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CHARACTERIZATIONS OF NEARLY LINDELÖF SPACES

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Abstract. A topological space (X, τ) is said to be nearly Lindelöf if every regular open cover of (X, τ) admits a countable subcover. In this paper, we obtain new characterizations and preservation theorems of nearly Lindelöf spaces.

1. Introduction

Among the various covering properties of topological spaces, a lot of attention has been made to those covers which involve open and regularly open sets. In [2], the notion of nearly Lindelöf spaces was introduced. In [3] and [4], the further properties of nearly Lindelöf spaces were studied. These spaces were considered as one of the main generalizations of Lindelöf spaces. The notion of regular open sets plays an important role in the study of nearly Lindelöf spaces. In this paper, first we introduce and study the notion of ω-regular open sets as a generalization of regular open sets. Then, by using ω-regular open sets, we obtain new characterizations and the further preservation theorems of nearly Lindelöf spaces. Throughout this paper, (X, τ) and (Y, σ) stand for topological spaces on which no separation axiom is assumed, unless otherwise stated. For a subset A of X, the closure of A and the interior of A will be denoted by $Cl(A)$ and $Int(A)$, respectively. A subset A of a topological space X is

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said to be regular open (resp. regular closed) if $A = Int(Cl(A))$ (resp. $A = Cl(Int(A))$. A point $x \in X$ is called a condensation point of A if for each $U \in \tau$ with $x \in U$, the set $U \cap A$ is uncountable. A is said to be ω-closed [5] if it contains all its condensation points. The complement of an ω-closed set is said to be ω-open. It is well known that a subset W of a space (X, τ) is ω-open if and only if for each $x \in W$, there exists $U \in \tau$ such that $x \in U$ and $U - W$ is countable.

2. ω-regular open sets

In this section we introduce the following notion:

Definition 2.1. A subset A is said to be *ω-regular open* if for each $x \in A$ there exists a regular open set U_x containing x such that $U_x - A$ is a countable set. The complement of an ω-regular open subset is said to be ω-regular closed.

The family of all ω-regular open (resp. regular open, regular closed) subsets of a space (X, τ) is denoted by $\omega RO(X)$ (resp. $RO(X)$, $RC(X)$).
For a subset of a topological space, the following implications hold and none of these implications is reveresible.

DIAGRAM I

$$\begin{array}{ccc} \text{regular open} & \longrightarrow & \omega\text{-regular open} \\ \downarrow & & \downarrow \\ \text{open} & \longrightarrow & \omega\text{-open} \end{array}$$

Example 2.1. *Let $\mathbb{R}$ be the set of all real numbers with the usual topology and $\mathbb{Q}$ the set of all rational numbers. Then $A = \mathbb{R} - \mathbb{Q}$ is an ω-regular open set but it is not open.*

Example 2.2. *Let X be a set and A be subsets of X such that A and $X - A$ are uncountable. Let τ=$\{\phi, X, A\}$. Then $\{A\}$ is an open set but it is not ω-regular open.*

Theorem 2.1. *Let (X, τ) be a topological space. Then $(X, \omega RO(X))$ is a topological space.*

Proof. (1): We have $\phi, X \in \omega RO(X)$.
(2): Let $U, V \in \omega RO(X)$ and $x \in U \cap V$. Then there exist regular open sets G, H of X containing x such that $G \setminus U$ and $H \setminus V$ are countable. And
$(G \cap H) \setminus (U \cap V) = (G \cap H) \cap ((X \setminus U) \cup (X \setminus V)) \subseteq (G \cap (X \setminus U)) \cup (H \cap (X \setminus V))$.
Thus $(G \cap H) \setminus (U \cap V)$ is countable. Since the intersection of two regular open sets is regular open, $U \cap V \in \omega RO(X)$.
(3): Let $\{U_i : i \in I\}$ be a family of ω-regular open subsets of X and $x \in \underset{i \in I}{\cup} U_i$. Then $x \in U_j$ for some $j \in I$. This implies that there exists a regular open subset V of X containing x such that $V \setminus U_j$ is countable. Since $V \setminus \underset{i \in I}{\cup} U_i \subseteq V \setminus U_j$, then $V \setminus \underset{i \in I}{\cup} U_i$ is countable. Thus $\underset{i \in I}{\cup} U_i \in \omega RO(X)$. □

Lemma 2.1. *A subset A of a space X is ω-regular open if and only if for every $x \in A$, there exist a regular open subset U_x containing x and a countable subset C such that $U_x - C \subseteq A$.*

Proof. Let A be ω-regular open and $x \in A$, then there exists a regular open subset U_x containing x such that $U_x - A$ is countable. Let $C = U_x - A = U_x \cap (X - A)$. Then $U_x - C \subseteq A$. Conversely, let $x \in A$. Then there exist a regular open subset U_x containing x and a countable subset C such that $U_x - C \subseteq A$. Thus $U_x - A \subseteq C$ and $U_x - A$ is a countable set. □

Theorem 2.2. *Let X be a space and $F \subseteq X$. If F is ω-regular closed, then $F \subseteq K \cup C$ for some regular closed subset K and a countable subset C.*

Proof. If F is ω-regular closed, then $X - F$ is ω-regular open and hence choose $x \in X - F$, there exist a regular open set U_x containing x and a countable set C_x such that $U_x - C_x \subseteq X - F$. Thus $F \subseteq X - (U_x - C_x) = X - (U_x \cap (X - C_x)) = (X - U_x) \cup C_x$. Let $K = X - U_x$. Then K is a regular closed set such that $F \subseteq K \cup C_x$. □

3. Nearly Lindelöf spaces

Definition 3.1. (1) A topological space X is said to be *nearly Lindelöf* [2] if every cover of X by regular open sets admits a countable subcover.

(2) A subset A of a space X is said to be *nearly Lindelöf relative to* X [3] if every cover of A by regular open sets of X admits a countable subcover.

Theorem 3.1. *For any space X, the following properties are equivalent:*

(1) *X is nearly Lindelöf;*

(2) *Every ω-regular open cover of X admits a countable subcover.*

Proof. (1) ⇒ (2): Let $\{U_\alpha : \alpha \in \Lambda\}$ be any ω-regular open cover of X. For each $x \in X$, there exists $\alpha(x) \in \Lambda$ such that $x \in U_{\alpha(x)}$. Since $U_{\alpha(x)}$ is ω-regular open, there exists a regular open set $V_{\alpha(x)}$ such that $x \in V_{\alpha(x)}$ and $V_{\alpha(x)} \backslash U_{\alpha(x)}$ is countable. The family $\{V_{\alpha(x)} : x \in X\}$ is a regular open cover of X. Since X is nearly Lindelöf, there exist $\{x_i : i < \omega\} \subseteq X$ such that $X = \cup\{V_{\alpha(x_i)} : i < \omega\}$. Now, we have

$$\begin{aligned} X &= \bigcup_{i<\omega} \left((V_{\alpha(x_i)} \backslash U_{\alpha(x_i)}) \cup U_{\alpha(x_i)}\right) \\ &= \left(\bigcup_{i<\omega} (V_{\alpha(x_i)} \backslash U_{\alpha(x_i)})\right) \cup \left(\bigcup_{i<\omega} U_{\alpha(x_i)}\right) \end{aligned}.$$

For each $\alpha(x_i)$, $V_{\alpha(x_i)} \backslash U_{\alpha(x_i)}$ is a countable set and there exists a countable subset $\Lambda_{\alpha(x_i)}$ of Λ such that $V_{\alpha(x_i)} \backslash U_{\alpha(x_i)} \subseteq \cup\{U_\alpha : \alpha \in \Lambda_{\alpha(x_i)}\}$. Therefore, we have

$$X \subseteq \left(\bigcup_{i<\omega} (\cup\{U_\alpha : \alpha \in \Lambda_{\alpha(x_i)}\})\right) \cup \left(\bigcup_{i<\omega} U_{\alpha(x_i)}\right).$$

(2) ⇒ (1): Since every regular open set is ω-regular open, the proof is obvious. □

We state the following proposition without proof.

Proposition 3.1. *A topological space X is nearly Lindelöf if and only if for every family of ω-regular closed sets $\{F_\alpha : \alpha \in \Lambda\}$ of X, $\cap_{\alpha\in\Lambda} F_\alpha = \phi$ implies that there exists a countable subset $\Lambda_0 \subseteq \Lambda$ such that $\cap_{\alpha\in\Lambda_0} F_\alpha = \phi$.*

Proposition 3.2. *A topological space X is nearly Lindelöf if and only if for every family $\{F_\alpha : \alpha \in \Lambda\}$ of ω-regular closed sets with countable intersection property, $\cap_{\alpha\in\Lambda} F_\alpha \neq \phi$.*

Proof. Necessity. Let X be a nearly Lindelöf space and suppose that $\{F_\alpha : \alpha \in \Lambda\}$ be a family of ω-regular closed subsets of X with countable intersection property such that $\cap_{\alpha\in\Lambda} F_\alpha = \phi$. Let us consider the ω-regular open sets $U_\alpha = X \setminus F_\alpha$, the family $\{U_\alpha : \alpha \in \Lambda\}$ is an ω-regular open cover of the space X. Since X is nearly Lindelöf, the cover $\{U_\alpha : \alpha \in \Lambda\}$ has a countable subcover $\{U_{\alpha_i} : \alpha_i \in \mathbb{N}\}$. Therefore $X = \cup\{U_{\alpha_i} : \alpha_i \in \mathbb{N}\} = \cup\{(X \setminus F_{\alpha_i}) : \alpha_i \in \mathbb{N}\} = X \setminus \cap\{F_{\alpha_i} : \alpha_i \in \mathbb{N}\}$ whence $\cap\{F_{\alpha_i} : \alpha_i \in \mathbb{N}\} = \phi$. Thus, if the family $\{F_\alpha : \alpha \in \Lambda\}$ of ω-regular closed sets with countable intersection property, then $\cap_{\alpha\in\Lambda} F_\alpha \neq \phi$.

Sufficiency . Let $\{U_\alpha : \alpha \in \Lambda\}$ be an ω-regular open cover of X and suppose that for every family $\{F_\alpha : \alpha \in \Lambda\}$ of ω-regular closed sets with countable intersection property, $\cap_{\alpha\in\Lambda} F_\alpha \neq \phi$. Then $X = \cup\{U_\alpha : \alpha \in \Lambda\}$. Therefore,
$\phi = X \setminus X = \cap\{(X \setminus U_\alpha) : \alpha \in \Lambda\}$ and $\{X \setminus U_\alpha : \alpha \in \Lambda\}$ is a family of
ω-regular closed sets with an empty intersection. By the hypothesis, there exists a countable subset $\{(X \setminus U_{\alpha_i}) : i \in \mathbb{N}\}$ such that $\{\cap(X \setminus U_{\alpha_i}) : i \in \mathbb{N}\} = \phi$; hence $X \setminus \{\cap(X \setminus U_{\alpha_i}) : i \in \mathbb{N}\} = X = \cup\{U_{\alpha_i} : i \in \mathbb{N}\}$. Thus, X is nearly Lindelöf. □

Theorem 3.2. *Every ω-regular closed subset of a nearly Lindelöf space X is nearly Lindelöf relative to X.*

Proof. Let A be an ω-regular closed subset of X. Let $\{U_\alpha : \alpha \in \Lambda\}$ be a cover of A by regular open sets of X. Now for each $x \in X - A$, there is a regular open set V_x such that $V_x \cap A$ is countable. Since X is nearly Lindelöf and the collection $\{U_\alpha : \alpha \in \Lambda\} \cup \{V_x : x \in X - A\}$ is a regular open cover of X, there exists a countable subcover $\{U_{\alpha_i} : i \in \mathbb{N}\} \cup \{V_{x_i} : i \in \mathbb{N}\}$. Since $\underset{i \in N}{\cup}(V_{x_i} \cap A)$ is countable, so for each $x_j \in \cup(V_{x_i} \cap A)$, there is $U_{\alpha(x_j)} \in \{U_\alpha : \alpha \in \Lambda\}$ such that $x_j \in U_{\alpha(x_j)}$ and $j \in \mathbb{N}$. Hence $\{U_{\alpha_i} : i \in \mathbb{N}\} \cup \{U_{\alpha(x_j)} : j \in \mathbb{N}\}$ is a countable subcover of $\{U_\alpha : \alpha \in \Lambda\}$ and it covers A. Therefore, A is nearly Lindelöf relative to X. □

Corollary 3.1. [2] *Every regular closed subset of a nearly Lindelöf space X is nearly Lindelöf relative to X.*

The topology generated by the regular open subsets of the space X is denoted by τ^* and it is called the semiregularization of X. If $\tau = \tau^*$, then X is said to be semi-regular.

Definition 3.2. A topological space (X, τ) is said to be *semi ω-regular* if for each $x \in X$ and each open set U_x containing x, there exists an ω-regular open set H_x such that $x \in H_x \subseteq U_x$.

Proposition 3.3. *A semi ω-regular space is nearly Lindelöf if and only if it is Lindelöf.*

Proof. Let X be a semi ω-regular space. Suppose that X is a nearly Lindelöf space and let $\mathcal{U} = \{U_\alpha : \alpha \in \Lambda\}$ be any open cover of X. For each $x \in X$, there exists $\alpha(x) \in \Lambda$ such that $x \in U_{\alpha(x)}$. Since X is semi ω-regular, $x \in H_{\alpha(x)} \subseteq U_{\alpha(x)}$. Then $\{H_{\alpha(x)} : x \in X\}$ is an ω-regular open cover of X. By Theorem 3.1, there exists a countable subcover $\{H_{\alpha(x_i)} : i \in \mathbb{N}\}$. Therefore, $\{U_{\alpha(x_i)} : i \in \mathbb{N}\}$ is a countable subcover of $\mathcal{U}$. Hence X is Lindelöf. The converse is obvious. □

Corollary 3.2. [2] *A semi-regular space is nearly Lindelöf if and only if it is Lindelöf.*

Definition 3.3. A topological space (X, τ) is said to be *weakly regular* if for each $x \in X$ and each open set U_x containing x, there exists an ω-regular open set V_x such that $x \in V_x \subseteq Cl(V_x) \subseteq U_x$.

Corollary 3.3. *A weakly regular, nearly Lindelöf space is Lindelöf.*

Proof. Every weakly regular space is semi ω-regular, the proof is obvious from Proposition 3.3. □

Definition 3.4. A topological space X is said to be *almost ω-regular* if for each $x \in X$ and each ω-regular open U_x containing x, there exists an open set V_x containing x such that $V_x \subseteq Cl(V_x) \subseteq U_x$, or equivalently, if for any ω-regular closed set C and any singleton $\{x\}$ disjoint from C, there exist disjoint open sets U and V such that $C \subseteq U$ and $x \in V$.

Proposition 3.4. *For a space (X, τ), the following are equivalent:*

(1) *(X, τ) almost ω-regular;*
(2) *for each $x \in X$ and each ω-regular open U_x containing x, there exists a regular open set V_x such that $x \in V_x \subseteq Cl(V_x) \subseteq U_x$.*

Proof. The proof follows from the definition of almost ω-regular spaces and the fact that if V is an open set in a space (X, τ) then $Int(Cl(V))$ is regular open. □

Theorem 3.3. *Let X be an almost ω-regular and nearly Lindelöf space. Then for every disjoint ω-regular closed sets C_1 and C_2, there exist two open sets U and V such that $C_1 \subseteq U$, $C_2 \subseteq V$ and $U \cap V = \phi$.*

Proof. Since X is almost ω-regular, for each $x \in C_1$ by Proposition 3.4 there exists a regular open set U_x containing x such that $Cl(U_x) \cap C_2 = \phi$. Then the family

$\{U_x : x \in C_1\} \cup \{X - C_1\}$ is an ω-regular open cover of X. Since X is nearly Lindelöf, by Theorem 3.1 there exists $\{x_i : i < \omega\} \subseteq X$ such that $X = \left(\bigcup_{i<\omega} U_{x_i}\right) \cup (X - C_1)$. It follows that for each $i < \omega$, $C_1 \subseteq \left(\bigcup_{i<\omega} U_{x_i}\right)$ and $Cl(U_{x_i}) \cap C_2 = \phi$. Analogously there exists a family of regular open sets V_{y_i} such that $C_2 \subseteq \left(\bigcup_{i<\omega} V_{y_i}\right)$ and $Cl(V_{y_i}) \cap C_1 = \phi$. Let $G_k = U_{x_k} \setminus \left(\bigcup_{i=1}^{k} Cl(V_{y_i})\right)$, $H_k = V_{y_k} \setminus \left(\bigcup_{i=1}^{k} Cl(U_{x_i})\right)$ and $U = \bigcup_{i<\omega} G_i$, $V = \bigcup_{i<\omega} H_i$ such that U and V are open, $U \cap V = \phi$ and $C_1 \subseteq U$, $C_2 \subseteq V$. □

4. **Preservation theorems**

Definition 4.1. [6] A function $f : X \to Y$ is said to be *δ-continuous* if for each $x \in X$ and each regular open set V of Y containing $f(x)$, there exists a regular open set U of X containing x such that $f(U) \subseteq V$.

Definition 4.2. A function $f : X \to Y$ is said to be *δ_ω-continuous* if for each $x \in X$ and each regular open set V of Y containing $f(x)$, there exists an ω-regular open set U of X containing x such that $f(U) \subseteq V$.

It is clear that every δ-continuous function is δ_ω-continuous but the converse is not true.

Example 4.1. *Let $X = \{a, b, c, d\}$ and $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$. Then the function $f : (X, \tau) \to (X, \tau)$, defined as follows: $f(a) = c$, $f(b) = d$, $f(c) = a$ and $f(d) = b$, is δ_ω-continuous but it is not δ-continuous.*

Theorem 4.1. *Let $f : X \to Y$ be a δ_ω-continuous surjection from X onto Y. If X is nearly Lindelöf, then Y is nearly Lindelöf.*

Proof. Let $\{V_\alpha : \alpha \in \Lambda\}$ be a regular open cover of Y. For each $x \in X$, there exists $\alpha(x) \in \Lambda$ such that $f(x) \in V_{\alpha(x)}$. Since f is δ_ω-continuous, there exists an ω-regular open set $U_{\alpha(x)}$ of X containing x such that $f(U_{\alpha(x)}) \subseteq V_{\alpha(x)}$. So $\{U_{\alpha(x)} : x \in X\}$ is an ω-regular open cover of the nearly Lindelöf space X, by Theorem 3.1 there exists a countable subset $\{x_k : k < \omega\} \subseteq X$ such that $X = \underset{k<\omega}{\cup} U_{\alpha(x_k)}$. Therefore

$Y = f(X) = f(\underset{k<\omega}{\cup} U_{\alpha(x_k)}) = \underset{k<\omega}{\cup} f\left(U_{\alpha(x_k)}\right) \subseteq \underset{k<\omega}{\cup} V_{\alpha(x_k)}$. This shows that Y is nearly Lindelöf. □

Corollary 4.1. [4] *Let $f : X \to Y$ be a δ-continuous surjection from X onto Y. If X is nearly Lindelöf, then Y is nearly Lindelöf.*

Definition 4.3. [8] A function $f : X \to Y$ is said to be *almost open in the sense of Singal*, written as *a.o.S.* if the image of each regular open set U of X is an open set in Y.

Proposition 4.1. *If $f : X \to Y$ is a.o.S., then the image of an ω-regular open set of X is ω-open in Y.*

Proof. Let $f : X \to Y$ be *a.o.S.* and W an ω-regular open subset of X. Let $y \in f(W)$, there exists $x \in W$ such that $f(x) = y$. Since W is ω-regular open, there exists a regular open set U such that $x \in U$ and $U - W = C$ is countable. Since f is *a.o.S.*, $f(U)$ is an open set in Y such that $y = f(x) \in f(U)$ and

$f(U) - f(W) \subseteq f(U - W) = f(C)$. Moreover, $f(C)$ is countable. Therefore, $f(W)$ is ω-open in Y. □

Definition 4.4. A function $f : X \to Y$ is said to be *ωR-continuous* if $f^{-1}(V)$ is ω-regular open in X for each open set V in Y.

Theorem 4.2. *Let f be an ωR-continuous function from a space X onto a space Y. If X is nearly Lindelöf, then Y is Lindelöf.*

Proof. Let $\{V_\alpha : \alpha \in \Lambda\}$ be an open cover of Y. Then $\{f^{-1}(V_\alpha) : \alpha \in \Lambda\}$ is an ω- regular open cover of X. Since X is nearly Lindelöf, by Theorem 3.1, X has a countable subcover, say $\{f^{-1}(V_\alpha) : \alpha \in \Lambda_0\}$, where Λ_0 is a countable subset of Λ. Hence $\{V_\alpha : \alpha \in \Lambda_0\}$ is a countable subcover of Y. Hence Y is Lindelöf. □

Definition 4.5. A function $f : X \to Y$ is said to be *completely continuous* [1] if $f^{-1}(V)$ is regular open for each open set V in Y.

Corollary 4.2. *Let f be a completely continuous function from a space X onto a space Y. If X is nearly Lindelöf, then Y is Lindelöf.*

Definition 4.6. A function $f : X \to Y$ is said to be *ωR-closed* if $f(A)$ is ω-regular closed in Y for each regular closed set A of X.

Definition 4.7. A topological space is called a *P_r^*-space* if every countable union of regular open sets is regular open.

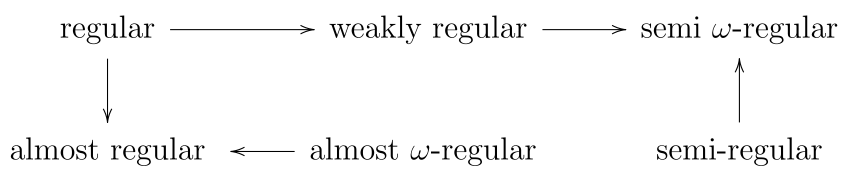
Theorem 4.3. *If $f : X \to Y$ is an ωR-closed surjection such that $f^{-1}(y)$ is nearly Lindelöf relative to X for each $y \in Y$, X is a P_r^*-space and Y is nearly Lindelöf, then X is nearly Lindelöf.*

Proof. Let $\{U_\alpha : \alpha \in \Lambda\}$ be any regular open cover of X. For each $y \in Y$, $f^{-1}(y)$ is nearly Lindelöf relative to X and there exists a countable subset $\Lambda(y)$ of Λ such that $f^{-1}(y) \subset \cup\{U_\alpha : \alpha \in \Lambda(y)\}$. Now we put $U(y) = \cup\{U_\alpha : \alpha \in \Lambda(y)\}$ and $V(y) = Y - f(X - U(y))$. Then, since f is ωR-closed, $V(y)$ is an ω- regular open set in Y containing y such that $f^{-1}(V(y)) \subset U(y)$. Since $\{V(y) : y \in Y\}$ is an ω- regular open cover of Y, by Theorem 3.1 there exist $\{y_k : k < \omega\} \subseteq Y$ such that $Y = \cup\{V(y_k) : k < \omega\}$. Therefore,
$X = f^{-1}(Y) = \cup\{f^{-1}(V(y_k)) : k < \omega\} \subseteq \cup\{U(y_k) : k < \omega\}$
$= \cup\{U_\alpha : \alpha \in \Lambda(y_k), k < \omega\}$. This shows that X is nearly Lindelöf. □

5. **Examples**

For modification of regular spaces, we have the following diagram:

DIAGRAM II



Remark 1. (1) The notion of almost ω-regular spaces is independent of each of the following notions: regular spaces, weakly regular spaces, semi ω-regular spaces and semi-regular spaces.

(2) If a topological space is semi ω-regular and almost ω-regular, then it is regular. However, the converse is not necessarily true.

Example 5.1. *Let* $X = \{a, b, c, d\}$ *with the topology* $\tau=\{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$*. Then* (X, τ) *is a semi* ω*-regular space which is not semi-regular.*

Example 5.2. *Let* $X = \{a, b, c, d\}$ *with the topology* $\tau=\{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$*. Then* (X, τ) *is a semi* ω*-regular space which is not weakly regular, since if* $c \in X$ *and each open set* U_c *containing* c*, there no an* ω*-regular open set* V *such that* $c \in V \subseteq Cl(V) \subseteq U_c$*.*

Example 5.3. *Let* $X = \{a, b, c\}$ *with the topology* $\tau=\{\phi, X, \{a\}, \{b, c\}\}$*. Then* (X, τ) *is regular space which is not almost* ω*-regular, since if* $b \in X$ *there is* ω*-regular open set* $U_b = \{b\}$ *containing* b*, with no an open set* V *such that* $b \in V \subseteq Cl(V) \subseteq U_b$*.*

Example 5.4. *Let* X *be a set and* A *be subsets of* X *such that* A *and* $X - A$ *are uncountable. Let* $\tau=\{\phi, X, A\}$*. Then* X *is almost* ω*-regular space since the set of all* ω*-regular open set is* $\{\phi, X\}$*, which is not semi* ω*-regular space.*

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