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FIXED POINT RESULTS ON A NONSYMMETRIC G- METRIC SPACES

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ABSTRACT. We prove some fixed point results for mappings that satisfy certain contractive conditions on a nonsymmetric complete G-metric space. Moreover, we prove the uniqueness of these fixed point results.

1. Introduction

The class of G-metric spaces introduced by Z. Mustafa and B. Sims ([2], [3]) was to provide a new class of generalized metric spaces and to extend the fixed point theory for a variety of mappings. Moreover, many theorems was proved in this new setting with most of them recognizable as a counterparts of a well-known metric space theorems.

Definition 1.1. *G-metric space is a pair (X, G), where X is a nonempty set, and G is a nonnegative real-valued function defined on $X \times X \times X$ such that for all $x, y, z, a \in X$ we have:*

(G1) $G(x, y, z) = 0$ *if* $x = y = z$;

(G2) $0 < G(x, x, y)$; *for all* $x, y \in X$, *with* $x \neq y$;

(G3) $G(x, x, y) \leq G(x, y, z)$, *for all* $x, y, z \in X$, *with* $z \neq y$;

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(G4) $G(x, y, z) = G(x, z, y) = G(y, z, x) = \ldots$, *(symmetry in all three variables); and*

(G5) $G(x, y, z) \leq G(x, a, a) + G(a, y, z)$, *(rectangle inequality).*

*The function G is called a G-*metric *on X.*

Definition 1.2. ([3]) *A sequence (x_n) in a G-metric space X is said to converge if there exists $x \in X$ such that $\lim_{n,m\to\infty} G(x, x_n, x_m) = 0$, and one say that the sequence (x_n) is G-*convergent *to x .*

Proposition 1.3. ([3]) *Let X be G-metric space. Then the following statements are equivalent.*

(1) *(x_n) is G-convergent to x.*

(3) *$G(x_n, x_n, x) \to 0$, as $n \to \infty$.*

(4) *$G(x_n, x, x) \to 0$, as $n \to \infty$.*

(5) *$G(x_m, x_n, x) \to 0$, as $m, n \to \infty$.*

In a G-metric space X, a sequence (x_n) is said to be G-Cauchy if given $\epsilon > 0$, there is $N \in \mathbf{N}$ such that $G(x_n, x_m, x_l) < \epsilon$, for all $n, m, l \geq N$. That is $G(x_n, x_m, x_l) \longrightarrow 0$ as $n, m, l \longrightarrow \infty$.

Proposition 1.4. ([3]) *In a G-metric space X, the following statements are equivalent.*

(1) *The sequence (x_n) is G-Cauchy.*

(2) *For every $\epsilon > 0$, there exists $N \in \mathbf{N}$ such that $G(x_n, x_m, x_m) < \epsilon$, for all $n, m \geq N$.*

Let (X, G) and (X', G') be two G-metric spaces, and let $f : (X, G) \to (X', G')$ be a function, then f is said to be *G-continuous at a point $a \in X$* if and only if, given $\epsilon > 0$, there exists $\delta > 0$ such that $x, y \in X$; and $G(a, x, y) < \delta$ implies $G'(f(a), f(x), f(y)) < \epsilon$. A function f is G-continuous on X if it is G-continuous at all $a \in X$.

Proposition 1.5. ([3]) *Let (X, G), and (X', G') be two G-metric spaces. Then a function $f : X \longrightarrow X'$ is G-continuous at a point $x \in X$ if and only if it is G-sequentially continuous at x; that is, whenever (x_n) is G-convergent to x we have $(f(x_n))$ is G'-convergent to $f(x)$.*

A G-metric space (X, G) is called symmetric G-metric space if $G(x, y, y) = G(y, x, x)$ for all $x, y \in X$, and called Nonsymmetric if it is not Symmetric.

Example **1.6.** ([2]) *Let $(\mathbf{R}, d)$ be the usual metric space. Define G_s and G_m by*

$$G_s(x, y, z) = d(x, y) + d(y, z) + d(x, z), \textit{ and}$$

$$G_m(x, y, z) = \max\{d(x, y), d(y, z), d(x, z)\}$$

for all $x, y, z \in \mathbf{R}$. Then $(\mathbf{R}, G_s)$ and $(\mathbf{R}, G_m)$ are symmetric G-metric spaces.

Example **1.7.** ([2]) *Let $X = \{a, b, c\}$ and define $G : X \times X \times X \longrightarrow \boldsymbol{R}^+$ by,*

$G(x, y, z) = 0$ *if* $x = y = z$

$G(a, b, b) = G(b, a, a) = 22$

$G(a, c, c) = G(c, a, a) = 27$

$G(b, c, c) = G(c, b, b) = 30,$

$G(a, b, c) = 35$

extended by symmetry in the variables. It is easily verified that G is a symmetric G-metric, but $G \neq G_s$ or G_m for any underlying metric.

Corollary 1.8. ([2]) *Let (X, G) be a symmetric G-metric space, then G satisfies :*

(1) $G(x, y, z) \leq G(z, y, y) + G(y, x, x)$.

(2) $G(x, y, z) \leq G(y, x, x) + G(y, z, z)$.

(3) $G(x, x, y) \leq G(x, x, a) + G(a, a, y) = G(x, a, a) + G(a, y, y)$.

(4) $|G(x, x, a) - G(x, x, y)| \leq G(y, y, a)$.

Example **1.9.** ([3]) *Let $X = \{a, b\}$, and let,*

$G(a, a, a) = G(b, b, b) = 0,$

$G(a, a, b) = 1,\ G(a, b, b) = 2$

and extend G to $X \times X \times X$ by symmetry in the variables. Then it is easily verified that G is a G-metric, but $G(a, b, b) \neq G(a, a, b)$. Note that the above G-metric does not arise from any metric.

Proposition 1.10. ([3]) *Let X be a G-metric space, then the function $G(x, y, z)$ is jointly continuous in all three of its variables.*

A G-metric space X is said to be complete if every G-Cauchy sequence in X is G-convergent in X. In ([4]) The following theorem has been proved

Theorem 1.11. ([4]) *Let (X, G) be a complete G-metric space, and let $T : X \longrightarrow X$ be a mapping satisfying one of the following conditions*

$$G(T(x), T(y), T(z)) \leq \{aG(x, y, z) + bG(x, T(x), T(x)) + cG(y, T(y), T(y)) + dG(z, T(z), T(z))\} \tag{1.1}$$

or

$$G(T(x), T(y), T(z)) \leq \{aG(x, y, z) + bG(x, x, T(x)) + cG(y, y, T(y)) + dG(z, z, T(z))\} \tag{1.2}$$

for all $x, y, z \in X$ where $0 \leq a + b + c + d < 1$, then T has a unique fixed point (say u, i.e., $Tu = u$), and T is G-continuous at u.

The authors divide the proof on the above theorem into two parts. In the first part the theorem proved where the G-metric space assumed to be Symmetric, while in the second part the theorem proved where the G-metric space assumed to be nonsymmetric.

2. **Main Results**

Now we present several fixed point results on a nonsymmetric complete G-metric space.

Theorem 2.1. *Let (X,G) be a nonsymmetric complete G-metric space, let $T : X \longrightarrow X$, be a mapping which satisfies the following condition for all $x, y \in X$.*

$$G(T(x),T(y),T(y)) \le k \max \left\{ \begin{array}{c} [G(x,x,T(x)) + 2G(y,y,T(y))], \\ [G(x,x,T(y)) + G(y,y,T(y)) + \\ G(y,y,T(x))], [G(Ty,Tx,Tx) + \\ G(y,Ty,Ty) + G(x,Ty,Ty)] \end{array} \right\} \tag{2.1}$$

where $k \in [0, 1/4)$, then T has unique fixed point, say u, also T is G-continuous at u.

Proof. Let $x_0 \in X$ be arbitrary and define the sequence (x_n), by $x_n = T^n(x_0)$ and assume $x_n \neq x_{n+1}$ for all n. Then by (2.1) we have

$$G(x_{n+1},x_n,x_n) \le k \max \left\{ \begin{array}{l} [G(x_n,x_n,x_{n+1}) + \ 2\,G(x_{n-1},x_{n-1},x_n)], \\ \\ [G(x_{n-1},x_{n-1},x_n) + G(x_{n-1},x_{n-1},x_{n+1})], \\ \\ [G(x_n,x_{n+1},x_{n+1}) + G(x_{n-1},x_n,x_n)] \end{array} \right\} \tag{2.2}$$

But since

$$G(x_{n-1},x_{n-1},x_{n+1}) \le G(x_{n-1},x_{n-1},x_n) + G(x_n,x_n,x_{n+1}),$$

it follows that

$$G(x_{n-1}, x_{n-1}, x_n) + G(x_{n-1}, x_{n-1}, x_{n+1}) \leq G(x_n, x_n, x_{n+1}) + \; 2\, G(x_{n-1}, x_{n-1}, x_n).$$

Then (2.2) reduces to

$$G(x_{n+1}, x_n, x_n) \leq k \max \begin{Bmatrix} [G(x_n, x_n, x_{n+1}) + \; 2\, G(x_{n-1}, x_{n-1}, x_n)], \\ [G(x_n, x_{n+1}, x_{n+1}) + G(x_{n-1}, x_n, x_n)] \end{Bmatrix}, \tag{2.3}$$

we see that inequality (2.3) implies two cases,

Case1:

$$G(x_{n+1}, x_n, x_n) \leq k[G(x_n, x_n, x_{n+1}) + \; 2\, G(x_{n-1}, x_{n-1}, x_n)].$$

Therefore,

$$G(x_{n+1}, x_n, x_n) \leq \frac{2k}{1-k} G(x_{n-1}, x_{n-1}, x_n). \tag{2.4}$$

Case2:

$$G(x_{n+1}, x_n, x_n) \leq k[G(x_n, x_{n+1}, x_{n+1}) + G(x_{n-1}, x_n, x_n)]. \tag{2.5}$$

But since

$$G(x_n, x_{n+1}, x_{n+1}) \leq 2G(x_n, x_n, x_{n+1})$$

and

$$G(x_{n-1}, x_n, x_n) \leq 2G(x_{n-1}, x_{n-1}, x_n).$$

Then the estimate of the inequality (2.5) will be

$$G(x_n, x_n, x_{n+1}) \leq \frac{2k}{1-2k} G(x_{n-1}, x_{n-1}, x_n). \tag{2.6}$$

Putting $a = \frac{2k}{1-k}$ and $q = \frac{2k}{1-2k}$, then the fact $a < q$ implies that the inequalities (2.4) and (2.6) reduce to

$$G(x_{n+1}, x_n, x_n) \leq qG(x_{n-1}, x_{n-1}, x_n).$$

Solving this recursive inequality we have

$$G(x_n, x_n, x_{n+1}) \le q^n G(x_0, x_0, x_1). \tag{2.7}$$

For all $n, m \in \mathbf{N}$; $n < m$, we use repeated rectangle inequality and (2.7) to obtain

$$\begin{aligned} G(x_n, x_n, x_m) &\le \sum_{j=n}^{m-1} G(x_j, x_j, x_{j+1}) \\ &\le \sum_{j=n}^{m-1} q^j G(x_0, x_0, x_1) \\ &\le \frac{q^n}{1-q} G(x_0, x_0, x_1) \end{aligned}$$

which implies that (x_n) is a G-Cauchy sequence in the complete G-metric space X. Thus, the sequence (x_n) is G-convergent to u in X.

Now let us prove that $T(u) = u$. To do so, we assume on the contrary that $T(u) \neq u$. Then,

$$G(T(u), x_n, x_n) \le k \max \left\{ \begin{array}{l} [G(u, u, Tu) + \ 2\, G(x_{n-1}, x_{n-1}, x_n)], \\ {[G(u, u, x_n) + G(x_{n-1}, x_{n-1}, x_n) +} \\ G(x_{n-1}, x_{n-1}, Tu)], [G(x_n, Tu, Tu) \\ \ + G(x_{n-1}, x_n, x_n) + G(u, x_n, x_n)] \end{array} \right\}$$

Letting $n \longrightarrow \infty$ and using the fact that the function G is continuous in its variables we obtain

$$G(u, u, Tu) \le k \max\{G(u, u, Tu), G(u, Tu, Tu)\}. \tag{2.8}$$

But since

$$G(u, Tu, Tu) \le 2G(u, u, Tu),$$

the inequality (2.8) implies that

$$G(u,u,Tu) \le 2kG(u,u,Tu).$$

This contradiction implies that $Tu = u$.

To Prove Uniqueness, suppose that $v \neq u$ such that $Tv = v$, then

$$G(u,v,v) = G(Tu,Tv,Tv) \le k \max \begin{Bmatrix} [G(u,u,u) + \ 2\,G(v,v,v], \\ [G(u,u,v) + G(v,v,u], \\ [G(v,u,u) + G(u,v,v)] \end{Bmatrix}$$

then,

$$G(u,v,v) \le k[G(v,u,u) + G(u,v,v)].$$

Therefore, $G(u,v,v) \le \frac{k}{1-k}G(v,u,u)$.

Similarly we get

$$G(v,u,u) \le \frac{k}{1-k}G(u,v,v).$$

Consequently,

$$G(u,v,v) \le (\frac{k}{1-k})^2 G(u,v,v)$$

which implies that $u = v$, since $0 < (\frac{k}{1-k})^2 < 1$.

To show that T is G-continuous at u, let $(y_n) \subseteq X$ be a sequence such that $\lim(y_n) = u$, then, from (2.1) we deduce that

$$G(T(y_n),T(u),T(u)) \le k \max \begin{Bmatrix} [\,G(y_n,y_n,T(y_n)) + 2\,G(u,u,T(u))], \\ [G(y_n,y_n,T(u)) + G(u,u,T(u)) + \\ G(u,u,T(y_n))], [G(T(u),T(y_n),T(y_n)) \\ +G(u,T(u),T(u)) + G(y_n,u,u,)] \end{Bmatrix}. \tag{2.9}$$

By the rectangle inequality,

$$G(y_n,y_n,T(y_n)) \le G(y_n,y_n,u) + G(u,u,T(y_n)).$$

Hence, we deduce that the equation (2.9) becomes

$$G(T(y_n), T(u), T(u)) \leq k \max \left\{ \begin{array}{c} [G(y_n, y_n, u) + G(u, u, T(y_n))], \\ [G(T(y_n), T(y_n), u) + G(y_n, u, u)] \end{array} \right\}. \tag{2.10}$$

We see that equation (2.10) implies two cases

Case (1) :

$$G(T(y_n), T(u), T(u) \leq k\{ G(y_n, y_n, u) + G(u, u, T(y_n))\}, \tag{2.11}$$

then equation (2.11) becomes

$$G(T(y_n), T(u), T(u)) \leq \frac{k}{1-k} G(y_n, y_n, u), \tag{2.12}$$

but since $G(y_n, y_n, u) \leq 2G(y_n, u, u)$, we will have

$$G(T(y_n), T(u), T(u)) \leq \frac{2k}{1-k} G(y_n, u, u). \tag{2.13}$$

Case (2) :

$$G(T(y_n), T(u), T(u) \leq k\{G(T(y_n), T(y_n), u) + G(y_n, u, u)\}. \tag{2.14}$$

By rectangle inequality,

$$G(T(y_n), T(y_n), u) \leq 2G(Ty_n, u, u). \tag{2.15}$$

Therefore, the inequality (2.14) implies that

$$G(T(y_n), T(u), T(u) \leq k\{2G(Ty_n, u, u) + G(y_n, u, u)\},$$

which implies that

$$G(T(y_n), T(u), T(u) \leq \frac{k}{1-2k} G(y_n, u, u), \tag{2.16}$$

In equations, (2.13) and (2.16), taking the limit as $n \to \infty$, we see that $G(u, T(y_n), T(y_n)) \to 0$ and so, by Proposition 1.5, we have $T(y_n) \to u = Tu$. This implies that T is G-continuous at u. □

Corollary 2.2. *Let X be a complete nonsymmetric G-metric space, let $T : X \longrightarrow X$, be a mapping which satisfies the following condition, for all $x, y, z \in X$*

(2.17)
$$G(T(x), T(y), T(z)) \leq k \max \left\{ \begin{array}{l} [G(x, x, T(x)) + G(y, y, T(y)) + G(z, z, Tz)], \\ [G(x, x, T(y)) + G(y, y, T(z)) + G(z, z, T(x))], \\ [G(Tz, Tx, T(x)) + G(z, T(y), T(y)) + \\ G(x, T(z), T(z))] \end{array} \right\}$$

where $k \in [0, 1/4)$, then T has unique fixed point, say u, and T is G-continuous at u.

Proof. Taking $z = y$ in condition (2.17) it reduced to condition (2.1) in Theorem (2.1), so, the proof follows from Theorem (2.1). □

Theorem 2.3. *Let X be a complete nonsymmetric G-metric space, and let $T : X \longrightarrow X$, be a mapping which satisfies the following condition for all $x, y, z \in X$,*

$$G(T(x), T(y), T(z)) \leq k \max \left\{ \begin{array}{l} [G(x, x, T(y)) + G(y, y, T(x))], \\ [G(y, y, T(z)) + G(z, z, T(y))], \\ [G(x, x, T(z)) + G(z, z, T(x))] \end{array} \right\} \tag{2.18}$$

where $k \in [0, 1/2)$, then T has unique fixed point (say u), and T is G-continuous at u.

Proof. For $x_0 \in X$, define the sequence (x_n) by $x_n = T^n(x_0)$, assume that $x_n \neq x_{n+1}$ for all n, then by (2.18) we get

$$G(x_n,x_n,x_{n+1}) \leq \ k \ \max \left\{ \begin{array}{l} [G(x_{n-1},x_{n-1},x_n) + \ G(x_{n-1},x_{n-1},x_n)], \\ [\, G(x_{n-1},x_{n-1},x_{n+1}) + \ G(x_n,x_n,x_n)], \\ [\, G(x_{n-1},x_{n-1},x_{n+1}) + G(x_n,x_n,x_n)] \end{array} \right\} \tag{2.19}$$

$$= k \max\{G(x_{n-1},x_{n-1},x_{n+1}),\ 2\ G(x_{n-1},x_{n-1},x_n)\}$$

therefore equation (2.19) implies two cases:

Case1:

$$G(x_n,x_n,x_{n+1}) \leq kG(x_{n-1},x_{n-1},x_{n+1}) \tag{2.20}$$

but from rectangle inequality we have the estimate

$$G(x_{n-1},x_{n-1},x_{n+1}) \leq G(x_{n-1},x_{n-1},x_n) + G(x_n,x_n,x_{n+1}),$$

so equation (2.20) implies that

$$G(x_n,x_n,x_{n+1}) \ \leq \ \frac{k}{1-k} \ G(x_{n-1},x_{n-1},x_n), \tag{2.21}$$

Case 2:

$$G(x_n,x_n,x_{n+1}) \leq 2kG(x_{n-1},x_{n-1},x_n).$$

Putting $q = \max\{\frac{k}{1-k}, 2k\}$ and using the fact $k < \frac{1}{2}$, we will have $q < 1$. So,

$$G(x_n,x_n,x_{n+1}) \leq qG(x_{n-1},x_{n-1},x_n). \tag{2.22}$$

Solving this recursive inequality we have

$$G(x_n,x_n,x_{n+1}) \ \leq q^n G(x_0,x_0,x_1). \tag{2.23}$$

For all $n, m \in \mathbf{N}$; $n < m$, we use repeated rectangle inequality and equation (2.23) to obtain

$$\begin{aligned} G(x_n, x_n, x_m) &\leq \sum_{j=n}^{m-1} G(x_j, x_j, x_{j+1}) \\ &\leq \sum_{j=n}^{m-1} q^j G(x_0, x_0, x_1) \\ &\leq \frac{q^n}{1-q} G(x_0, x_0, x_1). \end{aligned}$$

Which implies that (x_n) is a G-Cauchy sequence in the complete G-metric space X. Thus, the sequence (x_n) is G-convergent to u in X.

Now suppose that $T(u) \neq u$, then

$$G(x_n, x_n, T(u)) \leq k \max \left\{ \begin{array}{l} [G(x_{n-1}, x_{n-1}, x_n) + G(x_{n-1}, x_{n-1}, x_n)], \\ \\ [G(x_n, x_n, T(u)) + G(u, u, x_n)], \\ \\ [G(x_{n-1}, x_{n-1}, T(u)) + G(u, u, x_n)] \end{array} \right\}.$$

Letting $n \longrightarrow \infty$, and using the fact that the function G is continuous in its variables, we get

$$G(u, u, T(u)) \leq kG(u, u, T(u)).$$

This contradiction implies that $u = T(u)$.

To prove uniqueness, suppose that $v \neq u$ such that $T(v) = v$, then

$$G(u, v, v) \leq k \max \left\{ \begin{array}{l} [G(u, u, v) + G(v, v, u)], \\ \\ [G(v, v, v) + G(v, v, v)], \\ \\ [G(u, u, v) + G(v, v, u)] \end{array} \right\}, \tag{2.24}$$

so we deduce that

$$G(u, v, v) \leq k [G(u, u, v) + G(v, v, u)].$$

Therefore,

$$G(u,v,v) \leq \frac{k}{1-k} G(v,u,u).$$

Similarly,

$$G(v,u,u) \leq \frac{k}{1-k} G(u,v,v),$$

thus we have,

$$G(u,v,v) \leq (\frac{k}{1-k})^2 G(u,v,v).$$

This contradiction implies that $u = v$.

To show that T is G-continuous at u, consider $(y_n) \subseteq X$ any sequence such that $\lim(y_n) = u$ in X, then

$$G(T(y_n),T(u),T(u)) \ \leq k \ \max \left\{ \begin{array}{l} [G(y_n,y_n,T(u)) + G(u,u,T(y_n))], \\ \\ [\,G(u,u,T(u)) + G(u,u,T(u))] \\ \\ [\,G(y_n,y_n,T(u)) + G(u,u,T(y_n))] \end{array} \right\}. \tag{2.25}$$

Thus, equation (2.25) reduces to

$$G(T(y_n),u,u) \ \leq k[G(y_n,y_n,u) + G(u,u,T(y_n))]. \tag{2.26}$$

Therefore the inequality (2.26) implies that

$$G(T(y_n),u,u) \leq \frac{k}{1-k} G(y_n,u,u). \tag{2.27}$$

Taking the limit of (2.27) as $n \to \infty$ and using Proposition 1.5, we have
$T(y_n) \to u = Tu$ which implies that T is G-continuous at u. □

Corollary 2.4. *Let X be a complete nonsymmetric G-metric space, and let $T : X \longrightarrow X$ be a mapping satisfies the following condition*

$$G(T(x),T(y),T(z)) \leq k\{G(x,x,T(x)) + G(y,y,T(y)) + G(z,z,T(z))\} \tag{2.28}$$

for all $x, y, z \in X$, where $k \in [0, 1/3)$. Then T has a unique fixed point (say u), and T is G-continuous at u.

Proof. By taking $a = 0, b = c = d = k$ in condition (1.2) of Theorem (1.11), then condition (2.28), becomes special case of Theorem (1.11), so the proof follows from Theorem (1.11) □

Corollary 2.5. *Let (X, G) be a complete nonsymmetric G-metric space, and let $T : X \longrightarrow X$, be a mapping satisfying the following condition*

$$G(T(x), T(y), T(z)) \leq \alpha G(x, y, z) + \beta \begin{Bmatrix} G(y, y, T(y)) + G(z, z, T(z)) + \\ G(x, x, T(x)) \end{Bmatrix} \tag{2.29}$$

for all $x, y, z \in X$, where $0 \leq \alpha + 3\beta < 1$. Then T has unique fixed point (say u), and T is G-continuous at u.

Proof. By taking $\alpha = a, b = c = d = \beta$ in condition (1.2) of Theorem (1.11), then condition (2.29), becomes special case of Theorem (1.11), so the proof follows from Theorem (1.11) □

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