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# ON TOPOLOGICAL CLOSED GRAPHS IN THE NEW FORM

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Abstract. The aim of this paper is to introduce $\widetilde{g}_\alpha$-closed graphs and strongly $\widetilde{g}_\alpha$-closed graphs using $\widetilde{g}_\alpha$-open sets and derive the relation between them.We also derive the basic properties of the strongly $\widetilde{g}_\alpha$-closed graphs in terms of $\widetilde{g}_\alpha$-continuous,weakly $\widetilde{g}_\alpha$-continuous ,$\widetilde{g}_\alpha$-irresolute, completely $\widetilde{g}_\alpha$-continuous and almost $\widetilde{g}_\alpha$-irresolute functions. As an application we introduce $\widetilde{g}_\alpha$-Hausdorff spaces, strongly G-closed spaces and present some of their properties with $\widetilde{g}_\alpha$-closed graphs.

## 1. Introduction

Jafari et al.[1] introduced $\widetilde{g}_\alpha$-closed sets in topological spaces and proved that the class of $\widetilde{g}_\alpha$-closed sets form a topology. Long[5] introduced closed graphs and Noiri introduced[4] strongly closed graphs. In this paper we have introduced $\widetilde{g}_\alpha$-closed graphs and strongly $\widetilde{g}_\alpha$-closed graphs and establihed the relationship between them. We have derived some of the properties of strongly $\widetilde{g}_\alpha$-closed graphs with the help of different types of $\widetilde{g}_\alpha$-maps. As an application we have introduced $\widetilde{g}_\alpha$-Hausdorff spaces and strongly G-closed spaecs.

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## 2. Preliminaries

We list some definitions which are useful in the following sections. The interior and the closure of a subset $A$ of $(X, \tau)$ are denoted by $Int(A)$ and $Cl(A)$, respectively. Throughout the present paper $(X, \tau)$ and $(Y, \sigma)$(or $X$ and $Y$) represent topological spaces on which no separation axiom is defined, unless otherwise mentioned.

**Definition 2.1.** A subset A of a space X is called

(i) a semi-open set [4] if $A \subseteq Cl(Int(A))$
(ii) an $\alpha$-open set [6] if $A \subseteq Int(Cl(Int(A)))$
(iii) regular open set [8] if $A = Int(Cl(A))$

The complement of a semi-open (resp $\alpha$-open and regular open) set is called a semi-closed (resp. $\alpha$-closed and regular closed) set. The $\alpha$-closure[6] (resp semi-closure[4]) of a subset A of X is denoted by $\alpha Cl(A)$ (resp sCl (A))and is defined to be the intersection of all $\alpha$-closed sets (resp. semi-closed sets) containing A.A subset A of a topological space $(X, \tau)$ is $\alpha$-closed (resp. semi-closed) if and only if $\alpha Cl(A) = A$(resp $sCl(A) = A$). The collection of all semi-open sets, $\alpha$-open sets and regular open sets are denoted by SO(X), $\alpha$O(X) and RO(X) respectively. The collection of all semi-closed sets, $\alpha$-closed sets and regular closed sets are denoted as SC(X),$\alpha$C(X) and RC(X) respectively.

**Definition 2.2.** A subset A of a topological space $(X, \tau)$ is called

(i) an $\omega$-closed set [11] if $Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in $(X, \tau)$,
(ii) a *g-closed set [15] if $Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\omega$-open in $(X, \tau)$ ,
(iii) a $^{\#}$g-semi-closed set(briefly $^{\#}$gs-closed)[16] if $sCl(A) \subseteq U$ whenever $A \subseteq U$ and U is *g-open in $(X, \tau)$,

(iv) $\widetilde{g}_\alpha$-closed set[2] if $\alpha Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $^{\#}$gs-open in $(X, \tau)$ and

(v) $\widetilde{g}$-closed set[1] if $Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $^{\#}$gs-open in $(X, \tau)$

The complement of $\omega$-closed(resp *g-closed,$^{\#}$gs-closed,$\widetilde{g}_\alpha$-closed and $\widetilde{g}$-closed)set is said to be $\omega$-open(resp *g-open,$^{\#}$gs-open,$\widetilde{g}_\alpha$-open and $\widetilde{g}$-open). The collection of all $\widetilde{g}_\alpha$-open sets and $\widetilde{g}_\alpha$-closed sets are denoted by $\widetilde{G}_\alpha O(X)$ and $\widetilde{G}_\alpha C(X)$ respectively.

**Theorem 2.1** (2). *(i) Every open set is $\widetilde{g}_\alpha$-open but not conversely.*

*(ii) Every $\alpha$-open set is $\widetilde{g}_\alpha$-open but not conversely.*

*(iii)Every $\widetilde{g}$-open set is $\widetilde{g}_\alpha$-open but not conversely.*

*(iv) Every $\widetilde{g}_\alpha$-open set is $g\alpha(\alpha g)$-open but not conversely.*

*(v) Every $\widetilde{g}_\alpha$-open set is sg(gs-open)-open but not conversely .*

**Definition 2.3.** Let $(X, \tau)$ be a topological space and E be subset of X.We define $\widetilde{g}_\alpha$-Interior of E denoted by $\widetilde{g}_\alpha$-Int(E)to be the union of all $\widetilde{g}_\alpha$-open sets contained in E.[2]

**Definition 2.4.** Let x be a point of $(X, \tau)$ and V be a subset of X. Then V is called a $\widetilde{g}_\alpha$-neighbourhood of x in $(X, \tau)$ if there exists a $\widetilde{g}_\alpha$-open set U of $(X, \tau)$ such that $x \in U \subseteq V$.[14]

**Proposition 2.1** (2). *For any two subsets A and B of $(X, \tau)$*

(1) $\widetilde{g}_\alpha\text{-}Int(A \cap B) = \widetilde{g}_\alpha\text{-}Int(A) \cap \widetilde{g}_\alpha\text{-}Int(B)$

(2) $\widetilde{g}_\alpha\text{-}Int(A \cup B) \subseteq \widetilde{g}_\alpha\text{-}Int(A) \cup \widetilde{g}_\alpha\text{-}Int(B)$

(3) *If*$A \subseteq B$*then* $\widetilde{g}_\alpha\text{-}Int(A) \subseteq \widetilde{g}_\alpha\text{-}Int(B)$

(4) $\widetilde{g}_\alpha\text{-}Int(X) = X.$

(5) $\widetilde{g}_\alpha\text{-}Int(\phi) = \phi.$

**Definition 2.5** (2)**.** Let $(X, \tau)$ be a topological space and $B \subseteq X$. We define the $\widetilde{g}_\alpha$-closure of B (briefly $\widetilde{g}_\alpha$-$Cl(B)$ to be the intersection of all $\widetilde{g}_\alpha$-closed sets containing B which is denoted by $\widetilde{g}_\alpha$-$Cl(B) = \bigcap\{A : B \subseteq A \; and A \in \widetilde{G}_\alpha C(X, \tau)\}$.

**Lemma 2.1.** *For any $B \subseteq X, B \subseteq \widetilde{g}_\alpha$-$Cl(B) \subseteq Cl(B)$[2]*

**Theorem 2.2.** *The $\widetilde{g}_\alpha$-closure is a Kuratowski closure operator on X.[2]*

**Proposition 2.2** (2)**.** *Let $(X, \tau)$ be a topological space and $B \subseteq A$. The following properties hold. i) $\widetilde{g}_\alpha$-$Cl(B)$ is the smallest $\widetilde{g}_\alpha$-closed set containing B.*
*ii) B is $\widetilde{g}_\alpha$-closed if and only if $\widetilde{g}_\alpha$-$Cl(B) = B$*

**Definition 2.6.** A function $f : (X, \tau) \to (Y, \sigma)$ is called

(i) $\widetilde{g}_\alpha$-continuous [14] if $f^{-1}(V)$ is a $\widetilde{g}_\alpha$ -open set $(X, \tau)$ for each open set V of $(Y, \sigma)$,
(ii) $\widetilde{g}_\alpha$-irresolute[12] if $f^{-1}(V)$ is a $\widetilde{g}_\alpha$-open set of $(X, \tau)$ for each $\widetilde{g}_\alpha$-open set V of $(Y, \sigma)$,

**Proposition 2.3** (14)**.** *A function $f : (X, \tau) \to (Y, \sigma)$ is said to be $\widetilde{g}_\alpha$-continuous if for each $x \in X$ and every $V \in O(Y, f(x))$ there exist a $U \in \widetilde{G}_\alpha O(X, x) \ni f(U) \subseteq V$.*

**Proposition 2.4.** *Every $\widetilde{g}_\alpha$-irresolute function is $\widetilde{g}_\alpha$-continuous.[12]*

**Definition 2.7** (5)**.** Let $f : (X, \tau) \to (Y, \sigma)$ be any function. Then the subset $G(f) = \{(x, f(x) : x \in X\}$ of the product space $(X \times Y, \tau \times \sigma)$ is called the graph of f.

**Definition 2.8** (5)**.** Let $(X, \tau)$ and $(Y, \sigma)$ are topological spaces. A function $f : (X, \tau) \to (Y, \sigma)$ is said to have the closed graph if its graph G(f) is closed in the product space $(X \times Y, \tau \times \sigma)$.

**Definition 2.9** (7)**.** Let $(X, \tau)$ and $(Y, \sigma)$ are topological spaces. A function $f : (X, \tau) \to (Y, \sigma)$ is said to have the strongly closed graph if for each $(x, y) \in X \times Y - G(f)$, there exist $U \in O(X, x), V \in O(Y, y)$ such that $U \times Cl(V) \cap G(f) = \phi$.

Remark 1. The set of all open sets containing the point x is denoted by O(X,x). Similarly $\widetilde{G}_\alpha O(X, x)$ and $\alpha O(X, x)$ denote the set of all $\widetilde{g}_\alpha$-open sets and $\alpha$-open sets containing the point x respectively. Also $RO(X) = \{V \in RO(X)/x \in V forx \in X\}$.

**Lemma 2.2** (5)**.** *A function $f : (X, \tau) \to (Y, \sigma)$ has a closed graph if for each $(x, y) \in X \times Y - G(f)$, there exist $U \in O(X, x), V \in O(Y, y)$ such that $f(U) \cap V = \phi$.*

**Lemma 2.3** (7)**.** *A function $f : (X, \tau) \to (Y, \sigma)$ has a strongly closed graph if for each $(x, y) \in X \times Y - G(f)$, there exist $U \in O(X, x), V \in O(Y, y)$ such that $f(U) \cap Cl(V) = \phi$.*

**Definition 2.10** (10)**.** A space $(X, \tau)$ is called

(i) $\widetilde{g}$-$T_0$ if for every pair of distinct points x,y in X there exists a $\widetilde{g}$-open set U containing one of the points but not the other.

(ii) $\widetilde{g}$-$T_1$ if for every pair of distinct points x,y in X there exists a $\widetilde{g}$-open set U containing x but not y and a $\widetilde{g}$-open set V containing y but not x.

(iii) $\widetilde{g}$-$T_2$ if for every pair of distinct points x,y in X there exist disjoint $\widetilde{g}$-open sets U and V containing x and y respectively.[10]

## 3. $\widetilde{g}_\alpha$-CLOSED GRAPHS

**Definition 3.1.** A function $f : (X, \tau) \to (Y, \sigma)$ is said to have a $\widetilde{g}_\alpha$-closed graph if for each $(x, y) \in X \times Y - G(f)$, there exist $U \in \widetilde{G}_\alpha O(X, x), V \in \widetilde{G}_\alpha O(Y, y)$ such that $U \times V \cap G(f) = \phi$

**Lemma 3.1.** *Let $f : (X, \tau) \to (Y, \sigma)$ be a function then the graph G(f) is $\widetilde{g}_\alpha$-closed in $X \times Y$ if and only if for each $(x, y) \in X \times Y - G(f)$, there exist $U \in \widetilde{G}_\alpha O(X, x), V \in \widetilde{G}_\alpha O(Y, y)$ such that $f(U) \cap V = \phi$*

*Proof.* It follows from the defnition. □

**Definition 3.2.** A space $(X, \tau)$ is called

(i) $\widetilde{g}_\alpha$-$T_0$ if for every pair of distinct points x,y in X there exists a $\widetilde{g}_\alpha$-open set U containing one of the points but not the other.

(ii) $\widetilde{g}_\alpha$-$T_1$ if for every pair of distinct points x,y in X there exists a $\widetilde{g}_\alpha$-open set U containing x not y and a $\widetilde{g}_\alpha$-open set V containing y but not x.

(iii) $\widetilde{g}_\alpha$-$T_2$ if for every pair of distinct points x,y in X there exist disjoint $\widetilde{g}_\alpha$-open sets U and V containing x and y respectively.

Remark 2. Every $\widetilde{g}$-$T_i$-space is $\widetilde{g}_\alpha$-$T_i$-space for $i = 0, 1, 2$.

**Example 3.1.** *Let $X = \{a, b, c\}, \tau = \{\phi, X, \{a, b\}\}$. Here X is a $\widetilde{g}_\alpha$-$T_0$-space but not a $\widetilde{g}$-$T_0$-space.*

**Theorem 3.1.** *If $f : (X, \tau) \to (Y, \sigma)$ is an injective function with the $\widetilde{g}_\alpha$-closed graph G(f) then X is $\widetilde{g}_\alpha$-$T_1$*

*Proof.* Let x and y be two distinct points of X then $f(x) \neq f(y)$. Thus $(x, f(y)) \in X \times Y - G(f)$. But G(f) is $\widetilde{g}_\alpha$-closed. So there exist $\widetilde{g}_\alpha$-open sets U and V containing x and f(y) respectively such that $f(U) \cap V = \phi$.Hence $y \notin U$. Similarly there exist $\widetilde{g}_\alpha$-open sets M and N containing y and f(x) such that $f(M) \cap N = \phi$. Hence $x \notin M$. It follows that X is $\widetilde{g}_\alpha$-$T_1$ □

**Theorem 3.2.** *If $f : (X, \tau) \to (Y, \sigma)$ is a surjective function with the $\widetilde{g}_\alpha$-closed graph G(f) then Y is $\widetilde{g}_\alpha$-$T_1$.*

*Proof.* Let y and z be two distinct points of Y. Since f is surjective there exist a point x in X such that $f(x) = z$. Therefore $(x, y) \notin G(f)$, by lemma 3.2 there exist $\widetilde{g}_\alpha$-open sets U and V containing x and y respectively such that $f(U) \cap V = \phi$. It follows that $z \notin V$. Similarly there exist $w \in X$ such that f(w)=y. Hence $(w, z) \notin G(f)$.

Similarly there exist $\widetilde{g}_\alpha$-open sets M and N containing w and z respectively such that such that $f(M) \cap N = \phi$.Thus $y \notin N$. Hence the space Y is $\widetilde{g}_\alpha$-$T_1$. □

**Definition 3.3.** A function $f : (X, \tau) \to (Y, \sigma)$ is said to be weakly $\widetilde{g}_\alpha$-continuous if for each $x \in X$ and each open set V of Y containing f(x),there exists a $\widetilde{g}_\alpha$-open set U in X containing x such that $f(U) \subseteq \widetilde{g}_\alpha$-$Cl(V)$

Remark 3. Any $\widetilde{g}_\alpha$-continuous function is weakly $\widetilde{g}_\alpha$-continuous.

**Example 3.2.** *Let $X = \{a, b, c\}, Y = \{p, q, r\}, \tau = \{\phi, X, \{a\}, \{b, c\}\}$, $\sigma = \{\phi, Y, \{p\}, \{q\}, \{p, q\}, \{p, r\}\}$. The function $f : (X, \tau) \to (Y, \sigma)$ is defined as $f(a) = p, f(b) = q, f(c) = r$. The function f is weakly $\widetilde{g}_\alpha$-continuous but not $\widetilde{g}_\alpha$-continuous.*

**Theorem 3.3.** *If a function $f : (X, \tau) \to (Y, \sigma)$ is weakly $\widetilde{g}_\alpha$-continuous and Y is Hausdorff, then G(f) is $\widetilde{g}_\alpha$-closed.*

*Proof.* Suppose that $(x, y) \notin G(f)$ then $f(x) \neq y$. Since Y is Hausdroff ,there exist open sets W and V such that $f(x) \in W, y \in V$ and $W \cap V = \phi$. It follows that $\widetilde{g}_\alpha$-$Cl(W) \cap V = \phi$.Since f is weakly $\widetilde{g}_\alpha$-continuous there exist $U \in \widetilde{G}_\alpha O(X, x)$ such that $f(U) \subseteq \widetilde{g}_\alpha - Cl(W)$. Hence $f(U) \cap V = \phi$.Thus G(f) is $\widetilde{g}_\alpha$-closed. □

**Corollary 3.1.** *If a function $f : (X, \tau) \to (Y, \sigma)$ is $\widetilde{g}_\alpha$-continuous and Y is Hausdroff, then G(f) is $\widetilde{g}_\alpha$-closed.*

**Theorem 3.4.** *If a function $f : (X, \tau) \to (Y, \sigma)$ has a $\widetilde{g}_\alpha$-closed graph then for each $x \in X, \{f(x)\} = \bigcap_{A \in \widetilde{G}_\alpha O(X,x)} \widetilde{g}_\alpha$-$Cl(f(A))$*

*Proof.* Suppose that $y \neq f(x)$ and $y \in \bigcap_{A \in \widetilde{G}_\alpha O(X,x)} \widetilde{g}_\alpha$-$Cl(f(A))$. Then $y \in \widetilde{g}_\alpha$-$Cl(f(A))$ for each $x \in A \in \widetilde{G}_\alpha O(X, x)$. This implies that for each $\widetilde{g}_\alpha$-open set B containing y, $B \cap f(A) \neq \phi$(By lemma 8.3[14]). Since$(x, y) \notin G(f)$ and G(f) is a $\widetilde{g}_\alpha$-closed graph,this is a contradiction.Hence the result. □

**Definition 3.4.** A function $f : (X, \tau) \to (Y, \sigma)$ is said to be strongly $\widetilde{g}_\alpha$-open if the image of every $\widetilde{g}_\alpha$-open set in X is $\widetilde{g}_\alpha$-open in Y.

**Theorem 3.5.** *If $f : (X, \tau) \to (Y, \sigma)$ is a surjective strongly $\widetilde{g}_\alpha$-open function with $\widetilde{g}_\alpha$-closed graph G(f) then Y is $\widetilde{g}_\alpha - T_2$.*

*Proof.* Let $y_1, y_2$ be distinct points of Y.Since f is surjective $f(x) = y_1$ for some $x \in X$ and $(x, y_2) \in X \times Y - G(f)$. This implies that there exist a $\widetilde{g}_\alpha$-open set A of X and a $\widetilde{g}_\alpha$-open set B of Y such that $(x, y_2) \in A \times B$ and $(A \times B) \cap G(f) = \phi$. We have $f(A) \cap B = \phi$. Since f is strongly $\widetilde{g}_\alpha$-open then f(A) is $\widetilde{g}_\alpha$-open such that $f(x) = y_1 \in f(A)$.Thus Y is $\widetilde{g}_\alpha - T_2$. □

**Theorem 3.6.** *If $f : (X, \tau) \to (Y, \sigma)$ is a $\widetilde{g}_\alpha$-continuous injective function and Y is $T_2$ then X is $\widetilde{g}_\alpha$-$T_2$.*

*Proof.* Let x and y in X be any pair of distinct points. Then there exist disjoint open sets U and V in Y such that $f(x) \in U, f(y) \in V$. Since f is $\widetilde{g}_\alpha$-continuous,$f^{-1}(U), f^{-1}(V)$ are $\widetilde{g}_\alpha$-open in X containing x and y respectively. We have $f^{-1}(U) \cap f^{-1}(V) = \phi$. Thus X is $\widetilde{g}_\alpha - T_2$. □

## 4. Strongly $\widetilde{g}_\alpha$-closed graphs

**Definition 4.1.** A function $f : (X, \tau) \to (Y, \sigma)$ is said to have a strongly $\widetilde{g}_\alpha$-closed graph if and only if for each $(x, y) \in X \times Y - G(f)$, there exist $U \in \widetilde{G}_\alpha O(X, x), V \in \widetilde{G}_\alpha O(Y, y)$ such that $U \times \widetilde{g}_\alpha\text{-}Cl(V) \cap G(f) = \phi$.

**Lemma 4.1.** *A function $f : (X, \tau) \to (Y, \sigma)$ has a strongly $\widetilde{g}_\alpha$-closed graph if and only for each $(x, y) \in X \times Y - G(f)$, there exist $U \in \widetilde{G}_\alpha O(X, x), V \in \widetilde{G}_\alpha O(Y, y)$ such that $f(U) \cap \widetilde{g}_\alpha\text{-}Cl(V) = \phi$.*

Remark 4. Any strongly $\widetilde{g}_\alpha$-closed graph is a $\widetilde{g}_\alpha$-closed graph but not conversely.

**Example 4.1.** *Let $X = \{a, b, c\}, \tau = \{\phi, X, \{a, b, c\}\}, Y = \{p, q, r\}$, $\sigma = \{\phi, Y, \{p, q\}\}$. The function f is defined as $f(a) = f(c) = p, f(b) = q$. The function f has a $\widetilde{g}_\alpha$-closed graph but not a strongly $\widetilde{g}_\alpha$-closed graph.*

Remark 5. If $(X, \tau)$ is $\widetilde{g}_\alpha$-$T_i$ then it is $\widetilde{g}_\alpha$-$T_{i-1}$ for i=1,2.

**Example 4.2.** *Let $X = \{a, b, c\}, \tau = \{\phi, X, \{a, b, c\}\}$. The space X is $\widetilde{g}_\alpha$-$T_0$ but not $\widetilde{g}_\alpha$-$T_1$ and $\widetilde{g}_\alpha$-$T_2$.*

**Theorem 4.1.** *If $f : (X, \tau) \to (Y, \sigma)$ is $\widetilde{g}_\alpha$-irresolute and Y is $\widetilde{g}_\alpha$-$T_2$ then G(f) is strongly $\widetilde{g}_\alpha$-closed.*

*Proof.* Let $(x, y) \in X \times Y - G(f)$. Since Y is $\widetilde{g}_\alpha$-$T_2$ there exist a $V \in \widetilde{G}_\alpha O(Y, y)$ such that $f(x) \notin \widetilde{g}_\alpha$-$Cl(V)$. Then $Y - \widetilde{g}_\alpha$-$Cl(V) \in \widetilde{G}_\alpha O(Y, f(x))$. Since f is $\widetilde{g}_\alpha$-irresolute, there exist a $U \in \widetilde{G}_\alpha O(X, x) \ni f(U) \subseteq Y - \widetilde{g}_\alpha$-$Cl(V)$(By theorem 3.11[14]) then $f(U) \cap \widetilde{g}_\alpha$-$Cl(V) = \phi$. Hence G(f) is strongly $\widetilde{g}_\alpha$-closed. □

**Theorem 4.2.** *If $f : (X, \tau) \to (Y, \sigma)$ is $\widetilde{g}_\alpha$-open and has a closed graph G(f) then G(f) is strongly $\widetilde{g}_\alpha$-closed.*

*Proof.* Let $(x, y) \in X \times Y - G(f)$. Since G(f) is closed there exist $U \in O(X, x), V \in O(Y, y)$ such that $f(U) \cap V = \phi$—-(i). Since f is $\widetilde{g}_\alpha$-open $f(U) \in \widetilde{G}_\alpha O(Y)$. From (i) we get $V \subseteq (f(U))^c$ and $f(U)^c$ is $\widetilde{g}_\alpha$-closed in Y. Therefore $\widetilde{g}_\alpha$-$Cl(V) \subseteq (f(U))^c$ and hence $f(U) \cap \widetilde{g}_\alpha$-$Cl(V) = \phi$. Hence G(f) is strongly $\widetilde{g}_\alpha$-closed. □

**Theorem 4.3.** *If $f : (X, \tau) \to (Y, \sigma)$ is surjective and has a strongly $\widetilde{g}_\alpha$-closed graph G(f) then Y is $\widetilde{g}_\alpha$-$T_2$.*

*Proof.* Let $y_1 \neq y_2 \in Y$. Since f is surjective there exist $x_1 \in X \ni f(x_1) = y_1$. Now $(x_1, y_2) \in X \times Y - G(f)$. Since G(f) is strongly $\widetilde{g}_\alpha$-closed it implies that there exist $U \in \widetilde{G}_\alpha O(X, x_1), V \in \widetilde{G}_\alpha O(Y, y_2) \ni f(U) \cap \widetilde{g}_\alpha$-$Cl(V) = \phi$ which implies that

$y_1 \notin \widetilde{g}_\alpha\text{-}Cl(V)$. This means that there exist $W \in \widetilde{G}_\alpha O(Y, y_1) \ni W \cap V = \phi$. So Y is $\widetilde{g}_\alpha\text{-}T_2$. □

**Theorem 4.4.** *A space X is $\widetilde{g}_\alpha\text{-}T_2$ if and only if the identity function $i : X \to X$ has a srtongly $\widetilde{g}_\alpha$-closed graph.*

*Proof.* Necessity.Let X be $\widetilde{g}_\alpha\text{-}T_2$. Since the identity function is $\widetilde{g}_\alpha$-irresolute by theorem 4.1 G(i) is strongly $\widetilde{g}_\alpha$-closed.
Sufficiency. Let G(i) be strongly $\widetilde{g}_\alpha$-closed. Since f is surjective by theorem 4.3 X is $\widetilde{g}_\alpha\text{-}T_2$. □

**Theorem 4.5.** *If $f : (X, \tau) \to (Y, \sigma)$ is a bijective function with strongly $\widetilde{g}_\alpha$-closed graph then both X and Y are $\widetilde{g}_\alpha\text{-}T_1$*

*Proof.* It follows from the Theorems 4.3 and 3.1. □

**Definition 4.2.** A function $f : (X, \tau) \to (Y, \sigma)$ is said to be weakly $\widetilde{g}_\alpha$-irresolute if for each point x in X and each $V \in \widetilde{G}_\alpha O(Y, f(x))$ there exists $U \in \widetilde{G}_\alpha O(X, x)$ such that $f(U) \subseteq \widetilde{g}_\alpha\text{-}Cl(V)$.

Remark 6. Any $\widetilde{g}_\alpha$-irresolute function is weakly $\widetilde{g}_\alpha$-irresolute.

**Example 4.3.** *Let $X = Y = \{a, b, c\}, \tau = \{\phi, X, \{a, c\}, \{b\}\}, \sigma = \{\phi, Y, \{a, b\}\}$. The function f is the identity function. Here f is weakly $\widetilde{g}_\alpha$-irresolute but not $\widetilde{g}_\alpha$-irresolute.*

**Theorem 4.6.** *If $f : (X, \tau) \to (Y, \sigma)$ is a weakly $\widetilde{g}_\alpha$-irresolute injective function with strongly $\widetilde{g}_\alpha$-closed graph G(f) then X is $\widetilde{g}_\alpha\text{-}T_2$*

*Proof.* Since f is injective for any pair of distinct points $x_1, x_2 \in X, f(x_1) \neq f(x_2)$. Therefore $(x_1, f(x_2)) \in X \times Y - G(f)$. Since G(f) is strongly $\widetilde{g}_\alpha$-closed there exist $U \in \widetilde{G}_\alpha O(X, x_1), V \in \widetilde{G}_\alpha O(Y, f(x_2))$ such that $f(U) \cap \widetilde{g}_\alpha\text{-}Cl(V) = \phi$ and hence $U \cap f^{-1}(\widetilde{g}_\alpha\text{-}Cl(V)) = \phi$ which gives $f^{-1}((\widetilde{g}_\alpha\text{-}Cl(V)) \subseteq X - U$. Since f is weakly

$\widetilde{g}_\alpha$-irresolute, there exist $W \in \widetilde{G}_\alpha O(X, x_2) \ni f(W) \subseteq \widetilde{g}_\alpha$-$Cl(V)$. Hence $W \subseteq f^{-1}(\widetilde{g}_\alpha$-$Cl(V)) \subseteq X - U$ which implies that $W \cap U = \phi$. Hence X is $\widetilde{g}_\alpha$-$T_2$. □

**Corollary 4.1.** *If $f : (X, \tau) \to (Y, \sigma)$ is a $\widetilde{g}_\alpha$-irresolute function with strongly $\widetilde{g}_\alpha$-closed graph then G(f) then X is $\widetilde{g}_\alpha$-$T_2$*

**Definition 4.3.** A function $f : (X, \tau) \to (Y, \sigma)$ is said to be almost $\widetilde{g}_\alpha$-irresolute if for each x in X and each $\widetilde{g}_\alpha$ neighbourhood V of f(x), $\widetilde{g}_\alpha$-$Cl(f^{-1}(V))$ is a $\widetilde{g}_\alpha$ neighbourhood of x.

Remark 7. Every $\widetilde{g}_\alpha$-irresolute function is almost $\widetilde{g}_\alpha$-irresolute.

**Example 4.4.** *Let $X = Y = \{a, b, c\}, \tau = \{\phi, X, \{a, b\}, \{c\}\}, \sigma = \{\phi, Y, \{a, b\}\}$. The function f is the identity function. Here f is almost $\widetilde{g}_\alpha$-irresolute but not $\widetilde{g}_\alpha$-irresolute.*

**Theorem 4.7.** *If a function $f : (X, \tau) \to (Y, \sigma)$ is almost $\widetilde{g}_\alpha$-irresolute ,injective with strongly $\widetilde{g}_\alpha$-closed graph then X is $\widetilde{g}_\alpha$-$T_2$.*

*Proof.* As in Theorem 4.6 we get$f(U) \cap \widetilde{g}_\alpha$-$Cl(V) = \phi$ and hence $U \cap f^{-1}(\widetilde{g}_\alpha$-$Cl((V))) = \phi$ which implies that $f^{-1}(\widetilde{g}_\alpha$-$Cl((V))) \subseteq X - U$. Since X-U is a $\widetilde{g}_\alpha$-closed set containing $f^{-1}(\widetilde{g}_\alpha$-$Cl((V)))$ and any $\widetilde{g}_\alpha$-$Cl(f^{-1}(\widetilde{g}_\alpha$-$Cl((V))))$ is the smallest $\widetilde{g}_\alpha$-closed containing $f^{-1}(\widetilde{g}_\alpha$-$Cl((V)))$, we have $\widetilde{g}_\alpha$-$Cl(f^{-1}(\widetilde{g}_\alpha$-$Cl((V)))) \subseteq X - U$. Since f is almost $\widetilde{g}_\alpha$-irresolute $\widetilde{g}_\alpha$-$Cl(f^{-1}(\widetilde{g}_\alpha$-$Cl((V))))$ is a $\widetilde{g}_\alpha$ neighbourhood of $x_2$. This implies that there exist $H \in \widetilde{G}_\alpha O(X, x_2)$ such that $H \subseteq \widetilde{g}_\alpha$-$Cl(f^{-1}(\widetilde{g}_\alpha$-$Cl((V)))) \subseteq X - U$ which implies that $U \cap H = \phi$. Thus X is $\widetilde{g}_\alpha$-$T_2$. □

**Theorem 4.8.** *If a function $f : (X, \tau) \to (Y, \sigma)$ is almost $\widetilde{g}_\alpha$-irresolute, bijective with strongly $\widetilde{g}_\alpha$-closed graph then both X and Y are $\widetilde{g}_\alpha$-$T_2$.*

*Proof.* It follws from Theorems 4.5 and 4.7. □

**Definition 4.4.** A function $f : (X, \tau) \to (Y, \sigma)$ is said to be nearly $\widetilde{g}_\alpha$-continuous if and only if for each x in X and for each $V \in RO(Y, f(x))$ there exist a $U \in \widetilde{G}_\alpha O(X, x)$ such that $f(U) \subseteq V$.

Remark 8. Every $\widetilde{g}_\alpha$-continuous function is nearly $\widetilde{g}_\alpha$-continuous.

**Example 4.5.** *Let* $X = Y = \{a, b, c\}, \tau = \{\phi, X, \{a\}\}, \sigma = \{\phi, Y, \{a, b\}\}$. *The function f is the identity function. Here f is* $\widetilde{g}_\alpha$*-irresolute but not nearly* $\widetilde{g}_\alpha$*-irresolute.*

**Theorem 4.9.** *If* $f : (X, \tau) \to (Y, \sigma)$ *is nearly* $\widetilde{g}_\alpha$*-continuous and Y is* $T_2$*, then G(f) is strongly* $\widetilde{g}_\alpha$*-closed.*

*Proof.* Let $(x, y) \in X \times Y - G(f)$. Since Y is $T_2$ there exist an open set V in Y containing y such that $f(x) \notin Cl(V)$. Cl(V) is a regular closed set in Y. So $Y - Cl(V) \in RO(Y, f(x))$.By the nearly $\widetilde{g}_\alpha$-continuity of f there exist $U \in \widetilde{G}_\alpha O(X, x)$ such that $f(U) \subseteq Y - Cl(V)$ hence $f(U) \cap Cl(V) = \phi$. Hence $f(U) \cap \widetilde{g}_\alpha\text{-}Cl(V) = \phi$. □

**Corollary 4.2.** *If* $f : (X, \tau) \to (Y, \sigma)$ *is* $\widetilde{g}_\alpha$*-continuous and Y is* $T_2$*, then G(f) is strongly* $\widetilde{g}_\alpha$*-closed.*

## 5. Applications

**Definition 5.1.** A space $(X, \tau)$ is called strongly G-closed if every $\widetilde{g}_\alpha$-open cover of X has a finite subfamily such that the union of their $\widetilde{g}_\alpha$-closures cover X.

**Definition 5.2.** A subset A of X is said to be strongly G-closed relative to X,if every cover of A by $\widetilde{g}_\alpha$-open sets of X has a finite subfamily such that the union of their $\widetilde{g}_\alpha$-closures cover X.

**Definition 5.3.** A subset A of $(X, \tau)$ is said to be $\widetilde{g}_\alpha$-clopen if A is both $\widetilde{g}_\alpha$-open and $\widetilde{g}_\alpha$-closed.

**Lemma 5.1.** *Every $\widetilde{g}_\alpha$-clopen subset of a strongly G-closed space X is strongly G-closed relative to X.*

*Proof.* Let A be any $\widetilde{g}_\alpha$-clopen subset of a storngly G-closed space X.Let $\{G_\alpha : \alpha \in I\}$ be any cover of A by $\widetilde{g}_\alpha$-open sets in X. Then $\Omega = \{G_\alpha : \alpha \in I\} \cup \{A^c\}$ is a cover of X by $\widetilde{g}_\alpha$-open sets in X. Because of the strong G closedness of X there exist a finite subfamily $E = \{G_{\alpha_i} : i = 1, 2, ...n\} \cup \{A^c\}$ of $\Omega$ that covers X. Since A is $\widetilde{g}_\alpha$-clopen we find that $\{\widetilde{g}_\alpha\text{-}Cl(G_{\alpha_i}) : i = 1, 2, ..n\}$ covers A. Hence A is strongly G-closed relative to X. □

**Definition 5.4.** A space $(X, \tau)$ is called extremally $\widetilde{g}_\alpha$-disconnected if the $\widetilde{g}_\alpha$ closure of every $\widetilde{g}_\alpha$-open set is $\widetilde{g}_\alpha$-open.

**Example 5.1.** *Let* $X = \{a, b, c, d\}, \tau = \{\phi, X, \{b\}, \{a, b\}, \{b, c, d\}\}$. $\widetilde{G}_\alpha O(X) = \{\phi, X, \{b\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}\}$. *The space X is extremally $\widetilde{g}_\alpha$-disconnected.*

**Theorem 5.1.** *If Y is strongly G-closed and extremally $\widetilde{g}_\alpha$-disconnected and $\widetilde{g}_\alpha$-$T_2$ then the function $f : (X, \tau) \to (Y, \sigma)$ with G(f) strongly $\widetilde{g}_\alpha$-closed is weakly $\widetilde{g}_\alpha$-irresolute.*

*Proof.* Let $x \in X$, $V \in \widetilde{G}_\alpha O(Y, f(x))$. Take any $y \in Y - \widetilde{g}_\alpha\text{-}Cl(V)$. Then $(x, y) \in X \times Y - G(f)$. Since G(f) is strongly $\widetilde{g}_\alpha$-closed , there exist $U_y \in \widetilde{G}_\alpha O(X, x), V_y \in \widetilde{G}_\alpha O(Y, y)$ such that $f(U_y) \cap \widetilde{g}_\alpha\text{-}Cl(V_y) = \phi$——(i). Since Y is $\widetilde{g}_\alpha$-$T_2$, there exist $V_y \in \widetilde{G}_\alpha O(Y, y)$ such that $f(x) \notin \widetilde{g}_\alpha\text{-}Cl(V_y)$. Since Y is extremally $\widetilde{g}_\alpha$-disconnected implies the $\widetilde{g}_\alpha$-clopennes of $\widetilde{g}_\alpha\text{-}Cl(V)$ and hence $Y - \widetilde{g}_\alpha\text{-}Cl(V)$ is also $\widetilde{g}_\alpha$-clopen. Now $\{V_y : y \in Y - \widetilde{g}_\alpha\text{-}Cl(V)\}$ is a cover of $Y - \widetilde{g}_\alpha\text{-}Cl(V)\}$ by $\widetilde{g}_\alpha$-open sets in Y. By the Lemma 5.1 there exist a finite subfamily $\{V_{y_i} : i = 1, 2..n\}$ such that $Y - \widetilde{g}_\alpha\text{-}Cl(V) \subseteq \bigcup_{i=1}^{n} \widetilde{g}_\alpha\text{-}Cl(V_{y_i})$. Let $W = \bigcap_{i=1}^{i=n} U_{y_i}(x)$ where $U_{y_i}(x)$ are $\widetilde{g}_\alpha$-open sets in X satisfying (i). Since finite intersection of $\widetilde{g}_\alpha$-open set is $\widetilde{g}_\alpha$-open[1], we have $f(W) \cap (Y - \widetilde{g}_\alpha\text{-}Cl(V)) \subseteq f(\bigcap_{i=1}^{n} U_{y_i}(x)) \cap \bigcup_{i=1}^{n}(\widetilde{g}_\alpha\text{-}Cl(V_{yi})) = \bigcup_{i=1}^{n}(f(U_{y_i}(x)) \cap \widetilde{g}_\alpha\text{-}Cl(V_{yi}) = \phi$ by (i). Therefore $f(W) \subseteq \widetilde{g}_\alpha\text{-}Cl(V))$ and hence f is weakly $\widetilde{g}_\alpha$-irresolute. □

**Corollary 5.1.** *If Y is strongly G-closed,extremally $\widetilde{g}_\alpha$-disconnected space then the surjective function $f : (X, \tau) \to (Y, \sigma)$ with strongly $\widetilde{g}_\alpha$-closed graph is weakly $\widetilde{g}_\alpha$-irresolute.*

*Proof.* By the Theorem 4.3 Y is $\widetilde{g}_\alpha$-$T_2$ and by the Theorem 5.1 f is weakly $\widetilde{g}_\alpha$-irresolute. □

**Theorem 5.2.** *Let the function $f : (X, \tau) \to (Y, \sigma)$ have a strongly $\widetilde{g}_\alpha$-closed graph G(f). Then f satisfies the following property.*
*$P^*$. For each set A strongly G-closed relative to Y, $f^{-1}(A)$ is $\widetilde{g}_\alpha$-closed in X.*

*Proof.* If $f^{-1}(A)$ is not $\widetilde{g}_\alpha$-closed in X. Then there exists $x \in \widetilde{g}_\alpha Cl(f^{-1}(A)) - f^{-1}(A)$. Let $y \in A$. Then $(x, y) \in X \times Y - G(f)$. Since G(f) is strongly $\widetilde{g}_\alpha$-closed, there exist of $U_y(x) \in \widetilde{G}_\alpha O(X, x), V_y \in \widetilde{G}_\alpha O(Y, y)$ such that $f(U_y(x)) \cap \widetilde{g}_\alpha\text{-}Cl(V_y) = \phi$—-(i). $\{V_y : y \in A\}$ is a cover of A by $\widetilde{g}_\alpha$-open sets in Y. The strong G-closedness of A relative to Y implies the existence of $\widetilde{g}_\alpha$-open sets $V_{y_1}, V_{y_2}, ...V_{y_n}$ in Y such that $A \subseteq \bigcup_{i=1}^{n} \widetilde{g}_\alpha\text{-}Cl(V_{y_i})$. Let$\{U_{y_i}(x) : i = 1, 2, ..n\}$ be the corresponding $\widetilde{g}_\alpha$-open sets in X satisfying (i). Let $U = \cap_{i=1}^{n}\{U_{y_i}(x) : i = 1, 2, ..n\}$. Then $U \in \widetilde{G}_\alpha O(X, x)$. Since the finite intersection of $\widetilde{g}_\alpha$-open sets is $\widetilde{g}_\alpha$-open. We have, $f(U) \cap A \subseteq f(\cap_{i=1}^{n} U_{y_i}(x)) \cap (\bigcup_{i=1}^{n})\widetilde{g}_\alpha\text{-}Cl(V_{y_i}) = \bigcup_{i=1}^{n} f(U_{y_i}(x)) \cap \widetilde{g}_\alpha\text{-}Cl(V_{y_i}) = \phi$. But $x \in \widetilde{g}_\alpha\text{-}Cl(f^{-1}(A)) \Rightarrow U \cap f^{-1}(A) \neq \phi$ a contradiction to our assumption to the above result. Hence our assumption is wrong. □

**Theorem 5.3.** *For a $\widetilde{g}_\alpha$-$T_2$ space X ,every strongly G-closed subset relative to X is $\widetilde{g}_\alpha$-closed.*

*Proof.* Let X be a $\widetilde{g}_\alpha$-$T_2$ space and A be a strongly G-closed subset relative to X. Let $x \in A$ and $y \in X - A$. Since X is $\widetilde{g}_\alpha$-$T_2$ and $x \neq y$ there exist $U_x \in \widetilde{G}_\alpha O(X, x), V \in \widetilde{G}_\alpha O(Y, y)$ such that $U_x \cap V = \phi \Rightarrow \widetilde{g}_\alpha$-$Cl(U_x) \cap V = \phi \Rightarrow y \notin \widetilde{g}_\alpha - Cl(U_x) \Rightarrow \widetilde{g}_\alpha$-$Cl(U_x) \subseteq X - \{y\}$. $\{U_x : x \in A\}$ is a cover of A by $\widetilde{g}_\alpha$-open sets in X. Since A is strongly G-closed relative to X there exist a finite subcover $\{U_{x_i} : i = 1, 2..n\}$ such that $A \subseteq \bigcup_{i=1}^{n} \widetilde{g}_\alpha$-$Cl(U_{x_i}) \subseteq \widetilde{g}_\alpha$-$Cl(\bigcup_{i=1}^{n} U_{x_i}) = \widetilde{g}_\alpha$-$Cl(U)$ where $U = \bigcup_{i=1}^{n} U_{x_i}$. Hence $y \notin \widetilde{g}_\alpha$-$Cl(U)$. If $y \in \widetilde{g}_\alpha$-$Cl(U)$ then $y \in \widetilde{g}_\alpha - Cl(\bigcup_{i=1}^{n} U_{x_i}) \Rightarrow (\bigcup_{i=1}^{n} U_{x_i}) \cap V \neq \phi \Rightarrow \bigcup_{i=1}^{n} (U_{x_i} \cap V) \neq \phi$ which is a contradiction. Therefore $A \subseteq \widetilde{g}_\alpha$-$Cl(U) \subseteq X - \{y\}$. Therefore $A = \bigcap_{y \in X - A} (X - \{y\}) = \bigcap_{y \in X - A} \widetilde{g}_\alpha$-$Cl(U_y)$. Hence A is $\widetilde{g}_\alpha$-closed in Y. □

## References


[1] Jafari S,Noiri T,Rajesh N and Thivagar M.L.,Another generalized closed sets,Kochi J.Math., 3(2008),25-38.

[2] Jafari S,Thivagar M.L and Nirmala Rebecca P,Remarks on $\widetilde{g}_\alpha$-closed sets,International Mathematical Forum,**5**(2010), 1167-1178.

[3] Jancovic D.S.,A note on mappings of extremally disconnectd spacs,Acta Math. Hung.,**46** (1985),83-92.

[4] Levine N,Semi-open sets and semi-conitnuity in topological spaces, Amer. Math. Monthly, 70(1963),36-41.

[5] Long P.E,Functions with closed graphs,Amer. Math. Monthly,**76**(1969),930-932.

[6] Njastad O,On some classes of nearly open sets,Pacific J.Math., 15(1965),961-970.

[7] Noiri T,On functions with strongly closed graphs,Acta Sci. Hungar. Tomus,**32**(1978),1-4.

[8] Palaniappan N and Chandrasekhara Rao,Regular Generalised closed set,Kyungpook Mathematical Journal,**33**(1980),161-169.

[9] Reilly I.L and Vamanamurthy M.K,On $\alpha$-continuity in topological spaces,Acta Math. Hung.,**45**(1985),27-32.

[10] , Sarsak M and Rajesh N,Weak separation axioms via $\widetilde{g}$-open sets,Far East Journal of Mathematical Sciences,Vol30, Issue 3,(2008),457-471.

[11] Sundaram P and Sheik John M,On $\omega$-closed sets in Topology,Acta Ciencia Indica,**4**(2000), 389-392.

[12] Thivagar M.L and Nirmala Rebecca P,Remarks on $\widetilde{g}_\alpha$-irresolute maps,Bol.Soc.Paran.Mat.,v29(1)(2011),49-60.

[13] Thivagar M.L and Nirmala Rebecca P,On Topological $\widetilde{g}_\alpha$-quotient mappings,Journal of Advanced studies in Topology,1(2010),9-16.

[14] Thivagar M.L and Nirmala Rebecca P,New Types of Topological mappings ,International J.of Math.Sci and Engg.Appls.,Vol NoIV,(2011),43-61.

[15] Veera Kumar M.K.R.S,Between g* and g-closed sets,Mem.Fac. Sci.Kochi Univ Math.,**21** (2000),1-19.

[16] Veera Kumar M.K.R.S,$^{\#}$g-semi closed sets in Topological spaces,Antarctica J.Math.,2(2005),201-222.

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