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DECOMPOSITION OF α-CONTINUITY AND $\tilde{g}_\alpha$-CONTINUITY

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ABSTRACT. The main purpose of this paper is to introduce the concepts of $\mathcal{C}\eta^*$-sets, $\mathcal{C}\eta^{**}$-sets, $\mathcal{C}\eta^*$-continuity and $\mathcal{C}\eta^{**}$-continuity and to obtain decomposition of α-continuity and $\tilde{g}_\alpha$- continuity in topological spaces.

1. Introduction

Tong [17] introduced the notions of A-sets and A-continuity in topological spaces and established a decomposition of continuity. In [18], he also introduced the notions of B-sets and B-continuity and used them to obtain a decomposition of continuity and Ganster and Reilly [3] improved Tong's decomposition result. Moreover, Noiri and Sayed [11] introduced the notions of η-sets and obtained some decompositions of continuity. Quite recently, Jafari et al [5] introduced and studied the notions of $\tilde{g}_\alpha$-closed set and Ravi et al [13] introduced and studied the notions of $\tilde{g}$-preclosed sets.

In this paper, we introduce the notions of $\mathcal{C}\eta^*$-sets, $\mathcal{C}\eta^{**}$-sets, $\mathcal{C}\eta^*$-continuity and $\mathcal{C}\eta^{**}$-continuity and obtain decomposition of α-continuity and $\tilde{g}_\alpha$-continuity.

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2. Preliminaries

Throughout the present paper, spaces mean topological spaces on which no separation axioms are assumed unless explicitely stated. Let A be a subset of a space X. The closure and the interior of A in X are denoted by cl(A) and int(A), respectively.

Definition 2.1. A subset A of a space X is called

(a) a preopen set [7] if A $\subset$ int(cl(A)) and a preclosed set if cl(int(A)) $\subset$ A,

(b) a semi-open set [6] if A $\subset$ cl(int(A)) and a semi-closed set if int(cl(A)) $\subset$ A,

(c) an α-open set [9] if A $\subset$ int(cl(int(A))) and an α-closed set if cl(int(cl(A))) $\subset$ A,

(d) a t-set [18] if int(cl(A))=int(A),

(e) an α^*-set [4] if int(A)=int(cl(int(A))),

(f) an A-set [17] if A=V $\cap$ T where V is open and T is a regular closed set, (i.e., T=cl(int(T))),

(g) a B-set [18] if A=V $\cap$ T where V is open and T is a t-set,

(h) an αB-set [1] if A=V $\cap$ T where V is α-open and T is a t-set,

(i) an η-set [11] if A=V $\cap$ T where V is open and T is an α-closed set,

(j) a locally closed set [2] if A=V $\cap$ T where V is open and T is closed.

The preinterior (resp. the α-interior) of a subset A of X is, denoted by pint(A) (resp. αint(A)), defined to be the union of all preopen sets (resp. α-open sets) contained in A.

The α-closure (resp. semi-closure, preclosure) of a subset A of X is, denoted by αcl(A) (resp. scl(A), pcl(A)), defined to be the intersection of all α-closed sets (resp. semi-closed sets, preclosed sets) containing A.

The collection of A-sets (resp. B-sets, αB-sets, η-sets, locally closed sets) in X is denoted by A(X) (resp. B(X), αB(X), η(X), LC(X)).

Definition 2.2. A subset A of a space X is called

(a) $\hat{g}$-closed [19] if cl(A)$\subseteq$ U whenever A$\subseteq$ U and U is semi-open in X. The complement of a $\hat{g}$-closed set is called $\hat{g}$-open.

(b) $^*g$-closed [20] if cl(A)$\subseteq$ U whenever A$\subseteq$ U and U is $\hat{g}$-open in X. The complement of a $^*g$-closed set is called $^*g$-open.

(c) a $^\sharp$g-semiclosed [21] (briefly $^\sharp$gs-closed) if scl(A)$\subseteq$ U whenever A$\subseteq$ U and U is $^*g$-open in X. The complement of a $^\sharp gs$-closed set is called $^\sharp gs$-open.

(d) a $\tilde{g}_\alpha$-closed [5] if αcl(A)$\subseteq$ U whenever A$\subseteq$ U and U is $^\sharp gs$-open in X. The complement of a $\tilde{g}_\alpha$-closed set is called $\tilde{g}_\alpha$-open.

(e) a $\tilde{g}$-preclosed [13] if pcl(A)$\subseteq$ U whenever A$\subseteq$ U and U is $^\sharp gs$-open in X. The complement of a $\tilde{g}$-preclosed set is called $\tilde{g}$-preopen.

The collection of all $\tilde{g}_\alpha$-open (resp. $\tilde{g}$-preopen) sets in X will be denoted by $\tilde{g}_\alpha$O(X) (resp. $\tilde{g}$PO(X)).

Remark 2.3. In a space X, the followings hold:

(a) Every $\tilde{g}_\alpha$-closed set is $\tilde{g}$-preclosed but not conversely [13].

(b) Every $\tilde{g}_\alpha$-continuous map is $\tilde{g}$-precontinuous but not conversely [14].

(c) Every open set is $^\sharp$gs-open but not conversely [16].

(d) Every α-open set is $\tilde{g}_\alpha$-open but not conversely [5].

(e) The intersection of two t-sets is a t-set [18].

Remark 2.4. In a space X, the followings hold:

(a) A is α-closed set if and only if A=αcl(A).

(b) The collection of all $\tilde{g}_\alpha$-open sets in X forms a topology.

(c) Every regular closed set is closed but not conversely.

(d) Every regular closed set is semi-closed (=t-set) but not conversely.

(e) Every closed set is α-closed but not conversely.

(f) Every α-closed set is semi-closed (=t-set) but not conversely.

3. $\mathcal{C}\eta^*$-sets and $\mathcal{C}\eta^{**}$-sets

In this section we introduce and study the notions of $\mathcal{C}\eta^*$-sets and $\mathcal{C}\eta^{**}$-sets in topological spaces.

Definition 3.1. A subset A of a space X is said to be

(a) an $\mathcal{C}\eta^*$-set if A=U $\cap$ T where U is $^\sharp$gs-open and T is α-closed in X.

(b) an $\mathcal{C}\eta^{**}$-set if A=U $\cap$ T where U is $\tilde{g}_\alpha$-open and T is a t-set in X.

The collection of all $\mathcal{C}\eta^*$-sets (resp. $\mathcal{C}\eta^{**}$-sets) in X will be denoted by $\mathcal{C}\eta^*$(X) (resp. $\mathcal{C}\eta^{**}$(X))

Theorem 3.2. For a subset A of a space X, the following are equivalent:

(1) A is an $\mathcal{C}\eta^*$-set.

(2) A=U $\cap$ αcl(A) for some $^\sharp$gs-open set U.

Proof. (1) $\rightarrow$ (2) Since A is an $\mathcal{C}\eta^*$-set, A=U $\cap$ T, where U is $^\sharp$gs-open and T is α-closed. We have A $\subset$ U, A $\subset$ T and αcl(A) $\subset$ αcl(T). Therefore A $\subset$ U $\cap$ αcl(A) $\subset$ U $\cap$ αcl(T)=U $\cap$ T=A. Thus, A=U $\cap$ αcl(A).
(2) $\rightarrow$ (1) It is obvious because αcl(A) is α-closed by Remark 2.4 (a).

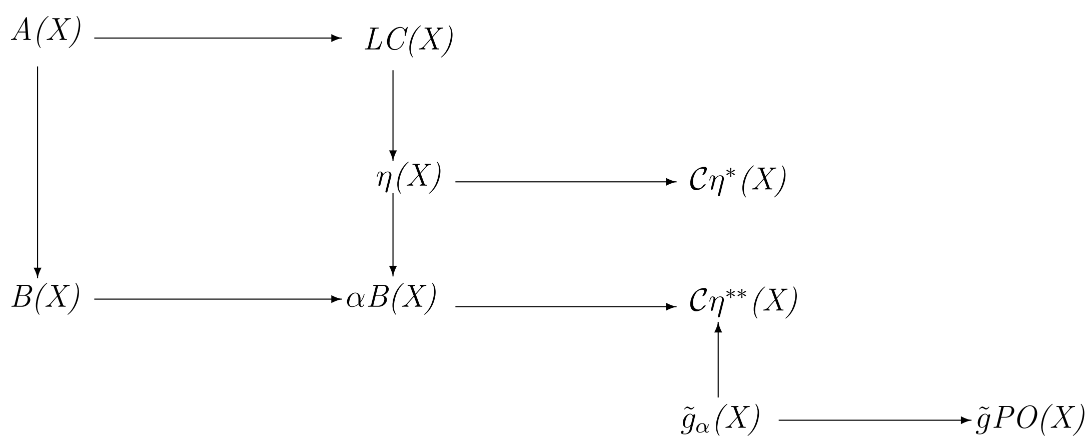
Remark 3.3. In a space X, the intersection of two $\mathcal{C}\eta^{**}$-sets is an $\mathcal{C}\eta^{**}$-set.

Proof. Let A and B be two $\mathcal{C}\eta^{**}$-sets. Then A=U $\cap$ T and B=V $\cap$ S where U, V are $\tilde{g}_\alpha$-open sets and S, T are t-sets. By Remark 2.3 (e), T $\cap$ S is a t-set and by Remark 2.4 (b), U $\cap$ V is a $\tilde{g}_\alpha$-open set. Therefore A $\cap$ B=(U $\cap$ V) $\cap$ (T $\cap$ S) is a $\mathcal{C}\eta^{**}$-set.

Remark 3.4. Union of two $\mathcal{C}\eta^{**}$-sets need not be an $\mathcal{C}\eta^{**}$-set as seen from the following example.

Example 3.5. Let X={a,b,c}, τ ={X,$\emptyset$,{b,c}}. Then the sets {a} and {b} are $\mathcal{C}\eta^{**}$-sets in (X,τ) but their union {a,b} is not an $\mathcal{C}\eta^{**}$-set in (X,τ).

Remark 3.6. Using the definitions of the subsets we discussed above, Remark 2.3 and Remark 2.4, the following implications are easily obtained.



where none of these implications is reversible as shown in [11, 13] and by the following examples.

Example 3.7. (a) Let X={a,b,c} and τ={X,$\emptyset$,{a}}. Clearly the set {a,b} is an $\mathcal{C}\eta^{*}$-set but not an η-set in (X,τ).

(b) Let X={a,b,c} and τ={X,$\emptyset$,{a,b}}. Clearly the set {c} is an $\mathcal{C}\eta^{**}$-set but not an $\tilde{g}_\alpha$-open set in (X,τ).

(c) In Example 3.5, the set {b} is an $\mathcal{C}\eta^{**}$-set but not an αB-set in (X,τ).

Remark 3.8. (1) The notions of $\mathcal{C}\eta^{*}$-sets and $\tilde{g}_\alpha$-closed sets are independent.

(2) The notions of $\mathcal{C}\eta^{**}$-sets and $\tilde{g}$-preopen sets are independent.

Example 3.9. In Example 3.5, the set {a,b} is $\tilde{g}_\alpha$-closed but not a $\mathcal{C}\eta^*$-set and the set {b,c} is an $\mathcal{C}\eta^*$-set but not a $\tilde{g}_\alpha$-closed in (X,τ).

Example 3.10. In Example 3.7 (b), the set {c} is an $\mathcal{C}\eta^{**}$-set but not a $\tilde{g}$-preopen set and also the set {a,c} is an $\tilde{g}$-preopen set but not a $\mathcal{C}\eta^{**}$-set in (X,τ).

Theorem 3.11. For a subset A of a space X, the following are equivalent:

(a) A is α-closed

(b) A is a $\mathcal{C}\eta^*$-set and $\tilde{g}_\alpha$-closed.

Proof. (a) $\rightarrow$ (b) It follows from Remark 2.3 (d) and Definition 3.1 (a).
(b) $\rightarrow$ (a) Since A is an $\mathcal{C}\eta^*$-set, then by Theorem 3.2, A=U $\cap$ αcl(A) where U is $^\sharp$gs-open in X. We have A $\subset$ U. Since A is $\tilde{g}_\alpha$-closed, then αcl(A) $\subset$ U. Therefore, αcl(A) $\subset$ U $\cap$ αcl(A)=A. But A $\subset$ αcl(A) always. Hence by Remark 2.4 (a), A is α-closed.

Proposition 3.12. Let A and B be subsets of a space X. If B is an α^*-set, then αint(A $\cap$ B)=αint(A) $\cap$ int(B) [**10**].

Theorem 3.13. For a subset S of a space X, the following are equivalent:

(a) S is $\tilde{g}_\alpha$-open.

(b) S is a $\mathcal{C}\eta^{**}$-set and $\tilde{g}$-preopen.

Proof. Necessity: It follows from Remark 2.3 (a) and Definition 3.1 (b).
Sufficiency: Assume that S is $\tilde{g}$-preopen and an $\mathcal{C}\eta^{**}$-set in X. Then S=A $\cap$ B where A is $\tilde{g}_\alpha$-open and B is a t-set in X. Let F $\subset$ S, where F is $^\sharp$gs-closed in X. Since S is $\tilde{g}$-preopen in X, F $\subset$ pint(S)=S $\cap$ int(cl(S))=(A $\cap$ B) $\cap$ int[cl(A $\cap$ B)] $\subset$ A $\cap$ B $\cap$ int(cl(A)) $\cap$ int(cl(B))=A $\cap$ B $\cap$ int(cl(A)) $\cap$ int(B), since B is a t-set. This implies,

F $\subset$ int(B). Note that A is $\tilde{g}_\alpha$-open and that F $\subset$ A. So, F $\subset$ αint(A). Therefore, F $\subset$ αint(A) $\cap$ int(B)=αint(S) by Proposition 3.12. Hence S is $\tilde{g}_\alpha$-open.

4. $\mathcal{C}\eta^*$-continuity and $\mathcal{C}\eta^{**}$-continuity

Definition 4.1. A function f : X$\rightarrow$ Y is said to be $\mathcal{C}\eta^*$-continuous (resp. $\mathcal{C}\eta^{**}$-continuous) if $f^{-1}(V)$ is an $\mathcal{C}\eta^*$-set (resp. $\mathcal{C}\eta^{**}$-set) in X for every open subset V of Y.

Definition 4.2. A function f : X$\rightarrow$ Y is said to be $\mathcal{C}^*\eta^*$-continuous if $f^{-1}(V)$ is an $\mathcal{C}\eta^*$-set in X for every closed subset V of Y.

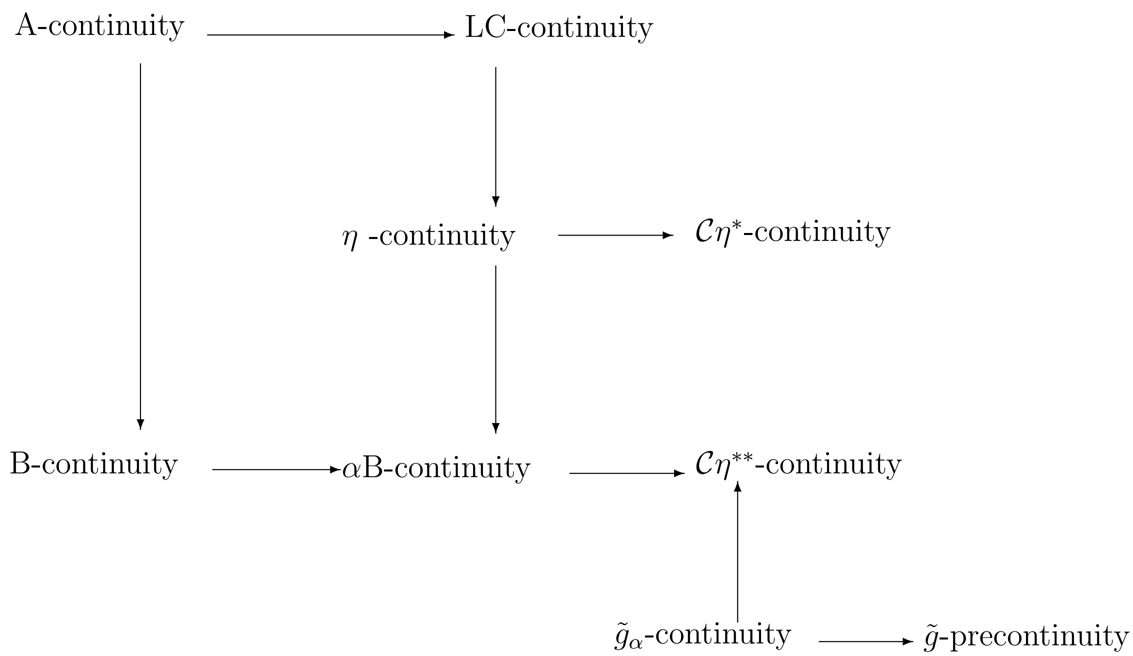
We shall recall the definitions of some functions used in the sequel.

Definition 4.3. A function f : X$\rightarrow$ Y is said to be

(a) A-continuous [17] if $f^{-1}(V)$ is an A-set in X for every open set V of Y,
(b) B-continuous [18] if $f^{-1}(V)$ is an B-set in X for every open set V of Y,
(c) α-continuous [8] if $f^{-1}(V)$ is an α-open set in X for every open set V of Y,
(d) LC-continuous [2] (resp. αB-continuous [1]) if $f^{-1}(V)$ is an locally closed (resp. αB-set) in X for every open set V of Y,
(e) η-continuous [11] if $f^{-1}(V)$ is an η-set in X for every open set V of Y,
(f) $\tilde{g}_\alpha$-continuous [14] (resp. $\tilde{g}$-precontinuous [14]) if $f^{-1}(V)$ is an $\tilde{g}_\alpha$-open set (resp. $\tilde{g}$-preopen set) in X for every open set V of Y.

Remark 4.4. It is clear that, a function f : X$\rightarrow$ Y is α-continuous if and only if $f^{-1}(V)$ is an α-closed set in X for every closed set V of Y.

From the definitions stated above, we obtain the following diagram



Remark 4.5. None of the implications is reversible as shown in [11, 13] and by the following examples.

Example 4.6. Let X=Y={a,b,c}, τ={X,$\emptyset$,{a}} and σ={Y,$\emptyset$,{a},{b},{a,b}}. Then the identity function f : X $\rightarrow$ Y is $\mathcal{C}\eta^*$-continuous but not η-continuous.

Example 4.7. Let X=Y={a,b,c}, τ={X,$\emptyset$,{b,c}} and σ ={Y,$\emptyset$,{c},{b,c}}. Then the identity function f : X $\rightarrow$ Y is $\mathcal{C}\eta^{**}$-continuous but not αB-continuous.

Example 4.8. Let X=Y={a,b,c}, τ={X,$\emptyset$,{b,c}} and σ={Y,$\emptyset$,{a}}. Then the identity function f : X $\rightarrow$ Y is $\mathcal{C}\eta^{**}$-continuous and not $\tilde{g}_\alpha$-continuous.

Remark 4.9. The following examples show the concepts of

(1) $\mathcal{C}\eta^{**}$-continuity and $\tilde{g}$-precontinuity are independent.

(2) $\tilde{g}_\alpha$-continuity and $\mathcal{C}^*\eta^*$-continuity are independent.

(3) $\mathcal{C}\eta^*$-continuity and $\mathcal{C}^*\eta^*$-continuity are independent.

Example 4.10. Let X=Y={a,b,c}, τ={X,$\emptyset$,{a,b}} and σ={Y,$\emptyset$,{c}}. Let f : X $\rightarrow$ Y be the identity function on X. Then f is $\mathcal{C}\eta^{**}$-continuous but not $\tilde{g}$-precontinuous.

Example 4.11. Let X=Y={a,b,c}, τ={X,$\emptyset$,{a,c}} and σ={Y,$\emptyset$,{a,b}}. Let f : X $\rightarrow$ Y be the identity function on X. Then f is $\tilde{g}$-precontinuous but not $\mathcal{C}\eta^{**}$-continuous.

Example 4.12. Let X=Y={a,b,c}, τ={X,$\emptyset$,{b,c}} and σ={Y,$\emptyset$,{c}}. Let f : X$\rightarrow$ Y be the identity function on X. Then f is $\tilde{g}_\alpha$-continuous but not $\mathcal{C}^*\eta^*$-continuous.

Example 4.13. Let X=Y={a,b,c}, τ={X,$\emptyset$,{a,b}} and σ={Y,$\emptyset$,{b,c}}. Let f : X$\rightarrow$ Y be the identity function on X. Then f is $\mathcal{C}^*\eta^*$-continuous but not $\tilde{g}_\alpha$-continuous.

Example 4.14. Let X=Y={a,b,c}, τ={X,$\emptyset$,{a,c}} and σ={Y,$\emptyset$,{a,b}}. Let f : X $\rightarrow$ Y be the identity function on X. Then f is $\mathcal{C}^*\eta^*$-continuous but not $\mathcal{C}\eta^*$-continuous.

Example 4.15. Let X=Y={a,b,c}, τ={X,$\emptyset$,{b,c}} and σ={Y,$\emptyset$,{c}}. Let f : X $\rightarrow$ Y be the identity function on X. Then f is $\mathcal{C}\eta^*$-continuous but not $\mathcal{C}^*\eta^*$-continuous.

Theorem 4.16. For a function f : X → Y, the following are equivalent:

(a) f is α-continuous.

(b) f is $\mathcal{C}^*\eta^*$-continuous and $\tilde{g}_\alpha$-continuous.

Proof. The proof follows from Definitions 4.2 and 4.3 (f), Remark 4.4 and Theorem 3.11.

Theorem 4.17. For a function f : X → Y, the following are equivalent:

(a) f is $\tilde{g}_\alpha$-continuous.

(b) f is $\mathcal{C}\eta^{**}$-continuous and $\tilde{g}$-precontinuous.

Proof. The proof follows from Theorem 3.13

References

[1] AL-NASHEF B., A decomposition of α-continuity and semicontinuity, Acta Math Hungar., 97(1-2)(2002), 115-120.

[2] GANSTER M., REILLY I. L., Locally closed sets and LC-continuous functions, Internat. J. Math. Math. Sci., 12(1989), 417-424.

[3] GANSTER M., REILLY I. L., A decomposition of continuity, Acta Math Hungar., 56(1990), 299-301.

[4] HATIR E., NOIRI T., YUKSEL S., A decomposition of continuity, Acta Math Hungar., 70(1996), 145-150.

[5] JAFARI S., THIVAGAR M. L., NIRMALA REBECCA PAUL, Remarks on $\tilde{g}_\alpha$-closed sets in Topological spaces, International Mathematical Forum, 5(24) (2010), 1167-1178.

[6] LEVINE N., Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly, 70(1963), 36-41.

[7] MASHHOUR A. S., ABD EL-MONSEF M. E., EL-DEEP S. N., On precontinuous mappings and weak pre-continuous mappings, Proc. Math. Phys. Soc. Egypt., 53(1982), 47-53.

[8] MASHHOUR A. S, HASANEIN I. A., EL-DEEP S. N., α-continuous and α- open mappings, Acta Math. Hungar., 41(1983), 213-218.

[9] NJASTAD O., On some classes of nearly open sets, Pacific J. Math., 15(1965), 961-970.

[10] NOIRI T., RAJAMANI M., SUNDARAM P., A decomposition of a weaker form of continuity, Acta Math. Hungar., 93(1-2)(2001), 109-114.

[11] NOIRI T., SAYED O. R., On decomposition of continuity, Acta Math. Hungar., 111(1-2)(2006), 1-8.

[12] PIPITONE V., RUSSO G., spazi semiconnessi e spazi semiaperti, Rend. Circ. Mat. Palermo 24(2)(1975), 273-285.

[13] RAVI O., GANESAN S., CHANDRASEKAR S., $\tilde{g}$-preclosed sets in topological spaces (submitted).

[14] RAVI O., GANESAN S., CHANDRASEKAR S., $\tilde{g}$-precontinuity in topological spaces (submitted).

[15] REILLY I. L., VAMANAMURTHY M. R., On α-continuity in topological spaces, Acta Math Hungar., 45(1985), 27-32.

[16] SUNDARAM P., RAJESH N., THIVAGAR M. L., DUSZYNSKI Z., $\tilde{g}$- semiclosed sets in topological spaces, Mathematica Pannonica 18/1 (2007), 51-61.

[17] TONG J., A decomposition of continuity, Acta Math. Hungar., 48 (1986), 11-15.

[18] TONG J., A decomposition of continuity in topological spaces, Acta Math. Hungar, 54(1-2)(1989), 51-55.

[19] VEERAKUMAR M. K. R. S., $\hat{g}$-closed sets in topological spaces, Bull. Allahabad Math Soc., 18(2003), 99-112.

[20] VEERAKUMAR M. K. R. S., Between g*-closed sets and g-closed sets, Antarctica J. Math, Vol(3)(1)(2006), 43-65.

[21] VEERAKUMAR M. K. R. S., $^\sharp$g-semiclosed sets in topological spaces, Antarctica J. Math, 2(2)(2005), 201-222.

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