \geq ab, \forall a,b \in [0,1]$. Let A , B : X × X → X and S , T : X → X be four mappings satisfying following conditions: A(X × X) ⊆ T(X), B(X × X) ⊆ S(X),

$$M(A(x,y), B(u,v), kt)) \geq [M(Sx, Tu, t)]^{\frac{1}{2}} * [M(Sy, Tv, t)]^{\frac{1}{2}}$$

S and T are continuous, the pairs (A, S) and (B, T) are compatible, $\forall x, y, u, v \in X, t > 0, 0 < k < 1$.

Then there exists unique x in X such that A(x, x) = T(x) = B(x, x) = S(x) = x.

Proof :

By (3.1), for arbitrary points x0, y0 in X, we can choose x1, y1 in X such that T(x1) = A(x0, y0), T(y1) = A(y0, x0). Again, by (3.1), we can choose x2, y2 in X such that S(x2) = B(x1, y1) and S(y2) = B(y1, x1).

Continuing in this way, we can construct two sequences $\{z_n\}$ and $\{z'_n\}$ in X such that

(3.5) $z_{2n+1} = A(x_{2n}, y_{2n}) = T(x_{2n+1})$, $z_{2n+2} = B(x_{2n+1}, y_{2n+1}) = S(x_{2n+2})$ and

(3.6) $z'_{2n+1} = A(y_{2n}, x_{2n}) = T(y_{2n+1})$, $z'_{2n+2} = B(y_{2n+1}, x_{2n+1}) = S(y_{2n+2})$, for all

$Let, \delta_n(t) = M(z_n, z_{n+1}, t) * M(z'_n, z'_{n+1}, t) = [M(Sx_n, Tx_{n+1}, t]^{\frac{1}{2}} * [M(Sy_n, Ty_{n+1}, t]^{\frac{1}{2}}$ n ≥ 0.

Now, from (3.1),(3.5) and (3.6)

$$M(z_{2n+1}, z_{2n+2}, kt) = M(Tx_{2n+1}, Sx_{2n+2}, kt)$$

$$= M(A(x_{2n}, y_{2n}), B(x_{2n+1}, y_{2n+1}), kt) \tag{3.7}$$

$$M(z'_{2n+1}, z'_{2n+2}, kt) = M(Ty_{2n+1}, Sy_{2n+2}, kt)$$

$$\geq [M(Sx_{2n}, Tx_{2n+1}, t)]^{\frac{1}{2}} * [M(Sy_{2n}, Ty_{2n+1} t)]^{\frac{1}{2}} = \delta_{2n}(t)$$

$$= M(A(y_{2n}, x_{2n}), B(y_{2n+1}, x_{2n+1}), kt) \tag{3.8}$$

$$\geq [M(Sy_{2n}, Ty_{2n+1}, t)]^{\frac{1}{2}} * [M(Sx_{2n}, Tx_{2n+1} t)]^{\frac{1}{2}} = \delta_{2n}(t)$$

Then, operating t-norm $*$ on (3.7) and (3.8) , we get $\delta_{2n+1}(kt) \geq \delta_{2n}(t)$,Similarly , we can show $\delta_{2n+2}(kt) \geq \delta_{2n+1}(t) \geq \delta_{2n}(\frac{t}{k})$.

In general, we have

$$\delta_n(t) \geq \delta_{n-1}(\frac{t}{k}) \geq \delta_{n-2}(\frac{t}{k^2}) \geq \delta_{n-3}(\frac{t}{k^3}) \ldots\ldots\ldots\ldots \geq \delta_0(\frac{t}{k^n}) \tag{3.9}$$

Since, $\lim_{n\to\infty} \delta_0(\frac{t}{k^n}) = 1$, for all t>0 , so by (3.9) , we have (3.10) $\lim_{n\to\infty} \delta_n(t) = 1$, for all t>0. Now , we claim that, for any p (3.11) $M(Sx_n, Tx_{n+1}, t) \geq *^p \delta_{n-1}(t-kt)$ and $M(Sy_n, Ty_{n+1}, t) \geq *^p \delta_{n-1}(t-kt)$ In fact, this is true for p=1. Assume that (3.11) holds for some p. By (3.7), we have $M(z_n, z_{n+1}, kt) \geq \delta_{n-1}(t)$ and $M(z'_n, z'_{n+1}, t-kt) \geq \delta_{n-1}(t-kt)$

From (3.2) and $a*b \geq ab$, we have $M(z_{n+1}, z_{n+p+1}, kt) = M(A(x_n, y_n), B(x_{n+p}, y_{n+p}), kt)$

$$\geq [M(Sx_n, Tx_{n+p}, t)]^{\frac{1}{2}} * [M(Sy_n, Ty_{n+p}, t)]^{\frac{1}{2}} \geq *^p \delta_{n-1}(t-kt)$$

Hence by monotonicity of $*$, we have

$$M(z_n, z_{n+p+1}, t) = M(z_n, z_{n+p+1}, t-kt+kt)$$

$$\geq M(z_n, z_{n+1}, t-kt) * M(z_{n+1}, z_{n+p+1}, kt)$$

$$\geq \delta_{n-1}(t-kt) * (*^p \delta_{n-1}(t-kt)) = *^{p+1} \delta_{n-1}(t-kt)$$

Similarly, we have

$$M(z'_n, z'_{n+p+1}, t) \geq *^{p+1} \delta'_{n-1}(t-kt)$$

Therefore, by induction (3.11) holds for all p .

Suppose that t > 0 and $\varepsilon \in (0,1]$ are given. By hypothesis, $*$ is a t-norm of H-type , there exists $\eta > 0$, such that $*^p(s) > 1-\varepsilon$ (3.12) for all $s \in (1-\eta, 1)$ and for all p.

By (3.10), there exists η_0 such that $\delta_{n-1}(t-kt)>1-\eta, \forall n>\eta_0$. Hence from (3.11) and (3.12), we get $M(z_n, z_{n+p+1}, t)>1-\varepsilon$ and $M(z'_n, z'_{n+p+1}, t)>1-\varepsilon$, for all $n\geq\eta_0$ and for all p.

Therefore $\{z_n\}$ and $\{z'_n\}$ are all Cauchy sequence.

Step 1: We now show that the pairs (B, S) and (A, T) have common coupled coincidence point.

Since X is complete, there exists points a, b in X such that

$$\lim_{n\to\infty} z_n = a \text{ and } \lim_{n\to\infty} z'_n = b,$$

that is,

$$\lim_{n\to\infty} z_{2n+1} = \lim_{n\to\infty} A(x2n, y2n) = \lim_{n\to\infty} T(x2n+1) = a,$$

$$\lim_{n\to\infty} z_{2n} = \lim_{n\to\infty} B(x2n+1, y2n+1) = \lim_{n\to\infty} S(x2n+2) = a$$

and

$$\lim_{n\to\infty} z'_{2n+1} = \lim_{n\to\infty} A(y2n, x2n) = \lim_{n\to\infty} T(y2n+1) = b,$$

$$\lim_{n\to\infty} z'_{2n} = \lim_{n\to\infty} B(y2n+1, x2n+1) = \lim_{n\to\infty} S(y2n+2) = b.$$

Since, the pair (A, S) is compatible, we have

$$\lim_{n\to\infty} M(SA(x_{2n}, y_{2n}), A(Sx_{2n}, Sy_{2n}), t) = 1,$$

$$\lim_{n\to\infty} M(SA(y_{2n}, x_{2n}), A(Sy_{2n}, Sx_{2n}), t) = 1, \text{ for all } t > 0.$$

Also, the pair (B, T) is compatible, thus we have

$$\lim_{n\to\infty} M(TB(x_{2n+1}, y_{2n+1}), B(Tx_{2n+1}, Ty_{2n+1}), t) = 1,$$

$$\lim_{n\to\infty} M(TB(y_{2n+1}, x_{2n+1}), B(Ty_{2n+1}, Tx_{2n+1}), t) = 1, \text{ for all } t > 0.$$

We first show that S(a) = T(a).

Since * is a t-norm of H-type, for any $\in > 0$, there exists $\delta > 0$ such that

$$\underbrace{(1-\delta) * (1-\delta) * \dots * (1-\delta)}_{p} \geq (1-\in), \text{ for all } p \in N.$$

Since $\lim_{t\to\infty} M(x, y, t) = 1$, for all x, y in X, there exists t0 > 0 such that

M(S(a), T(a), t0) ≥$(1-\delta)$ and M(S(b), T(b), t0) ≥$(1-\delta)$.

Using condition (3.2), we have

M(S(a), T(a),kt) = $\lim_{n\to\infty}$ [M(S(Ax2n, Ay2n,), T(Bx2n+1, By2n+1), kt))]

= $\lim_{n\to\infty}$ [M(A(Sx2n, Sy2n,), B(Tx2n+1, Ty2n+1), kt)]

$$= \lim_{n\to\infty} \left[M\left(SSx_{2n},\ TTx_{2n+1},\ t_0\right)\right]^{\frac{1}{2}} * [M\left(SSy_{2n},\ TTy_{2n+1},\ t_0\right)]^{\frac{1}{2}},$$

using lemma(3.1) , with the continuity of S and T, we have

$$M(S(a), T(a), kt) \geq \left[M\left(S(a),\ T(a),\ t_0\right)\right]^{\frac{1}{2}} * \left[M\left(S(b),\ T(b),\ t_0\right)\right]^{\frac{1}{2}}$$

$$\geq (1-\delta)^{\frac{1}{2}} * (1-\delta)^{\frac{1}{2}} \geq (1-\delta) \geq 1-\in$$

Thus we have S(a) = T(a), similarly S(b) = T(b).

Now, for all t > 0, using condition (3.2), we have

M(S(a), B(a, b), kt) = M(SA(x2n, y2n), B(a, b), kt)

= M(A(Sx2n, Sy2n), B(a, b), kt)

$$\geq [M\left(SSx_{2n},\ T(a),\ t\right)]^{\frac{1}{2}} * [M\left(SSy_{2n},\ T(b),\ t\right)]^{\frac{1}{2}}$$

Letting n $\to\infty$, since the pair (A, S) is compatible and S is continuous, we get M(S(a), B(a, b), kt) ≥1, which implies that S(a) = B(a, b). Similarly, we can get that S(b) = B(b, a).

In a similar way, it is easy to check that T(a) = A(a, b) and T(b) = A(b, a).

Thus, B(a, b) = S(a) = T(a) = A(a, b) and B(b, a) = S(b) = T(b) = A(b, a).

Step 2: Next, we show that S(a) = a and S(b) = b.

Since * is a t-norm of H-type, for any ∈ > 0, there exists δ > 0 such that

$$\underbrace{(1-\delta) * (1-\delta) * \ldots * (1-\delta)}_{p} \geq (1-\epsilon), \text{ for all } p \in N.$$

Since $\lim_{t\to\infty} M(x,y,t)$ = 1, for all x, y in X, there exists t0 > 0 such that

M(a, S(a), t0) $\geq(1-\delta)$ and M(b, S(b), t0) $\geq(1-\delta)$.

Using condition (3.2), we have M(A(a, b), B(x2n+1, y2n+1), kt)

$$\geq [M(Sa,\ Tx_{2n+1},\ t)]^{\frac{1}{2}} * [M(Sb,\ Ty_{2n+1},\ t)]^{\frac{1}{2}}$$

letting n $\to\infty$, we have M(S(a), a, kt) $\geq [M(Sa,\ a,\ t)]^{\frac{1}{2}} * [M(Sb,\ b,\ t)]^{\frac{1}{2}}$

$$\geq (1-\delta)^{\frac{1}{2}} * (1-\delta)^{\frac{1}{2}} \geq (1-\delta) \geq 1-\in$$, for all t > 0 and for any ∈ > 0.

Hence, we get S(a) = a. Similarly, S(b) = b.

Therefore, B(a, b) = S(a) = a = T(a) = A(a, b) and B(b, a) = S(b) = b = T(b) = A(b, a).

Step 3: Now , we show that a = b.

Since * is a t-norm of H-type, for any ∈ > 0, there exists δ > 0 such that

$$\underbrace{(1-\delta) * (1-\delta) * \ldots * (1-\delta)}_{p} \geq (1-\epsilon), \text{ for all } p \in N.$$

Since $\lim_{t\to\infty} M(x,y,t)$ = 1, for all x, y in X, there exists t0 > 0 such that

M(a, b, t0) $\geq(1-\delta)$.

Using condition (3.2), we have M(Tx2n+1, Sy2n+2, kt) = M(A(x2n, y2n), B(y2n+1, x2n+1), kt)

$$\geq [M(Sx_{2n},\ Ty_{2n+1},\ t)]^{\frac{1}{2}} * [M(Sy_{2n},\ Tx_{2n+1},\ t)]^{\frac{1}{2}}.$$

Letting n → ∞,

M(a, b, t) $\geq (1-\delta)^{\frac{1}{2}} * (1-\delta)^{\frac{1}{2}} \geq (1-\delta) \geq 1-\in$, for all t > 0 and for any ∈ > 0.

This implies a = b. This shows that A, B, S, T have a common fixed point.

Uniqueness follows immediately from (3.2).

Remark:

On putting A = B = F and S = T = g with ϕ(t) = kt , where 0 < k < 1, in Theorem 3.2 we get the result of Xin - Qi Hu [6].

Cor. 3.1. Let (X, M, *) be a Fuzzy Metric Space, * being continuous t-norm of H-type. Let:

A , B : X × X → X and S , T : X → X be four mappings satisfying (3.1), (3.3), (3.4) with the following condition:

$M(A(x,y),B(u,v),\phi t)) \geq [M(Sx,Tu,t)]^{\frac{1}{2}} * [M(Sy,Tv,t)]^{\frac{1}{2}}$, for all x, y, u, v in X and t > 0 and: R+ → R+ is non decreasing , upper-semicontinuous from right and $\sum_{n=0}^{\infty} \phi^n(t)$ < + ∞ for all t > 0.Then there exists a unique point a in X such that

A(a, a) = S(a) = a =T(a) = B(a, a).

Taking A = B = F and S = T = g in theorem 3.2 , we have the following result:

Cor. 3.2. Let (X, M, *) be a Fuzzy Metric Space, * being continuous t-norm of H-type. Let:

F: X × X → X and g: X → X be two mappings such that

$M(F(x,y),F(u,v),\phi t)) \geq [M(gx,gu,t)]^{\frac{1}{2}} * [M(gy,gv,t)]^{\frac{1}{2}}$, for all x, y, u, v in X and t >0.

Suppose that F(X × X) ⊆ g(X) and F and g are compatible and ϕ : R+ → R+ is non decreasing , upper-semicontinuous from right and $\sum_{n=0}^{\infty} \phi^n(t)$ < + ∞ for all t > 0.If one of the range spaces of F or g is complete, then there exits a unique x in X such that x = g(x) = F(x, x).

Taking A = B = F and S = T = I in theorem 3.2 , we have the following result:

Cor. 3.3. Let (X, M, *) be a Fuzzy Metric Space, * being continuous t-norm of H-type. Let:

F: X × X → X and there exists $\phi \in \phi$ such that

$M(F(x,y),F(u,v),kt)) \geq [M(x,u,t)]^{\frac{1}{2}} * [M(y,v,t)]^{\frac{1}{2}}$ for all x, y, u, v in X, $0 < k < 1$ and t > 0.

If F(X × X) is complete, then F has a unique fixed point in X.

Theorem 3.3. Theorem 3.2 remains true if the 'compatible property' is replaced by any one (retaining the rest of the hypothesis) of the following:

- compatible of type (A) property,
- compatible of type (P) property,

Proof.

In case (A, S) is compatible of type (A), then

$\lim_{n\to\infty} M(A(Sx_n, Sy_n), S^2 x_n, t) = 1$, $\lim_{n\to\infty} M(A(Sy_n, Sx_n), S^2 y_n, t) = 1$ and

$\lim_{n\to\infty} M\big(SA(x_n, y_n), A\big(A(x_n, y_n), A(y_n, x_n)\big), t\big) = 1$,

$\lim_{n\to\infty} M\big(SA(y_n, x_n), A\big(A(y_n, x_n), A(x_n, y_n)\big), t\big) = 1$,

Whenever {xn} and {yn} are sequences in X such that $\lim_{n\to\infty} A(x_n, y_n) = \lim_{n\to\infty} S(x_n)$=x, $\lim_{n\to\infty} A(y_n, x_n) = \lim_{n\to\infty} S(y_n)$=y for some x,y ∈ X and t > 0.

Let a, b be two points in X so that A(a, b) = Sa and A(b, a) = Sb. Taking xn = a, yn = b it is easy to show that A(Sa, Sb) = SA(a, b) and A(Sb, Sa) = SA(b, a). Similarly, (B, T) commutes at all of its coincidence points. Now applying Theorem 3.2, we can conclude that A, B, S, T have a unique common fixed point. Similarly, the theorem follows if pairs are compatible of type (P).

4. An Application

Theorem 4.1. Let (X, M, *) be a fuzzy metric space, * being continuous t-norm defined by a * b = min.{a, b} for all a, b in X. Let f, g be weakly compatible self maps on X satisfying the following conditions:

(4.1) f(X) ⊆ g(X),

(4.2) M(fx, fy, k(t)) ≥M(gx, gy, t) for all x, y in X and t > 0 and 0 < k < 1.

If range space of any one of the maps f or g is complete, then f and g have a unique common fixed point in X.

Proof.

By taking A(x, y) = B(x, y) = f(x) and S(x) = T(x) = g(x) for all x, y ∈ X in theorem (3.2), we get the desired result.

REFERENCES

[1]. T. G. Bhaskar and V. Lakshmikantham, Fixed point theorems in partially ordered metric spaces and applications, Nonlinear Analysis: Theory, Methods and Applications, Vol.65, no.7, pp. 1379-1393, (2006).

[2]. J.X.Fang, Common fixed point theorems of compatible and weakly compatible maps in Menger Spaces,Nonlinear Analysis: Theory, Methods and Applications, Vol.71, no. 5-6, pp.1833-1843, (2009).

[3]. A.George and P. Veeramani, On some results in fuzzy metric spaces, Fuzzy Sets and Systems, Vol.64, no.3, pp.395-399, (1994).

[4]. M.Grabiec, Fixed points in fuzzy metric spaces, Fuzzy Sets and Systems, Vol.27, no.3, pp. 385- 389, (1988).

[5]. O.Hadžić and E.Pap, Fixed Point Theory in Probabilistic Metric Spaces, Vol.536 of Mathematics and its Applications, Kluwer Academic, Dordrecht, The Netherlands, (2001).

[6]. Xin-Qi Hu, Common Coupled Fixed Point Theorems for Contractive Mappings in Fuzzy Metric Spaces, Fixed Point Theory and Applications, Vol. 2011, article id 363716, 14 pages.

[7]. O.Kaleva and S.Seikkala, On Fuzzy Metric Spaces, Fuzzy Sets and Systems, Vol.12, pp. 215-229, (1984).

[8]. I.Kramosil and J.Michalek, Fuzzy Metric and Statistical metric Spaces, Kybernetica, Vol.11, pp. 326-334, (1975).

[9]. V. Lakshmikantham and L. Ćirić , Coupled fixed point theorems for nonlinear contractions in partially ordered metric spaces, Nonlinear Analysis: Theory, Methods and Applications, Vol.70, no.12, pp. 4341-4349, (2009).

[10]. Zhu, Xing-Hua, Xiao, Jian-Zhong , Note on "Coupled fixed point theorems for contractions in fuzzy metric spaces", Nonlinear Analysis: Theory, Methods and Applications , Ser A , Theory Methods 74 No 16 , 5475-5479 (2011) Vol .

[11]. B.Schweizer and A.Sklar, Probabilistic Metric Spaces, North Holland Series in Probability and Applied Math., Vol. 5, (1983).

[12]. L.A.Zadeh, Fuzzy Sets, Information and Control, Vol. 89, pp. 338-353, (1965).

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