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# ON $\mathcal{I}_g^{*\alpha}$-CLOSED SETS AND $\mathcal{I}_g^{*\alpha}$-CONTINUITY

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ABSTRACT. In this paper, we introduce and study the notions of $\mathcal{I}_g^{*\alpha}$-closed sets, $\mathcal{I}_g^{*\alpha}$-continuity and $\mathcal{I}_g^{*\alpha}$-normal spaces in ideal topological spaces. Also we obtain a decomposition of $*^{\alpha}$-continuity in ideal topological spaces.

## 1. INTRODUCTION AND PRELIMINARIES

An *ideal* $\mathcal{I}$ on a topological space $(X, \tau)$ is a non-empty collection of subsets of $X$ which satisfies the following properties: (1) $A \in \mathcal{I}$ and $B \subseteq A$ implies $B \in \mathcal{I}$, (2) $A \in \mathcal{I}$ and $B \in \mathcal{I}$ implies $A \cup B \in \mathcal{I}$. An ideal topological space is a topological space $(X, \tau)$ with an ideal $\mathcal{I}$ on $X$ and is denoted by $(X, \tau, \mathcal{I})$. For a subset $A \subseteq X$, $A^*(\mathcal{I}, \tau) = \{x \in X : A \cap U \notin \mathcal{I}$ for every $U \in \tau(x)\}$ is called the *local function* of $A$ with respect to $\mathcal{I}$ and $\tau$ [3]. We simply write $A^*$ in case there is no chance for confusion. A Kuratowski closure operator $cl^*(.)$ for a topology $\tau^*(\mathcal{I}, \tau)$ called the $*$-topology, finer than $\tau$ is defined by $cl^*(A) = A \cup A^*$ [9]. A subset $A$ of an ideal topological space $(X, \tau, \mathcal{I})$ is $*$-*closed* [2], if $A^* \subseteq A$. A subset $A$ of an ideal topological space $(X, \tau, \mathcal{I})$ is $\mathcal{I}_g$-*closed* [1], if $A^* \subseteq U$ whenever $A \subseteq U$ and $U$ is open in $X$.

A subset $A$ of a topological space $(X, \tau)$ is $\alpha$-*open* [7], if $A \subseteq int(cl(int(A)))$. The family of all $\alpha$-open sets denoted by $\tau^{\alpha}$. A subset $A$ of a topological space $(X, \tau)$ is *g-closed* [5], if $cl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open in $X$. A subset $A$

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of a topological space $(X, \tau)$ is *$\alpha g$-closed* [6], if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open in $X$. Let $A$ be a subset of an ideal topological space $(X, \tau, \mathcal{I})$. Then $A^{*\alpha}(\mathcal{I}, \tau) = \{x \in X : A \cap U \notin \mathcal{I}$ for every $U \in \tau^{\alpha}\{x\}$ is called *$\alpha$-local function* of $A$ with respect to $\mathcal{I}$ and $\tau^{\alpha}$. We simply write $A^{*\alpha}$ in case there is no chance for confusion. A Kuratowski $\alpha$-closure operator $cl^{*\alpha}(.)$ for a topology $\tau^{*}(\mathcal{I}, \tau)$ called the $\tau^{*\alpha}$-topology, finer than $\tau^{*}$, $\tau^{\alpha}$ and $\tau$ is defined by $cl^{*\alpha}(A) = A \cup A^{*\alpha}$. A subset $A$ of an ideal topological space $(X, \tau, \mathcal{I})$ is *$*^{\alpha}$-closed* [8], if $A^{*\alpha} \subseteq A$.

## 2. $\mathcal{I}_g^{*\alpha}$-CLOSED SETS AND $\mathcal{I}_g^{*\alpha}$-CONTINUITY

**Definition 2.1.** A subset $A$ of an ideal topological space $(X, \tau, \mathcal{I})$ is said to be *$\mathcal{I}_g^{*\alpha}$-closed*, if $A^{*\alpha} \subseteq U$ whenever $A \subseteq U$ and $U$ is open in $(X, \tau, \mathcal{I})$

**Proposition 2.2.** *Let $(X, \tau, \mathcal{I})$ be an ideal topological space and $A \subseteq X$. If $A \subseteq A^{*\alpha}$, then $A$ is $\mathcal{I}_g^{*\alpha}$-closed if and only if $A$ is $\alpha g$-closed.*

**Proof.** Proof is trivial, since $A \subseteq A^{*\alpha}$, $A^{*\alpha} = \alpha cl(A) = cl^{*\alpha}(A)$.

**Proposition 2.3.** *Let $A$ be a subset of an ideal topological space $(X, \tau, \mathcal{I})$ and $A$ is $\mathcal{I}_g^{*\alpha}$-closed. If $\mathcal{I} = \{\emptyset\}$, then $A$ is $\alpha g$-closed.*

**Proof.** Follows from the fact that $A^{*\alpha}(\{\emptyset\}) = \alpha cl(A)$. [[8], Remark 2.3 (2)]

**Remark 2.4.** *For a subsets of an ideal topological space, the following implications hold.*

$$\begin{array}{ccccc} closed & \Longrightarrow & *\text{-}closed & \Longrightarrow & *^{\alpha}\text{-}closed \\ \Downarrow & & \Downarrow & & \Downarrow \\ g\text{-}closed & \Longrightarrow & \mathcal{I}_g\text{-}closed & \Longrightarrow & \mathcal{I}_g^{*\alpha}\text{-}closed \end{array}$$

*where none of these implications is reversible as shown in* [5], [1] *and the following examples.*

**Example 2.5.** *Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{c\}, \{a, c\}, X\}$ and $\mathcal{I} = \{\emptyset, \{c\}\}$. Then*

(1) *$A = \{a\}$ is $\mathcal{I}_g^{*\alpha}$-closed but not $\mathcal{I}_g$-closed.*

(2) $A = \{c\}$ *is* $\mathcal{I}_g^{*\alpha}$*-closed but not* $\alpha g$*-closed.*

(3) $A = \{a, c\}$ *is* $\mathcal{I}_g^{*\alpha}$*-closed but not* $g$*-closed.*

**Example 2.6.** *Let* $X = \{a, b, c, d\}$, $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$ *and* $\mathcal{I} = \{\emptyset, \{c\}\}$. *Then in* $(X, \tau, \mathcal{I})$, $A = \{a, c\}$ *is* $\mathcal{I}_g^{*\alpha}$*-closed but not* $*^\alpha$*-closed.*

**Theorem 2.7.** *If a subset* $A$ *of* $(X, \tau, \mathcal{I})$ *is* $\mathcal{I}_g^{*\alpha}$*-closed, then* $A^{*\alpha} - A$ *contains no nonempty closed set.*

**Proof.** Let $A$ be an $\mathcal{I}_g^{*\alpha}$-closed set and $U$ be a closed subset of $A^{*\alpha} - A$, then $A \subseteq U^c$. Since $A$ is $\mathcal{I}_g^{*\alpha}$-closed, we have $A^{*\alpha} \subseteq U^c$. Consequently, $U \subseteq (A^{*\alpha})^c$. Hence $U \subseteq A^{*\alpha} \cap (A^{*\alpha})^c = \emptyset$.

**Theorem 2.8.** *For a subset* $A$ *of an ideal topological space* $(X, \tau, \mathcal{I})$*, the following are equivalent.*

(1) $A$ *is* $\mathcal{I}_g^{*\alpha}$*-closed.*

(2) $cl^{*\alpha}(A) \subseteq U$ *whenever* $A \subseteq U$ *and* $U$ *is open in* $X$.

(3) *For all* $x \in cl^{*\alpha}(A)$; $cl(\{x\}) \cap A \neq \emptyset$.

**Proof.** (1)⇒(2): If $A$ is $\mathcal{I}_g^{*\alpha}$-closed, then $A^{*\alpha} \subseteq U$ whenever $A \subseteq U$ and $U$ is open in $(X, \tau, \mathcal{I})$. Therefore, $cl^{*\alpha}(A) = A \cup A^{*\alpha} \subseteq U$ whenever $A \subseteq U$ and $U$ is open in $(X, \tau, \mathcal{I})$.

(2)⇒ (3): Suppose $x \in cl^{*\alpha}(A)$. If $cl(\{x\}) \cap A = \emptyset$, then $A \subseteq (cl(\{x\}))^c$ and $(cl(\{x\}))^c$ is open. By assumption, we obtain $cl^{*\alpha}(A) \subseteq (cl(\{x\}))^c$. This is contrary to $x \in cl^{*\alpha}(A)$.

(3)⇒(1): Suppose that $A$ is not a $\mathcal{I}_g^{*\alpha}$-closed set. Then there exists an open set $U$ such that $A \subseteq U$ and $A^{*\alpha} \nsubseteq U$. Then there exists a point $x \in A^{*\alpha}$ such that $x \notin U$. Then we have $\{x\} \cap U = \emptyset$ and hence $cl(\{x\}) \cap U = \emptyset$. Since $A \subseteq U$, $cl(\{x\}) \cap A = \emptyset$. This is a contradiction and hence $A$ is $\mathcal{I}_g^{*\alpha}$-closed.

**Corollary 2.9.** *A subset* $A$ *of* $(X, \tau, \mathcal{I})$ *is* $\mathcal{I}_g^{*\alpha}$*-closed if and only if* $\alpha cl(A^{*\alpha}) \subseteq U$ *whenever* $A \subseteq U$ *and* $U$ *is open in* $X$.

**Proof.** Follows from the fact that $A^{*\alpha}=\alpha cl(A^{*\alpha})$.

**Corollary 2.10.** *If a subset $A$ of $(X,\tau,\mathcal{I})$ is $\mathcal{I}_g^{*\alpha}$-closed, then $cl^{*\alpha}(A)-A$ contains no nonempty closed set.*

**Proof.** Since $cl^{*\alpha}(A)-A=A^{*\alpha}-A$, This is obvious from Theorem 2.7.

**Lemma 2.11.** *[see Lemma 2.4 of [1]] Let $\{A_\gamma:\gamma\in\Omega\}$ be a locally finite family of sets in $(X,\tau,\mathcal{I})$. Then $\cup_{\gamma\in\Omega}(A_\gamma)^{*\alpha}=(\cup_{\gamma\in\Omega}A_\gamma)^{*\alpha}$.*

**Theorem 2.12.** *Let $(X,\tau,\mathcal{I})$ be an ideal topological space. If $\{A_\gamma:\gamma\in\Omega\}$ is a locally finite family of sets and each $A_\gamma$ is $\mathcal{I}_g^{*\alpha}$-closed, then $\cup_{\gamma\in\Omega}A_\gamma$ is $\mathcal{I}_g^{*\alpha}$-closed.*

**Proof.** Let $\cup_{\gamma\in\Omega}(A_\gamma)\subseteq U$, where $U$ is open in $X$. Since $A_\gamma$ is $\mathcal{I}_g^{*\alpha}$-closed for each $\gamma\in\Omega$, then $(A_\gamma)^{*\alpha}\subseteq U$. Hence $\cup_{\gamma\in\Omega}(A_\gamma)^{*\alpha}\subseteq U$. By Lemma 2.11., $(\cup_{\gamma\in\Omega}A_\gamma)^{*\alpha}\subseteq U$. Hence $\cup_{\gamma\in\Omega}A_\gamma$ is $\mathcal{I}_g^{*\alpha}$-closed.

**Remark 2.13.** *The following example shows that finite intersection of $\mathcal{I}_g^{*\alpha}$-closed sets need not be $\mathcal{I}_g^{*\alpha}$-closed.*

**Example 2.14.** *Let $X=\{a,b,c,d\}$, $\tau=\{\emptyset,\{a\},\{b\},\{a,b\},X\}$ and $\mathcal{I}=\{\emptyset,\{c\}\}$. For $A=\{a,b,c\}$ and $B=\{a,b,d\}$, $X$ is the only open set containing $A$ and $B$ hence $A$ and $B$ are $\mathcal{I}_g^{*\alpha}$-closed but $A\cap B=\{a,b\}$ is open and $(A\cap B)^{*\alpha}=X\nsubseteq\{a,b\}$. Therefore $A\cap B$ is not an $\mathcal{I}_g^{*\alpha}$-closed set.*

**Theorem 2.15.** *If $A$ is an $\mathcal{I}_g^{*\alpha}$-closed set of $(X,\tau,\mathcal{I})$ such that $A\subseteq B\subseteq A^{*\alpha}$, then $B$ is also $\mathcal{I}_g^{*\alpha}$-closed.*

**Proof.** Let $U$ be any open set of $X$ such that $B\subseteq U$. Then $A\subseteq U$, since $A$ is $\mathcal{I}_g^{*\alpha}$-closed, $A^{*\alpha}\subseteq U$. Since $B\subseteq A^{*\alpha}$, $B^{*\alpha}\subseteq(A^{*\alpha})^{*\alpha}\subseteq A^{*\alpha}\subseteq U$ and hence $B$ is $\mathcal{I}_g^{*\alpha}$-closed.

**Theorem 2.16.** *Let $(X,\tau,\mathcal{I})$ be an ideal topological space. Then every subset of $X$ is $\mathcal{I}_g^{*\alpha}$-closed if and only if every open set is $*^\alpha$-closed.*

**Proof.** Suppose that every subset of $X$ is $\mathcal{I}_g^{*\alpha}$-closed. If $U$ is open, then $U$ is $\mathcal{I}_g^{*\alpha}$-closed and so $U^{*\alpha} \subseteq U$. Hence $U$ is $*^\alpha$-closed. Conversely, suppose that every open set is $*^\alpha$-closed. If $A \subseteq X$ and $U$ is open such that $A \subseteq U$, then $A^{*\alpha} \subseteq U^{*\alpha} \subseteq U$ and so $A$ is $\mathcal{I}_g^{*\alpha}$-closed.

**Definition 2.17.** A subset $A$ of an ideal topological space $(X, \tau, \mathcal{I})$ is said to be $\mathcal{I}_g^{*\alpha}$*-open* if and only if $A^c$ is $\mathcal{I}_g^{*\alpha}$-closed.

**Theorem 2.18.** *Let $(X, \tau, \mathcal{I})$ be an ideal topological space and $A \subseteq X$. Then $A$ is $\mathcal{I}_g^{*\alpha}$-open if and only if $F \subseteq int^{*\alpha}(A)$ whenever $F \subseteq A$ and $F$ is closed.*

**Proof.** Suppose $A$ is $\mathcal{I}_g^{*\alpha}$-open. If $F$ is closed and $F \subseteq A$, then $X - A \subseteq X - F$ and so $cl^{*\alpha}(X - A) \subseteq X - F$. Therefore $F \subseteq int^{*\alpha}(A)$. Conversely, let $U$ be an open set such that $X - A \subseteq U$, then $X - U \subseteq A$ and so $X - U \subseteq int^{*\alpha}(A)$ which implies that $cl^{*\alpha}(X - A) \subseteq U$. Therefore $X - A$ is $\mathcal{I}_g^{*\alpha}$-closed and hence $A$ is $\mathcal{I}_g^{*\alpha}$-open.

**Definition 2.19.** A subset $A$ of an ideal topological space $(X, \tau, \mathcal{I})$ is an $\mathcal{I}^{*\alpha}$*-locally closed* (briefly $\mathcal{I}^{*\alpha}$-LC)-set, if $A = U \cap V$, where $U \in \tau$ and $V$ is $*^\alpha$-closed.

**Theorem 2.20.** *A subset $A$ of an ideal topological space $(X, \tau, \mathcal{I})$ is $*^\alpha$-closed if and only if it is $\mathcal{I}^{*\alpha}$-LC-set and $\mathcal{I}_g^{*\alpha}$-closed.*

**Proof.** Necessity is trivial. We prove only sufficiency. Let $A$ be an $\mathcal{I}^{*\alpha}$-LC-set and $\mathcal{I}_g^{*\alpha}$-closed. Since $A$ is an $\mathcal{I}^{*\alpha}$-LC-set, $A = U \cap V$, where $U$ is open and $V$ is $*^\alpha$-closed. So, we have $A = U \cap V \subseteq U$. Since $A$ is $\mathcal{I}_g^{*\alpha}$-closed, $A^{*\alpha} \subseteq U$. Also $A = U \cap V \subseteq V$ and $V$ is $*^\alpha$-closed, then $A^{*\alpha} \subseteq V^{*\alpha} \subseteq V$. Consequently, we have $A^{*\alpha} \subseteq U \cap V = A$ and hence $A$ is $*^\alpha$-closed.

**Remark 2.21.** *The following examples shows that notions of an $\mathcal{I}^{*\alpha}$-LC-set and $\mathcal{I}_g^{*\alpha}$-closed are independent.*

**Example 2.22.** *Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$ and $\mathcal{I} = \{\emptyset, \{c\}\}$. Then*

(1) $A = \{a\}$ *is an* $\mathcal{I}^{*\alpha}$*-LC-set but not* $\mathcal{I}_g^{*\alpha}$*-closed.*

(2) $A = \{a, b, c\}$ *is* $\mathcal{I}_g^{*\alpha}$*-closed but not an* $\mathcal{I}^{*\alpha}$*-LC-set.*

**Definition 2.23.** A function $f : (X, \tau, \mathcal{I}) \rightarrow (Y, \sigma)$ is said to be $*^{\alpha}$*-continuous* (resp.$\mathcal{I}_g^{*\alpha}$*-continuous*, $\mathcal{I}^{*\alpha}$*-LC-continuous*), if $f^{-1}(V)$ is $*^{\alpha}$-closed (resp. $\mathcal{I}_g^{*\alpha}$-closed, an $\mathcal{I}^{*\alpha}$-LC-set) in $(X, \tau, \mathcal{I})$ for every closed set $V$ in $(Y, \sigma)$.

**Theorem 2.24.** *For a function* $f : (X, \tau, \mathcal{I}) \rightarrow (Y, \sigma)$, *the following are equivalent.*

(1) $*^{\alpha}$*-continuous.*

(2) *For each* $x \in X$ *and each* $V \in \sigma$ *containing* $f(x)$, *there exists* $U \in \tau^{*\alpha}(\mathcal{I})$ *containing* $x$ *such that* $f(U) \subseteq V$.

(3) *inverse image of every closed set is* $*^{\alpha}$*-closed.*

**Theorem 2.25.** *Let* $f : (X, \tau, \mathcal{I}) \rightarrow (Y, \sigma)$ *and* $g : (Y, \sigma) \rightarrow (Z, \mu)$, *be any two functions. Then*

(1) $g \circ f$ *is* $*^{\alpha}$*-continuous, if* $f$ *is* $*^{\alpha}$*-continuous and* $g$ *is continuous.*

(2) $g \circ f$ *is* $\mathcal{I}_g^{*\alpha}$*-continuous, if* $f$ *is* $\mathcal{I}_g^{*\alpha}$*-continuous and* $g$ *is continuous.*

**Corollary 2.26.** *A function* $f : (X, \tau, \mathcal{I}) \rightarrow (Y, \sigma)$ *is* $*^{\alpha}$*-continuous if and only if it is* $\mathcal{I}^{*\alpha}$*-LC-continuous and* $\mathcal{I}_g^{*\alpha}$*-continuous.*

**Proof.** This is an immediate consequence of Theorem 2.20.

## 3. $\mathcal{I}_g^{*\alpha}$-NORMAL SPACES

**Definition 3.1.** An ideal topological space $(X, \tau, \mathcal{I})$ is said to be $\mathcal{I}_g^{*\alpha}$*-normal*, if for any two disjoint closed sets $A$ and $B$ in $(X, \tau, \mathcal{I})$, there exist disjoint $\mathcal{I}_g^{*\alpha}$-open sets $U$, $V$ such that $A \subseteq U$ and $B \subseteq V$.

**Theorem 3.2.** *Let* $(X, \tau, \mathcal{I})$ *be an ideal topological space. Then the following are equivalent.*

(1) $(X, \tau, \mathcal{I})$ *is* $\mathcal{I}_g^{*\alpha}$*-normal.*

(2) *For each closed set $A$ and for each open set $V$ containing $A$, there exists an $\mathcal{I}_g^{*\alpha}$-open set $U$ such that $A \subseteq U \subseteq cl^{*\alpha}(U) \subseteq V$.*

**Proof.** (1) $\Rightarrow$ (2) : Let $A$ be a closed subset of $X$ and $B$ be an open set such that $A \subseteq B$. Since $A$ and $X - B$ are disjoint closed sets in $X$, there exists disjoint $\mathcal{I}_g^{*\alpha}$-open sets $U$ and $V$ such that $A \subseteq U$ and $X - B \subseteq V$. Thus $A \subseteq U \subseteq X - V \subseteq B$. Since $B$ is open and $X - V$ is $\mathcal{I}_g^{*\alpha}$-closed, $A \subseteq U \subseteq cl^{*\alpha}(U) \subseteq cl^{*\alpha}(X - V) \subseteq B \subseteq V$.
(2) $\Rightarrow$ (1) : Let $A$ and $B$ be two disjoint closed subsets of $X$. By hypothesis, there exists an $\mathcal{I}_g^{*\alpha}$-open set $U$ such that $A \subseteq U \subseteq cl^{*\alpha}(U) \subseteq X - B$. If $W{=}X - cl^{*\alpha}(U)$, then $U$ and $W$ are the required disjoint $\mathcal{I}_g^{*\alpha}$-open sets containing $A$ and $B$ respectively. So, $(X, \tau, \mathcal{I})$ is $\mathcal{I}_g^{*\alpha}$-normal.

**Theorem 3.3.** *Let $(X, \tau, \mathcal{I})$ be an $\mathcal{I}_g^{*\alpha}$-normal space.*

(1) *If $F$ is closed and $A$ is a g-closed set such that $A \cap F{=}\emptyset$, then there exist disjoint $\mathcal{I}_g^{*\alpha}$-open sets $U$ and $V$ such that $A \subseteq U$ and $F \subseteq V$.*
(2) *If $A$ is closed and $B$ is a g-open set containing $A$, then there exist $\mathcal{I}_g^{*\alpha}$-open set $U$ such that $A \subseteq int^{*\alpha}(U) \subseteq U \subseteq B$.*
(3) *If $A$ is g-closed and $B$ is an open set containing $A$, then there exist $\mathcal{I}_g^{*\alpha}$-open set $U$ such that $A \subseteq U \subseteq cl^{*\alpha}(U) \subseteq B$.*

**Proof (1).** Since $A \cap F{=}\emptyset$ implies that $A \subseteq X - F$, where $X - F$is open. Therefore by hypothesis, $cl(A) \subseteq X - F$. Since $cl(A) \cap F{=}\emptyset$ and $X$ is $\mathcal{I}_g^{*\alpha}$-normal, there exist $\mathcal{I}_g^{*\alpha}$-open sets $U$ and $V$ such that $cl(A) \subseteq U$ and $F \subseteq V$. Hence the proof.
The proof of (2) and (3) are similar.

**Definition 3.4.** A function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ is said to be $\mathcal{I}_g^{*\alpha}$-irresolute, if $f^{-1}(V)$ is $\mathcal{I}_g^{*\alpha}$-open in $(X, \tau, \mathcal{I})$ for every $\mathcal{J}_g^{*\alpha}$-open set $V$ in $(Y, \sigma, \mathcal{J})$.

**Theorem 3.5.** *Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ be a function. If $f$ is an $\mathcal{I}_g^{*\alpha}$-irresolute (resp. $\mathcal{I}_g^{*\alpha}$-continuous) closed injection and $Y$ is $\mathcal{I}_g^{*\alpha}$-normal (resp. normal), then $X$ is $\mathcal{I}_g^{*\alpha}$-normal.*

**Proof.** Let $A$ and $B$ are disjoint closed sets of $X$. Since $f$ is closed injection, $f(A)$ and $f(B)$ are disjoint closed sets of $Y$. By the $\mathcal{I}_g^{*\alpha}$-normality (resp. normality) of $Y$, there exist disjoint $\mathcal{I}_g^{*\alpha}$-open (resp. open) sets $U$ and $V$ of $Y$ such that $f(A) \subseteq U$ and $f(B) \subseteq V$. Since $f$ is $\mathcal{I}_g^{*\alpha}$-irresolute (resp. $\mathcal{I}_g^{*\alpha}$-continuous), $f^{-1}(U)$ and$f^{-1}(V)$ are disjoint $\mathcal{I}_g^{*\alpha}$-open sets containing $A$ and $B$ respectively. It follows from Theorem 3.2 that $X$ is $\mathcal{I}_g^{*\alpha}$-normal.

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