

A REMARK ON A RESULT OF AL-OMARI

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ABSTRACT. This is a note to explain a mathematical correction in the paper entitled The Generalized Stieltjes and Fourier Transforms of Certain Spaces of Generalized Functions, by S. K. Q. Al-Omari in Jordan Journal of Mathematics and Statistics 2 (2) , 2009, 55-66. Complete mathematical derivation is given to explain and justify the correction.

1. The Corrected Result

Refer to Theorem 3.1, equation (3.2) of Section 3 of the paper, which is expressed as

$$\left| (1+x)^\alpha x^k D^k (z+x)^{-p} \right| \leq \frac{|(p)_k| (1+x)^\alpha x^k e^{\pi|\Im(p)|}}{[(\sigma+x)^2 + \omega^2]^{\frac{1}{2(\Re(p+k))}}}$$

The derivation for the corrected result is as follows. We refer to Pathak, R. S., Integral Transforms of Generalized Functions and their Applications, Gordon and Breach Science Publishers, Australia, Canada, India, 1997. Further, we refer to Lemma 5.3.6, p. 158 of the book that is cited. We have

$$\left| (1+x)^\alpha x^k \left(\frac{d}{dx} \right) \left[\frac{1}{(s+x)^p} \right] \right| = \frac{(1+x)^\alpha x^k (-p)(-p-1) \dots (-p-k)}{|(s+x)^{p+k}|}$$

i.e.

$$= \frac{(1+x)^\alpha x^k |(p)_k|}{|(s+x)^{p+k}|}$$

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i.e.

$$\begin{aligned}
 &= \frac{(1+x)^\alpha x^k |(p)_k|}{|(\sigma + i\omega + x)^{p+k}|} \\
 &= \frac{(1+x)^\alpha x^k |(p)_k|}{|(\sigma + i\omega + x)^k| \cdot |(\sigma + i\omega + x)^p|} \quad .
 \end{aligned}$$

Further, we have the right hand side of the above expression, which is obtained by invoking the inequality (3.1) of the paper

$$= \frac{(1+x)^\alpha x^k |(p)_k|}{[(\sigma + x)^2 + \omega^2]^{\frac{k}{2}}} \cdot \frac{e^{\pi|\Im(p)|}}{[(\sigma + x)^2 + \omega^2]^{\frac{\Re(p)}{2}}}$$

i.e.

$$(A) \quad \leq \frac{(1+x)^\alpha x^k |(p)_k| e^{\pi|\Im(p)|}}{[(\sigma + x)^2 + \omega^2]^{\frac{\Re(p+k)}{2}}} .$$

It may be mention that the inequality (3.1) of this paper (p. 58) is given in Byrne, A. and Love, E. R. Complex Inversion Theorems for Generalized Stieltjes Transforms, J. Aust. Math. Soc. 18 (1974), 328-358.

As can be observed, the major correction in relation (3.2) of the paper and relation (A) above. Without this proposed correction the result proved by the author will be incorrect. Similar corrections (modifications) are advised in the paper wherever this formulation is used, say Eqn. (3.8), p. 59 of the paper.

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