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ON (1,2)*-$r\omega$-CLOSED SETS AND (1,2)*-$r\omega$-OPEN SETS

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Abstract. The aim of this paper is to introduce the concept of (1,2)*-rω-closed sets in bitopological spaces and study some of its properties. Their corresponding (1,2)*-rω-open sets are also defined and studied in this paper.

1. Introduction

Regular open sets have been introduced and investigated by Stone [21]. Levine [10], Cameron [2], Sundaram and Sheik John [23], Nagaveni [12], Palaniappan and Rao [13], Mashhour et. al. [11] and Gnanambal [5] introduced and investigated semi-open sets, regular semiopen sets, weakly closed sets, weakly generalized closed sets, regular generalized closed sets, preopen sets and generalized pre-regular closed sets, respectively. Regular ω-closed sets have been introduced and investigated by Benchalli and Wali [1] which is properly placed in between the class of ω-closed sets [22] and the class of regular generalized closed sets [13]. The study of bitopological spaces was first initiated by Kelly [7] in the year 1963. Recently Ravi, Lellis Thivagar, Ekici and Many others [6, 8, 14-20] have defined weakly open sets in bitopological spaces. By using the topological notions, namely, semi-open, preopen, regular open

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and regular semi-open sets, many new bitopological sets are defined and studied by them.

In this paper, we introduce the notion of regular (1,2)*-ω-closed (briefly, (1,2)*-rω-closed) sets and investigate their properties. By using the class of (1,2)*-rω-closed sets, we study the properties of (1,2)*-rω-open sets and its relations with other bitopological sets called (1,2)*-rg-closed sets [19], (1,2)*-gpr-closed sets [19], (1,2)*-wg-closed sets [20] and (1,2)*-πg-closed sets [18]. In most of the occasions our ideas are illustrated and substantiated by suitable examples.

2. Preliminaries

Throughout this paper, X denote bitopological space (X, τ_1, τ_2) on which no separation axioms are assumed.

Definition 2.1. *Let S be a subset of a bitopological space X. Then S is called* $\tau_{1,2}$*-open* [8] *(or quazi-open* [3]*) if S=A∪B, where A*$\in\tau_1$ *and B*$\in\tau_2$.

The complement of $\tau_{1,2}$-open set is called $\tau_{1,2}$-closed.

The family of all $\tau_{1,2}$-open sets in X is denoted by (1,2)*-O(X).

Definition 2.2. *Let A be a subset of a bitopological space X. Then*

(1) *the* $\tau_{1,2}$*-closure of A* [3, 8]*, denoted by* $\tau_{1,2}$*-cl(A), is defined by* $\cap\{$*U: A*$\subseteq$*U and U is* $\tau_{1,2}$*-closed*$\}$*;*
(2) *the* $\tau_{1,2}$*-interior of A* [8]*, denoted by* $\tau_{1,2}$*-int(A), is defined by* $\cup\{$*U: U*$\subseteq$*A and U is* $\tau_{1,2}$*-open*$\}$*.*

Remark 2.3. *Notice that* $\tau_{1,2}$*-open subsets of X need not necessarily form a topology.*

We recall some definitions and results which are used in this paper.

Definition 2.4. *A subset S of a bitopological space X is said to be*

(1) *(1,2)*-semi-open* [17] *if S*$\subseteq\tau_{1,2}$*-cl(*$\tau_{1,2}$*-int(S));*

(2) *regular (1,2)*-open* [14] *if* $S=\tau_{1,2}$*-int*($\tau_{1,2}$*-cl(S));*

(3) *(1,2)*-preopen* [16] *if* $S\subseteq\tau_{1,2}$*-int*($\tau_{1,2}$*-cl(S));*

(4) *(1,2)*-*π*-open* [18] *if the finite union of regular (1,2)*-open sets.*

The complements of the above mentioned open sets are called their respective closed sets.

The family of all (1,2)*-semi-open (resp. (1,2)*-preopen, regular (1,2)*-open, (1,2)*-π-open) sets in X is denoted by (1,2)*-SO(X) (resp. (1,2)*-PO(X), (1,2)*-RO(X), (1,2)*-πO(X)).

The (1,2)*-semi-closure (resp. (1,2)*-preclosure) of a subset S of X is, denoted by (1,2)*-scl(S) (resp. (1,2)*-pcl(S)), defined as the intersection of all (1,2)*-semi-closed (resp. (1,2)*-preclosed) sets containing S [15].

Definition 2.5. *A subset S of a bitopological space X is said to be*

(1) *a regular (1,2)*-generalized closed (briefly, (1,2)*-rg-closed* [19]*) if* $\tau_{1,2}$*-cl(S)*$\subseteq U$ *whenever* $S\subseteq U$ *and* $U\in$*(1,2)*-RO(X).*

(2) *a (1,2)*-*ω*-closed or (1,2)*-*$\hat{g}$*-closed* [6] *if* $\tau_{1,2}$*-cl(S)*$\subseteq U$ *whenever* $S\subseteq U$ *and* $U\in$*(1,2)*-SO(X).*

(3) *a (1,2)*-gpr-closed* [19] *if (1,2)*-pcl(S)*$\subseteq U$ *whenever* $S\subseteq U$ *and* $U\in$*(1,2)*-RO(X).*

(4) *a (1,2)*-generalized closed (briefly, (1,2)*-g-closed* [19]*) if* $\tau_{1,2}$*-cl(S)*$\subseteq U$ *whenever* $S\subseteq U$ *and* $U\in$*(1,2)*-O(X).*

(5) *a weakly (1,2)*-generalized closed (briefly, (1,2)*-wg-closed* [20]*) if* $\tau_{1,2}$*-cl(*$\tau_{1,2}$*-int(S))*$\subseteq U$ *whenever* $S\subseteq U$ *and* $U\in$*(1,2)*-O(X).*

(6) *a (1,2)*-*πg*-closed* [18] *if* $\tau_{1,2}$*-cl(S)*$\subseteq U$ *whenever* $S\subseteq U$ *and* $U\in$*(1,2)*-*$\pi O(X)$*.*

The complements of the above mentioned closed sets are called their respective open sets.

Definition 2.6. *A subset S of a bitopological space X is called regular (1,2)*-semi-open if there is a regular (1,2)*-open set U such that* $U \subseteq S \subseteq \tau_{1,2}$*-cl(U).*

The family of all regular (1,2)*-semi-open sets in X is denoted by (1,2)*-RSO(X).

Definition 2.7. *A subset S of a bitopological space X is called* $\tau_{1,2}$*-clopen if it is both* $\tau_{1,2}$*-open and* $\tau_{1,2}$*-closed in X.*

Remark 2.8.
(1) *Every regular (1,2)*-semi-open set in (X,* τ_1*,* τ_2*) is (1,2)*-semi-open but not conversely.*
(2) *If A is regular (1,2)*-semi-open in (X,* τ_1*,* τ_2*), then* $X \backslash A$ *is also regular (1,2)*-semi-open.*
(3) *In a space (X,* τ_1*,* τ_2*), the regular (1,2)*-open sets and the regular (1,2)*-closed sets are regular (1,2)*-semi-open.*

Theorem 2.9. [16] *For a subset S of X, we have (1,2)*-scl(S)=*$S \cup \tau_{1,2}$*-int(*$\tau_{1,2}$*-cl(S)).*

3. Properties of (1,2)*-Rω-closed sets

Definition 3.1. *A subset S of a bitopological space X is said to be regular (1,2)*-ω-closed (briefly, (1,2)*-rω-closed) if* $\tau_{1,2}$*-cl(S)*$\subseteq U$ *whenever* $S \subseteq U$ *and U is regular (1,2)*-semi-open.*

The family of all (1,2)*-rω-closed sets in X is denoted by (1,2)*-RωC(X).

Example 3.2. *Let* $X=\{a, b, c\}$, $\tau_1= \{\emptyset, X, \{b\}\}$ *and* $\tau_2= \{\emptyset, X, \{c\}\}$*. Then*
(1) *the sets in* $\{\emptyset, X, \{b\}, \{c\}, \{b, c\}\}$ *are called* $\tau_{1,2}$*-open;*
(2) *the sets in* $\{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}\}$ *are called* $\tau_{1,2}$*-closed;*
(3) *the sets in* $\{\emptyset, X, \{b\}, \{c\}, \{a, b\}, \{a, c\}\}$ *are called regular (1,2)*-semi-open in X;*
(4) *the sets in* $\{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}, \{b, c\}\}$ *are called (1,2)*-rω-closed in X;*

(5) *the sets in* $\{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}\}$ *are called (1,2)*-semi-closed in X;*

(6) $\{b, c\}$ *is (1,2)*-rω-closed set but* $\{b, c\}$ *is neither* $\tau_{1,2}$*-closed nor (1,2)*-semi-closed in X;*

(7) $\{b\}$ *is (1,2)*-semi-closed set but* $\{b\}$ *is not (1,2)*-rω-closed in X;*

(8) *the set* $\{b, c\}$ *is (1,2)*-rω-closed set but* $\{b, c\}$ *is not (1,2)*-ω-closed in X.*

Example 3.3. *Let* $X=\{a, b, c, d\}$, $\tau_1=\{\emptyset, X, \{a\}\}$ *and* $\tau_2=\{\emptyset, X, \{b\}, \{a, b, c\}\}$. *Then*

(1) *the sets in* $\{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$ *are called* $\tau_{1,2}$*-open;*

(2) *the sets in* $\{\emptyset, X, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$ *are called* $\tau_{1,2}$*-closed;*

(3) *the sets in* $\{\emptyset, X, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$ *are called (1,2)*-rg-closed in X;*

(4) *the sets in* $\{\emptyset, X, \{a\}, \{b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, c, d\}, \{b, c, d\}\}$ *are called regular (1,2)*-semi-open in X;*

(5) *the sets in* $\{\emptyset, X, \{d\}, \{a, b\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$ *are called (1,2)*-rω-closed in X;*

(6) *the sets in* $\{\emptyset, X, \{c\}, \{d\}, \{c, d\}, \{a, d\}, \{b, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$ *are called (1,2)*-wg-closed in X;*

(7) *the sets in* $\{\emptyset, X, \{c\}, \{d\}, \{a, c\}, \{b, c\}, \{c, d\}, \{a, d\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$ *are called (1,2)*-πg-closed in X;*

(8) *the sets in* $\{\emptyset, X, \{d\}, \{c, d\}, \{a, d\}, \{b, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$ *are called (1,2)*-g-closed in X;*

(9) *the sets in* $\{\emptyset, X, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$ *are called (1,2)*-gpr-closed in X;*

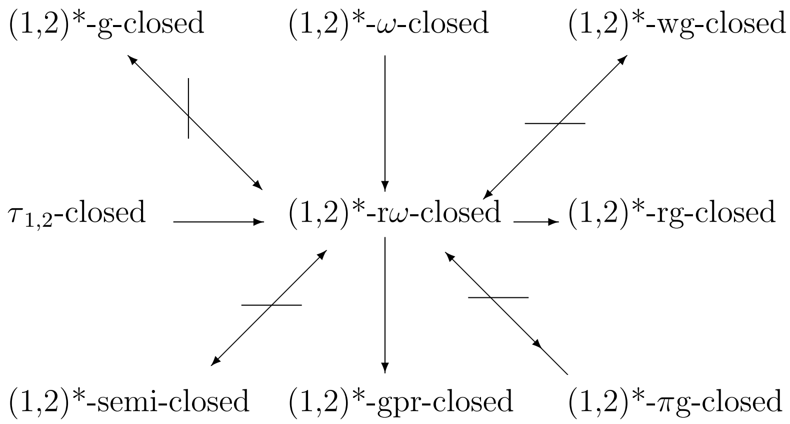
(10) $\{c\}$ *is (1,2)*-wg-closed, (1,2)*-πg-closed and (1,2)*-rg-closed set but* $\{c\}$ *is not (1,2)*-rω-closed in X;*

(11) $\{a, b\}$ *is (1,2)*-rω-closed set but it is neither (1,2)*-wg-closed nor (1,2)*-πg-closed in X;*

(12) $\{a, d\}$ *is (1,2)*-g-closed set but it is not (1,2)*-rω-closed in X. Also,* $\{a, b\}$ *is (1,2)*-rω-closed set but it is not (1,2)*-g-closed in X;*

(13) $\{c\}$ *is (1,2)*-gpr-closed set but it is not (1,2)*-rω-closed in X.*

Remark 3.4. *The following diagram follows immediately from the above definitions and the above examples. Where A→B (resp. A↮B) means A implies B but not conversely (resp. A and B are independent).*



Remark 3.5. *The following example shows that the intersection of two (1,2)*-rω-closed sets need not be an (1,2)*-rω-closed.*

Example 3.6. *Let X,* τ_1 *and* τ_2 *be as in Example 3.2. We have* $\{a, b\}$ *and* $\{b, c\}$ *are (1,2)*-rω-closed but their intersection* $\{a, b\}\cap\{b, c\}=\{b\}$ *is not (1,2)*- rω-closed in X.*

Theorem 3.7. *If a subset A of X is (1,2)*-rω-closed in X, then* $\tau_{1,2}$*-cl(A)\A does not contain any nonempty regular (1,2)*-semi-open set in X.*

Proof. Suppose that A is an (1,2)*-rω-closed set in X. We prove the result by contradiction. Let U be a regular (1,2)*-semi-open set such that $\tau_{1,2}$-cl(A)\A⊇U and

U$\neq\emptyset$. Since U$\nsubseteq$A, U$\subseteq$X\A which implies A$\subseteq$X\U. Since U is regular (1,2)*-semi-open , X\U is also regular (1,2)*-semi-open in X. Since A is an (1,2)*-rω-closed set in X, by definition, we have $\tau_{1,2}$-cl(A)$\subseteq$X\U. So U$\subseteq$X\$\tau_{1,2}$-cl(A). We already have U$\subseteq\tau_{1,2}$-cl(A). Therefore, U$\subseteq$($\tau_{1,2}$-cl(A)$\cap$(X\$\tau_{1,2}$-cl(A)))=$\emptyset$. This shows that U=$\emptyset$, which is a contradiction. Hence $\tau_{1,2}$-cl(A)\A does not contain any nonempty regular (1,2)*-semi-open set in X.

Remark 3.8. *The following example shows that the converse of Theorem 3.7 need not be true.*

Example 3.9. *Let X, τ_1 and τ_2 be as in Example 3.2. Then the sets in* {$\emptyset$, *X*, {*b*}, {*c*}, {*a*, *b*}, {*a*, *c*}} *are called regular (1,2)*-semi-open in X.*

If we put A={*c*}*, then $\tau_{1,2}$-cl(A)\A=*{*a*, *c*}\{*c*}=*{*a*} *does not contain any nonempty regular (1,2)*-semi-open set but A is not an (1,2)*-rω-closed in X.*

Corollary 3.10. *If a subset A of X is an (1,2)*-rω-closed set in X, then $\tau_{1,2}$-cl(A)\A does not contain any non-empty regular (1,2)*-open (resp. regular (1,2)*-closed) set in X.*

Proof. Follows from Theorem 3.7 and the fact that every regular (1,2)*-open (resp. regular (1,2)*-closed) set is regular (1,2)*-semi-open.

Remark 3.11. *The following example shows that the converse of Corollary 3.10 need not be true.*

Example 3.12. *Let X, τ_1 and τ_2 be as in Example 3.2. Then the sets in* {$\emptyset$, *X*, {*b*}, {*c*}} *are called regular (1,2)*-open in X;*

If we put A={*c*}*, then $\tau_{1,2}$-cl(A)\A=*{*a*, *c*}\{*c*}=*{*a*} *does not contain any non-empty regular (1,2)*-open set but A is not an (1,2)*-rω-closed in X.*

Theorem 3.13. *For an element x$\in$X, the set X*{*x*} *is (1,2)*-rω-closed or regular (1,2)*-semi-open.*

Proof. Suppose X\{x} is not regular (1,2)*-semi-open. Then X is the only regular (1,2)*-semi-open set containing X\{x}. This implies $\tau_{1,2}$-cl(X\{x})⊆X. Hence X\{x} is an (1,2)*-rω-closed in X.

Theorem 3.14. *If A is an (1,2)*-rω-closed subset of X such that A⊆B⊆$\tau_{1,2}$-cl(A), then B is an (1,2)*-rω-closed set in X.*

Proof. Let A be an (1,2)*-rω-closed subset of X such that A⊆B⊆$\tau_{1,2}$-cl(A). Let U be a regular (1,2)*-semi-open set of X such that B⊆U. Then A⊆U. Since A is (1,2)*-rω-closed, we have $\tau_{1,2}$-cl(A)⊆U. Now $\tau_{1,2}$-cl(B)⊆$\tau_{1,2}$-cl($\tau_{1,2}$-cl(A)) = $\tau_{1,2}$-cl(A)⊆U. Therefore, B is an (1,2)*-rω-closed set in X.

Theorem 3.15. *Let A be (1,2)*-rω-closed in X. Then A is $\tau_{1,2}$-closed if and only if $\tau_{1,2}$-cl(A)\A is regular (1,2)*-semi-open.*

Proof. Suppose A is $\tau_{1,2}$-closed in X. Then $\tau_{1,2}$-cl(A)=A and so $\tau_{1,2}$-cl(A)\A=∅, which is regular (1,2)*-semi-open in X. Conversely $\tau_{1,2}$-cl(A)\A is regular (1,2)*-semi-open in X. Since A is (1,2)*-rω-closed, by Theorem 3.7, $\tau_{1,2}$-cl(A)\A does not contain any nonempty regular (1,2)*-semi-open set in X. Then $\tau_{1,2}$-cl(A)\A=∅ and hence A is $\tau_{1,2}$-closed in X.

Theorem 3.16. *If A is regular (1,2)*-open and (1,2)*-rg-closed, then A is (1,2)*-rω-closed in X.*

Proof. Let A be regular (1,2)*-open and (1,2)*-rg-closed in X. We prove that A is an (1,2)*-rω-closed set in X. Let U be any regular (1,2)*-semi-open set in X such that A⊆U. Since A is regular (1,2)*-open and (1,2)*-rg-closed, we have $\tau_{1,2}$-cl(A)⊆A. Then $\tau_{1,2}$-cl(A)⊆A⊆U. Hence A is (1,2)*-rω-closed in X.

Theorem 3.17. *If a subset A of a bitopological space X is both regular (1,2)*-semi-open and (1,2)*-rω-closed, then it is $\tau_{1,2}$-closed.*

Proof. Since A is regular (1,2)*-semi-open and (1,2)*-rω-closed, $\tau_{1,2}$-cl(A)$\subseteq$A. Thus, A is $\tau_{1,2}$-closed.

Corollary 3.18. *If A is regular (1,2)*-open and (1,2)*-rω-closed, then A is regular (1,2)*-closed and hence $\tau_{1,2}$-clopen.*

Proof. Since A is regular (1,2)*-open, A is regular (1,2)*-semi-open by Remark 2.8. By Theorem 3.17, A is $\tau_{1,2}$-closed. Since A is regular (1,2)*-open, A is $\tau_{1,2}$-open. Thus A is $\tau_{1,2}$-clopen and regular (1,2)*-closed.

Corollary 3.19. *Suppose the collection of $\tau_{1,2}$-closed sets of X is closed under finite intersections. Let A be regular (1,2)*-semi-open and (1,2)*-rω-closed in X. Suppose that F is $\tau_{1,2}$-closed in X. Then A$\cap$F is an (1,2)*-rω-closed set in X.*

Proof. Let A be regular (1,2)*-semi-open and (1,2)*-rω-closed in X. By Theorem 3.17, A is $\tau_{1,2}$-closed. Since F is $\tau_{1,2}$-closed, A$\cap$F is $\tau_{1,2}$-closed in X. Hence A$\cap$F is (1,2)*-rω-closed set in X.

Theorem 3.20. *Let A be regular (1,2)*-open in a bitopological space X. Then the following are equivalent:*

(1) *A is (1,2)*-g-closed.*
(2) *A is (1,2)*-πg-closed.*
(3) *A is (1,2)*-rg-closed.*
(4) *A is (1,2)*-rω-closed.*

Proof. (1)$\Rightarrow$(2) It follows from the fact that every (1,2)*-g-closed set is (1,2)*-πg-closed [18].

(2)$\Rightarrow$(3) It follows from the fact that every (1,2)*-πg-closed set is (1,2)*-rg-closed [18].

(3)$\Rightarrow$(4) It follows from Theorem 3.16.

(4)⇒(1) It follows from Corollary 3.18 and the fact that every regular (1,2)*-closed set is $\tau_{1,2}$-closed [14] and every $\tau_{1,2}$-closed set is (1,2)*-g-closed [18].

Theorem 3.21. *Let A be regular (1,2)*-open in a bitopological space X. Then the following are equivalent*

1. *A is (1,2)*-g-closed.*
2. *A is (1,2)*-wg-closed.*
3. *A is (1,2)*-rω-closed.*

Proof. (1)⇒(2) It follows from the fact that every (1,2)*-g-closed set is (1,2)*-wg-closed [20].

(2)⇒(3) We know that every regular (1,2)*-open set is $\tau_{1,2}$-open. Let A⊆U where U is regular (1,2)*-semi-open in X. Since A is (1,2)*-wg-closed and $\tau_{1,2}$-open, $\tau_{1,2}$-cl($\tau_{1,2}$-int(A))⊆A. Since A=$\tau_{1,2}$-int(A), $\tau_{1,2}$-cl(A)⊆A⊆U. Thus, A is (1,2)*-rω-closed.

(3)⇒(1) It follows from Theorem 3.20.

Theorem 3.22. *In a bitopological space X, if (1,2)*-RSO(X)={X, ∅}, then every subset of X is an (1,2)*-rω-closed set.*

Proof. Let X be a bitopological space and (1,2)*-RSO(X)={X, ∅}. Let A be any subset of X. Suppose A=∅. Then ∅ is an (1,2)*-rω-closed set in X. Suppose A≠∅. Then X is the only regular (1,2)*-semi-open set containing A and so $\tau_{1,2}$-cl(A)⊆X. Hence A is an (1,2)*-rω-closed set in X.

Remark 3.23. *The following example shows that the converse of Theorem 3.22 need not be true.*

Example 3.24. *Let X={a, b, c, d}, τ_1={∅, X, {a, b}} and τ_2={∅, X, {c, d}}. Then*

1. *the sets in {∅, X, {a, b}, {c, d}} are called $\tau_{1,2}$-open;*
2. *the sets in {∅, X, {a, b}, {c, d}} are called $\tau_{1,2}$-closed;*
3. *the sets in {∅, X, {a, b}, {c, d}} are called regular (1,2)*-semi-open in X;*

(4) *Every subset of X is an (1,2)*-rω-closed set in X;*
(5) *(1,2)*-RSO(X)=$\{\emptyset, X, \{a, b\}, \{c, d\}\}$;*
(6) *(1,2)*-RSO(X)$\neq\{\emptyset, X\}$.*

Theorem 3.25. *In a bitopological space X, (1,2)*-RSO(X)$\subseteq\{F\subseteq X : F^c\in$(1,2)*-O(X)$\}$ if and only if every subset of X is (1,2)*-rω-closed.*

Proof. Suppose that (1,2)*-RSO(X)$\subseteq\{$F$\subseteq$X : F$^c\in$(1,2)*-O(X)$\}$. Let A be any subset of X such that A$\subseteq$U where U is regular (1,2)*-semi-open. Then U$\in$(1,2)*-RSO(X)$\subseteq\{$F$\subseteq$X : F$^c\in$(1,2)*-O(X)$\}$. That is U$\in\{$F$\subseteq$X : F$^c\in$(1,2)*-O(X)$\}$. Thus U is $\tau_{1,2}$-closed and hence $\tau_{1,2}$-cl(U)=U. Now, we have $\tau_{1,2}$-cl(A)$\subseteq\tau_{1,2}$-cl(U)=U. Hence A is a (1,2)*-rω-closed in X.

Conversely, suppose that every subset of X is (1,2)*-rω-closed. Let U$\in$(1,2)*-RSO(X). Since U is (1,2)*-rω-closed, we have $\tau_{1,2}$-cl(U)$\subseteq$U. Thus $\tau_{1,2}$-cl(U)=U and hence U$\in\{$F$\subseteq$X : F$^c\in$(1,2)*-O(X)$\}$. Therefore (1,2)*-RSO(X)$\subseteq\{$F$\subseteq$X : F$^c\in$(1,2)*-O(X)$\}$.

Definition 3.26. (1) *The intersection of all regular (1,2)*-semi-open subsets of (X, τ_1, τ_2) containing A is called the regular (1,2)*-semi-kernel of A and is denoted by (1,2)*-rsker(A).*
(2) *The intersection of all (1,2)*-semi-open subsets of (X, τ_1, τ_2) containing A is called the (1,2)*-semi-kernel of A and is denoted by (1,2)*-sker(A).*

Lemma 3.27. *Let X be a bitopological space and A be a subset of X. If A is regular (1,2)*-semi-open in X, then (1,2)*-rsker(A)=A but not conversely.*

Proof. It follows from Definition 3.26.

Example 3.28. *Let X=$\{a, b, c, d\}$, τ_1= $\{\emptyset, X, \{a\}, \{d\}, \{a, c\}, \{a, d\}, \{a, c, d\}\}$ and τ_2=$\{\emptyset, X, \{c, d\}, \{a, c, d\}\}$. Then*

(1) *the sets in* $\{\emptyset, X, \{a\}, \{d\}, \{a, c\}, \{a, d\}, \{c, d\}, \{a, c, d\}\}$ *are called* $\tau_{1,2}$*-open;*

(2) *the sets in* $\{\emptyset, X, \{b\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{b, c, d\}\}$ *are called* $\tau_{1,2}$*-closed;*

(3) *the sets in* $\{\emptyset, X, \{a\}, \{d\}, \{a, b\}, \{a, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{b, c, d\}\}$ *are called regular (1,2)*-semi-open;*

(4) *we have (1,2)*-rsker(*$\{b\}$*)=*$\{b\}$ *but* $\{b\}$ *is not regular (1,2)*- semi-open.*

Lemma 3.29. *For any subset A of X, (1,2)*-sker(A)⊆(1,2)*-rsker(A).*

Proof. It follows from Definition 3.26 and (1,2)*-RSO(X)⊆(1,2)*-SO(X).

Lemma 3.30. *For any subset A of X, A⊆(1,2)*-rsker(A).*

Proof. It follows from Definition 3.26.

Theorem 3.31. *A subset A of X is (1,2)*-r*ω*-closed if and only if* $\tau_{1,2}$*-cl(A)⊆(1,2)*-rsker(A).*

Proof. Suppose that A is (1,2)*-rω-closed. Then $\tau_{1,2}$-cl(A)⊆U whenever A⊆U and U is regular (1,2)*-semi-open. Let x∈$\tau_{1,2}$-cl(A). Suppose x∉(1,2)*-rsker(A), then there is a regular (1,2)*-semi-open set U containing A such that x is not in U. Since A is (1,2)*-rω-closed, $\tau_{1,2}$-cl(A)⊆U. We have x not in $\tau_{1,2}$-cl(A), which is a contradiction. Hence x∈(1,2)*-rsker(A) and so $\tau_{1,2}$-cl(A)⊆(1,2)*-rsker(A). Conversely, let $\tau_{1,2}$-cl(A)⊆(1,2)*-rsker(A). If U is any regular (1,2)*-semi-open set containing A, then (1,2)*-rsker(A)⊆U and hence $\tau_{1,2}$-cl(A)⊆(1,2)*-rsker(A)⊆U. Therefore, A is (1,2)*-rω-closed in X.

Definition 3.32. *A subset S of a bitopological space X is said to be regular (1,2)*-*ω*-open (briefly, (1,2)*-r*ω*-open) if* A^c *is (1,2)*-r*ω*-closed in X.*

We denote the family of all (1,2)*-rω-open sets in X by (1,2)*-RωO(X).

Theorem 3.33. *If a set A is (1,2)*-rω-open in X, then G=X, whenever G is regular (1,2)*-semi-open and $\tau_{1,2}$-int(A)$\cup$A$^c\subseteq$G.*

Proof. Suppose that A is (1,2)*-rω-open in X. Let G be regular (1,2)*-semi-open and $\tau_{1,2}$-int(A)$\cup$A$^c\subseteq$G. This implies G$^c\subseteq(\tau_{1,2}$-int(A)$\cup$A$^c)^c$ = $(\tau_{1,2}$-int(A))$^c\cap$A. That is G$^c\subseteq(\tau_{1,2}$-int(A))$^c\backslash$A^c. Thus G$^c\subseteq\tau_{1,2}$-cl(A^c)$\backslash$A^c, since $(\tau_{1,2}$-int(A))^c=$\tau_{1,2}$-cl(A^c). Now G^c is also regular (1,2)*-semi-open and A^c is (1,2)*-rω-closed, by Theorem 3.7, it follows that G^c=$\emptyset$. Hence G=X.

Remark 3.34. *The following example shows that the converse of Theorem 3.33 need not be true.*

Example 3.35. *Let X, τ_1 and τ_2 be as in Example 3.2. Then*

(1) *the sets in {$\emptyset$, X, {b}, {c}, {a, b}, {a, c}} are called regular (1,2)*-semi-open in X;*
(2) *the sets in {$\emptyset$, X, {a}, {b}, {c}, {b, c}} are called (1,2)*-rω-open in X;*
(3) *If we put A={a, b}, then the following conditions are satisfied*
   (a) *$\tau_{1,2}$-int({a, b})$\cup$({a, b})^c={b}$\cup${c}={b, c}$\subseteq$X=G;*
   (b) *G is regular (1,2)*-semi-open;*
   (c) *G=X. But A is not (1,2)*-rω-open in X.*

4. Properties of (1,2)*-rslc*-sets

Definition 4.1. *A subset A of X is said to be (1,2)*-rslc*-set if A=M$\cap$N where M is regular (1,2)*-semi-open and N is $\tau_{1,2}$-closed.*

Remark 4.2. (1) *Every $\tau_{1,2}$-closed set is (1,2)*-rslc*-set but not conversely.*
(2) *Every regular (1,2)*-semi-open set is (1,2)*-rslc*-set but not conversely.*

Example 4.3. *Let X, τ_1 and τ_2 be as in Example 3.3. Then*

(1) *the sets in {$\emptyset$, X, {a}, {b}, {a, c}, {a, d}, {b, c}, {b, d}, {b, c, d}, {a, c, d}} are called regular (1,2)*-semi-open in X;*

(2) *the sets in* $\{\emptyset, X, \{a\}, \{b\}, \{c\}, \{d\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$ *are called (1,2)*-rslc*-sets in X;*

(3) $\{a\}$ *is (1,2)*-rslc*-set but not* $\tau_{1,2}$*-closed set in X;*

(4) $\{c\}$ *is (1,2)*-rslc*-set but not regular (1,2)*-semi-open.*

Theorem 4.4. *Let A be a subset of a bitopological space X. Then A is* $\tau_{1,2}$*-closed if and only if A is (1,2)*-r*ω*-closed and (1,2)*-rslc*-set.*

Proof. Let A be a $\tau_{1,2}$-closed subset of X. Then A is (1,2)*-rω-closed and (1,2)*-rslc*-set.

Conversely, let A=M∩N where M is regular (1,2)*-semi-open and N is $\tau_{1,2}$-closed. Since A is (1,2)*-rω-closed, A⊆M and M is regular (1,2)*-semi-open, $\tau_{1,2}$-cl(A)⊆M. Moreover, since A⊆N, $\tau_{1,2}$-cl(A)⊆$\tau_{1,2}$-cl(N)=N. We have $\tau_{1,2}$-cl(A)⊆M∩N and so $\tau_{1,2}$-cl(A)⊆A. Hence A is $\tau_{1,2}$-closed.

Remark 4.5. *The concepts of (1,2)*-r*ω*-closed sets and (1,2)*-rslc*-sets are independent of each other.*

Example 4.6. *Let X,* τ_1 *and* τ_2 *be as in Example 4.3. Then*

(1) $\{a\}$ *is (1,2)*-rslc*-set but not (1,2)*-r*ω*-closed set;*

(2) $\{a, b\}$ *is (1,2)*-r*ω*-closed but not (1,2)*-rslc*-set.*

Definition 4.7. *A subset A of X is said to be (1,2)*-*Λ^b_{rs}*-set if A=(1,2)*-rsker(A).*

Definition 4.8. *A subset A of X is said to be (1,2)*-*λ^b_{rs}*-closed if A=L∩F where L is (1,2)*-*Λ^b_{rs}*-set and F is* $\tau_{1,2}$*-closed.*

Lemma 4.9. *For a bitopological space (X,* τ_1*,* τ_2*), the following conditions are equivalent.*

(1) *A is (1,2)*-*λ^b_{rs}*-closed.*

(2) *A=L∩*$\tau_{1,2}$*-cl(A) where L is (1,2)*-*Λ^b_{rs}*-set.*

(3) *A=(1,2)*-rsker(A)*$\cap\tau_{1,2}$*-cl(A).*

Remark 4.10. (1) *Every* $\tau_{1,2}$*-closed set is (1,2)*-*λ^b_{rs}*-closed but not conversely.*

(2) *Every (1,2)*-rslc*-set is (1,2)*-*λ^b_{rs}*-closed.*

Example 4.11. *Let X,* τ_1 *and* τ_2 *be as in Example 3.28. Then*

(1) *the sets in* $\{\emptyset, X, \{a\}, \{d\}, \{a, b\}, \{a, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{b, c, d\}\}$ *are called regular (1,2)*-semi-open in X;*

(2) *the sets in* $\{\emptyset, X, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{b, c, d\}\}$ *are called (1,2)*-*Λ^b_{rs}*-sets in X;*

(3) *the sets in* $\{\emptyset, X, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{b, c, d\}\}$ *are called (1,2)*-*λ^b_{rs}*-closed in X;*

(4) $\{c\}$ *is (1,2)*-*λ^b_{rs}*-closed set but not* $\tau_{1,2}$*-closed set in X.*

Theorem 4.12. *For a bitopological space (X,* τ_1*,* τ_2*), the following conditions are equivalent.*

(1) *A is* $\tau_{1,2}$*-closed.*

(2) *A is (1,2)*-r*ω*-closed and (1,2)*-rslc*-set.*

(3) *A is (1,2)*-r*ω*-closed and (1,2)*-*λ^b_{rs}*-closed.*

Proof. (1)$\Rightarrow$(2) Obvious.

(2)$\Rightarrow$(3) Obvious.

(3)$\Rightarrow$(1) Let A be (1,2)*-λ^b_{rs}-closed. Then, by Lemma 4.9, A=(1,2)*-rsker(A)$\cap\tau_{1,2}$-cl(A). Since A is (1,2)*-rω-closed, by Theorem 3.31, A=$\tau_{1,2}$-cl(A). Thus A is $\tau_{1,2}$-closed.

Remark 4.13. *The concepts of (1,2)*-r*ω*-closed sets and (1,2)*-*λ^b_{rs}*-closed sets are independent.*

Example 4.14. *Let X,* τ_1 *and* τ_2 *be as in Example 4.3. Then*

(1) *the sets in* $\{\emptyset, X, \{a\}, \{b\}, \{c\}, \{d\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$ *are called (1,2)*-*Λ^b_{rs}*-sets and (1,2)*-*λ^b_{rs}*-closed in X;*

(2) $\{a\}$ *is (1,2)*-*λ^b_{rs}*-closed set but not (1,2)*-r*ω*-closed set in X;*

(3) $\{a, b\}$ *is (1,2)*-r*ω*-closed but not (1,2)*-*λ^b_{rs}*-closed set in X.*

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