JJMS **18**, No. 4, 539-548 (2025)

Jordan Journal of Mathematics and Statistics.
Yarmouk University

DOI:https://doi.org/10.47013/18.4.4

On Coherent Filters of Pseudo-Complemented Almost Distributive Lattices

Noorbhasha Rafi[1,*], *M. Sambasiva Rao*[2]

[1] Department of Mathematics, Bapatla Engineering College, Bapatla, India-522 102
[2] Department of Mathematics, MVGR College of Engineering, Vizianagaram, India-535 005

Received: May 18, 2024 Accepted : Nov. 26, 2024

Abstract: In a pseudo-complemented Almost Distributive Lattice(pseudo-complemented ADL), the notions of coherent filters, strongly coherent filters, ⧫-closed filters are introduced and their properties are studied. A set of equivalent conditions for any filter of a pseudo- complemented ADL to become a coherent filter is given. Also, the concept of median filters is introduced and a set of equivalent conditions for any maximal filter of a pseudo-complemented ADL to become a median filter is derived which leads to the characterization of a stone ADL.

Keywords: strongly coherent filter; median filter; minimal prime filter; maximal filter; stone ADL.

2010 Mathematics Subject Classification. 06D99; 06D15.

Introduction

The concept of an Almost Distributive Lattice(ADL) was introduced by Swamy and Rao [7] as a common abstraction of many existing ring theoretic generalizations of a Boolean algebra on one hand and the class of distributive lattices on the other. In that paper, the concept of an ideal in an ADL was introduced analogous to that in a distributive lattice and it was observed that the set $\mathscr{I}^{P\mathscr{I}}(\mathscr{R})$ of all principal ideals of $\mathscr{R}$ forms a distributive lattice. This provided a path to extend many existing concepts of lattice theory to the class of ADLs. With this motivation, Swamy, Rao and Nanaji[8] introduced the concept of pseudo-complementation on an ADL and also in [9] studied stone Almost Distributive Lattices. In the paper by Rafi et al. [5], introduced the concept of δ-ideals in pseudo-complemented ADLs and characterized the stone ADLs in terms of δ-ideals. Rafi et al. [6] derived the properties of $\mathscr{D}$-filters in ADLs and characterized the minimal prime $\mathscr{D}$-filters. In this paper, the concepts of coherent filters and strongly coherent filters are introduced in pseudo-complemented ADLs. A set of equivalent conditions is given for any filter of an ADL become a coherent filter which characterizes a Boolean algebra. Observed that each strongly coherent filter in a pseudo-complemented ADL is coherent. Additionally, the concepts of ⧫-closed filters, semi stone ADLs in pseudo-complemented ADLs are introduced, and the semi stone ADLs are characterized with the help of ⧫-closed filters. Observed that classes of maximal filters and prime $\mathscr{D}$-filters coincide in a pseudo-complemented ADL. The median filters are characterized that every median filter in a pseudo-complemented ADL is a coherent filter. Equivalence conditions are derived for any maximal filter in an ADL to become a strongly coherent filter. Finally, equivalent conditions are given for every maximal filter in a pseudo-complemented ADL to become a median filter which leads to a characterization of stone ADLs.

1 Preliminaries

In this section, certain definitions and important results are collected and presented from [2], [7], [8], [9], those will be required in the text of the paper.

* Corresponding author e-mail: rafimaths@gmail.com

Definition 1.[7] *An Almost Distributive Lattice with zero or simply ADL is an algebra* $(\mathscr{R}, \vee, \wedge, 0)$ *of type* $(2,2,0)$ *satisfying:*

(1) $(\kappa \vee \upsilon) \wedge \sigma = (\kappa \wedge \sigma) \vee (\upsilon \wedge \sigma)$;
(2) $\kappa \wedge (\upsilon \vee \sigma) = (\kappa \wedge \upsilon) \vee (\kappa \wedge \sigma)$;
(3) $(\kappa \vee \upsilon) \wedge \upsilon = \upsilon$;
(4) $(\kappa \vee \upsilon) \wedge \kappa = \kappa$;
(5) $\kappa \vee (\kappa \wedge \upsilon) = \kappa$;
(6) $0 \wedge \kappa = 0$, *for any* $\kappa, \upsilon, \sigma \in \mathscr{R}$.

Example 1.[7] Every non-empty set $\mathscr{R}$ can be regarded as an ADL as follows. Let $\theta_0 \in \mathscr{R}$. Define the binary operations $\vee, \wedge$ on $\mathscr{R}$ by

$$\theta \vee \psi = \begin{cases} \theta & \text{if } \theta \neq \theta_0 \\ \psi & \text{if } \theta = \theta_0 \end{cases} \qquad \theta \wedge \psi = \begin{cases} \psi & \text{if } \theta \neq \theta_0 \\ \theta_0 & \text{if } \theta = \theta_0. \end{cases}$$

Then $(\mathscr{R}, \vee, \wedge, \theta_0)$ is an ADL (where θ_0 is the zero) and is called a discrete ADL.

If $(\mathscr{R}, \vee, \wedge, 0)$ is an ADL, for any $\theta, \psi \in \mathscr{R}$, define $\theta \leq \psi$ if and only if $\theta = \theta \wedge \psi$ (or equivalently, $\theta \vee \psi = \psi$), then $\leq$ is a partial ordering on $\mathscr{R}$. An element m in $\mathscr{R}$ is said to be *maximal* if it is maximal with respect to the partial ordering $\leq$ on $\mathscr{R}$. That is, for any $\kappa \in \mathscr{R}$, $m \leq \kappa \Rightarrow m = \kappa$. The set of all maximal elements of an ADL is denoted by $\mathscr{M}_{Max.elts}$.

In Swamy's work[7], it is noted that an ADL satisfies almost all properties of a distributive lattice except the right distributivity of $\vee$ over $\wedge$, commutativity of $\vee$, commutativity of $\wedge$. Any one of these properties make an ADL $\mathscr{R}$ a distributive lattice. A nonempty subset $\mathscr{I}$ of $\mathscr{R}$ is called an *ideal* (respectively a *filter*) of $\mathscr{R}$, if $\kappa \vee \upsilon, \kappa \wedge \theta \in \mathscr{I}$ (respectively $\kappa \wedge \upsilon, \theta \vee \kappa \in \mathscr{I}$) for all $\kappa, \upsilon \in \mathscr{I}$ and all $\theta \in \mathscr{R}$. A proper ideal(filter) P of $\mathscr{R}$ is called a prime ideal(filter) if, for any $\theta, \psi \in \mathscr{R}$, $\theta \wedge \psi \in P(\theta \vee \psi \in \mathscr{U}) \Rightarrow \theta \in \mathscr{U}$ or $\psi \in \mathscr{U}$. A proper ideal(filter) $\mathscr{N}$ of $\mathscr{R}$ is said to be maximal if it is not properly contained in any proper ideal(filter) of $\mathscr{R}$. It can be observed that every maximal ideal(filter) of $\mathscr{R}$ is a prime ideal(filter). For any subset $\mathscr{G}$ of $\mathscr{R}$ the smallest ideal containing $\mathscr{G}$ is given by $(\mathscr{G}] := \{(\bigvee_{i=1}^{n} \kappa_i) \wedge \theta \mid \kappa_i \in \mathscr{G}, \theta \in \mathscr{R} \text{ and } n \in \mathbb{N}\}$. If $\mathscr{G} = \{\kappa\}$, we write $(\kappa]$ instead of $(\mathscr{G}]$ and such an ideal is called the principal ideal of $\mathscr{R}$. In a similar way, for each $\mathscr{G} \subseteq \mathscr{R}$, $[\mathscr{G}) := \{\theta \vee (\bigwedge_{i=1}^{n} \kappa_i) \mid \kappa_i \in \mathscr{G}, \theta \in \mathscr{R} \text{ and } n \in \mathbb{N}\}$. If $\mathscr{G} = \{\kappa\}$, we write $[\kappa)$ instead of $[\mathscr{G})$ and such a filter is called the principal filter of $\mathscr{R}$.

For any $\kappa, \upsilon \in \mathscr{R}$, it can be verified that $(\kappa] \vee (\upsilon] = (\kappa \vee \upsilon]$ and $(\kappa] \cap (\upsilon] = (\kappa \wedge \upsilon]$. Hence the set $(\mathscr{I}^{P\mathscr{I}}(\mathscr{R}), \vee, \cap)$ of all principal ideals of $\mathscr{R}$ is a sublattice of the distributive lattice $(\mathscr{I}(\mathscr{R}), \vee, \cap)$ of all ideals of $\mathscr{R}$. Also, we have that the set $(\mathfrak{F}(\mathscr{R}), \vee, \cap)$ of all filters of $\mathscr{R}$ is a bounded distributive lattice. In an ADL[3], observe that the prime ideal $\mathscr{U}$ of $\mathscr{R}$ can only exist if $\mathscr{R} \setminus \mathscr{U}$ is a prime filter of $\mathscr{R}$.

Definition 2.[8] *Let* $(\mathscr{R}, \vee, \wedge, 0)$ *be an ADL. Then a unary operation* $\kappa \longrightarrow \kappa^*$ *on* $\mathscr{R}$ *is called a pseudo-complementation on* $\mathscr{R}$ *if, for any* $\kappa, \upsilon \in \mathscr{R}$, *it satisfies the following conditions:*

(1) $\kappa \wedge \upsilon = 0$ *implies* $\kappa^* \wedge \upsilon = \upsilon$;
(2) $\kappa \wedge \kappa^* = 0$;
(3) $(\kappa \vee \upsilon)^* = \kappa^* \wedge \upsilon^*$.

Theorem 1.[8] *Let* * *be a pseudo-complementation on an ADL* $\mathscr{R}$. *Then, for any* $\kappa, \upsilon \in \mathscr{R}$:

(1) $0^* \in \mathscr{M}_{Max.elts}$; (2) $\kappa \in \mathscr{M}_{Max.elts} \Rightarrow \kappa^* = 0$; (3) $0^{**} = 0$; (4) $\kappa^{**} \wedge \kappa = \kappa$; (5) $\kappa^{***} = \kappa^*$; (6) $\kappa \leq \upsilon \Rightarrow \upsilon^* \leq \kappa^*$;

(7) $\kappa^* \wedge \upsilon^* = \upsilon^* \wedge \kappa^*$; (8) $(\kappa \wedge \upsilon)^{**} = \kappa^{**} \wedge \upsilon^{**}$; (9) $\kappa^* \wedge \upsilon = (\kappa \wedge \upsilon)^* \wedge \upsilon^*$.

An element κ of a pseudo-complemented ADL $\mathscr{R}$ is called a dense if $\kappa^* = 0$ and the set $\mathscr{D}$ of all dense elements of $\mathscr{R}$ forms a filter in $\mathscr{R}$.

Definition 3.[9] *A pseudo-complemented ADL* $\mathscr{R}$ *is called a stone ADL if* $\kappa^* \vee \kappa^{**} = 0^*$, *for all* κ *in* $\mathscr{R}$.

Theorem 2.[9] *The following conditions in a pseudo-complemented ADL* $\mathscr{R}$ *are equivalent:*

(1) $\mathscr{R}$ *is stone;* (2) *for* $\kappa, \upsilon \in \mathscr{R}$, $(\kappa \wedge \upsilon)^* = \kappa^* \vee \upsilon^*$; (3) *for* $\kappa, \upsilon \in \mathscr{R}$, $(\kappa \vee \upsilon)^{**} = \kappa^{**} \vee \upsilon^{**}$.

A filter $\mathscr{K}$ of an ADL $\mathscr{R}$ is called a $\mathscr{D}$-filter [6] if $\mathscr{D} \subseteq \mathscr{K}$. If $\mathscr{G}$ is a any non-empty subset of an ADL $\mathscr{R}$, then we define the set $(\mathscr{G}, \mathscr{D}) = \{\theta \in \mathscr{R} \mid \theta \vee \kappa \in \mathscr{D} \text{ for all } \kappa \in \mathscr{G}\}$ is a $\mathscr{D}$-filter of $\mathscr{R}$. We just represent $(\{\kappa\}, \mathscr{D})$ by $(\kappa, \mathscr{D})$ in the case of $\mathscr{G} = \{\kappa\}$. A prime $\mathscr{D}$-filter of an ADL is minimal if it is the minimal element in the poset of all prime $\mathscr{D}$-filters. A prime $\mathscr{D}$-filter of an ADL is minimal [6] if and only if to each $\theta \in \mathscr{U}$, there exists $\psi \notin \mathscr{U}$ such that $\theta \vee \psi \in \mathscr{D}$. Throughout this article, ADL $\mathscr{R}$ represents a pseudo-complemented ADL with maximal elements unless otherwise mentioned.

2 Coherent filters

In this section, the concepts of coherent filters and strongly coherent filter are introduced in a pseudo-complemented ADL. Stone ADLs are characterized with the help of coherent filters. A set of equivalent conditions is proved for every filter of a pseudo-complemented ADL to become a coherent filter which leads to a characterization of a Boolean algebra.

Definition 4. *If a non-void subset $\mathscr{G}$ of $\mathscr{R}$, then define the set*

$$\mathscr{G}^{\blacklozenge} = \{\theta \in \mathscr{R} \mid \kappa^{**} \vee \theta^{**} = 0^* \textit{ for all } \kappa \in \mathscr{G}\}$$

It is observed that $\mathscr{D}^{\blacklozenge} = \mathscr{R}$ and $\mathscr{R}^{\blacklozenge} = \mathscr{D}$. For any $\kappa \in \mathscr{R}$, we denote $(\{\kappa\})^{\blacklozenge}$ by $(\kappa)^{\blacklozenge}$. It is obvious that $(0)^{\blacklozenge} = \mathscr{D}$, $(m)^{\blacklozenge} = \mathscr{R}$, for any $m \in \mathscr{M}_{Max.elts.}$. For any $\emptyset \neq \mathscr{G} \subseteq \mathscr{R}, \mathscr{G} \cap \mathscr{G}^{\blacklozenge} \subseteq \mathscr{D}$.

Proposition 1. *Let $\mathscr{G}$ be a non-void subset of $\mathscr{R}$. Then $\mathscr{G}^{\blacklozenge}$ is a $\mathscr{D}$-filter in $\mathscr{R}$.*

Proof. Clearly $\mathscr{D} \subseteq \mathscr{G}^{\blacklozenge}$. Let $\theta, \psi \in \mathscr{G}^{\blacklozenge}$. Then $\kappa^{**} \vee \theta^{**} = 0^* = \kappa^{**} \vee \psi^{**}$, for all $\kappa \in \mathscr{G}$. Now $\kappa^{**} \vee (\theta \wedge \psi)^{**} = \kappa^{**} \vee (\theta^{**} \wedge \psi^{**}) = (\kappa^{**} \vee \theta^{**}) \wedge (\kappa^{**} \vee \psi^{**}) = 0^* \wedge 0^* = 0^*$. Hence $\theta \wedge \psi \in \mathscr{G}^{\blacklozenge}$. Let $\theta \in \mathscr{G}^{\blacklozenge}$. Then $\kappa^{**} \vee \theta^{**} = 0^*$, for all $\kappa \in \mathscr{G}$. Let $\psi \in \mathscr{R}$. Since $\theta \leq (\theta \vee \psi)$, we get $\theta^{**} \leq (\theta \vee \psi)^{**} = (\psi \vee \theta)^{**}$ and hence $0^* = \kappa^{**} \vee \theta^{**} \leq \kappa^{**} \vee (\psi \vee \theta)^{**}$. Which gives $\kappa^{**} \vee (\psi \vee \theta)^{**} = 0^*$. Therefore $\psi \vee \theta \in \mathscr{G}^{\blacklozenge}$. Thus, $\mathscr{G}^{\blacklozenge}$ is a $\mathscr{D}$-filter.

The definition above leads directly to the following result.

Lemma 1. *Let $\mathscr{G}$ and $\mathscr{H}$ be any non-void subsets of $\mathscr{R}$. Then:*

(1) *$\mathscr{G} \subseteq \mathscr{H}$ implies $\mathscr{H}^{\blacklozenge} \subseteq \mathscr{G}^{\blacklozenge}$;*
(2) *$\mathscr{G} \subseteq \mathscr{G}^{\blacklozenge\blacklozenge}$;*
(3) *$\mathscr{G}^{\blacklozenge\blacklozenge\blacklozenge} = \mathscr{G}^{\blacklozenge}$;*
(4) *$\mathscr{G}^{\blacklozenge} = \mathscr{R}$ if and only if $\mathscr{G} = \mathscr{D}$.*

Proposition 2. *Let $\mathscr{K}, \mathscr{L}$ be any filters of $\mathscr{R}$. Then $(\mathscr{K} \vee \mathscr{L})^{\blacklozenge} = \mathscr{K}^{\blacklozenge} \cap \mathscr{L}^{\blacklozenge}$.*

Proof. Clearly, It is observed that $(\mathscr{K} \vee \mathscr{L})^{\blacklozenge} \subseteq \mathscr{K}^{\blacklozenge} \cap \mathscr{L}^{\blacklozenge}$. Let $\theta \in \mathscr{K}^{\blacklozenge} \cap \mathscr{L}^{\blacklozenge}$. Then $\imath^{**} \vee \theta^{**} = 0^*$, for all $\imath \in A$ and $\blacksquare^{**} \vee \theta^{**} = 0^*$, for all $\blacksquare \in B$. Take $\sigma = \imath \wedge \blacksquare$ for all $\imath \in \mathscr{K}$, $\blacksquare \in \mathscr{L}$. Now $\sigma^{**} \vee \theta^{**} = (\imath \wedge \blacksquare)^{**} \vee \theta^{**} = (\imath^{**} \wedge \blacksquare^{**}) \vee \theta^{**} = (\imath^{**} \vee \theta^{**}) \wedge (\blacksquare^{**} \vee \theta^{**}) = 0^*$. Therefore $\theta \in (\mathscr{K} \vee \mathscr{L})^{\blacklozenge}$ and hence $(\mathscr{K} \vee \mathscr{L})^{\blacklozenge} = \mathscr{K}^{\blacklozenge} \cap \mathscr{L}^{\blacklozenge}$.

As an immediate consequence of the above results, the following corollary can be proved easily.

Corollary 1. *For any $\kappa, \upsilon \in \mathscr{R}$, we have:*

(1) *$\kappa \leq \upsilon$ implies $(\kappa)^{\blacklozenge} \subseteq (\upsilon)^{\blacklozenge}$;*
(2) *$(\kappa \wedge \upsilon)^{\blacklozenge} = (\kappa)^{\blacklozenge} \cap (\upsilon)^{\blacklozenge}$;*
(3) *$(\kappa)^{\blacklozenge} = \mathscr{R}$ iff κ is dense;*
(4) *$\kappa \in (\upsilon)^{\blacklozenge}$ implies $\kappa \vee \upsilon \in \mathscr{D}$;*
(5) *$\kappa^* = \upsilon^*$ implies $(\kappa)^{\blacklozenge} = (\upsilon)^{\blacklozenge}$.*

Clearly $\mathscr{G}^{\blacklozenge} \subseteq (\mathscr{G}, \mathscr{D})$. On the other hand, we derive a collection of comparable requirements that essentially lead to a characterization of a stone ADL by satisfying each filter to the reverse inclusion.

Theorem 3. *The following conditions are equivalent in an ADL $\mathscr{R}$:*

(1) *$\mathscr{R}$ is a stone ADL;*
(2) *for any filter $\mathscr{K}$ of $\mathscr{R}$, $\mathscr{K}^{\blacklozenge} = (\mathscr{K}, \mathscr{D})$;*
(3) *for any $\kappa \in \mathscr{R}$, $(\kappa)^{\blacklozenge} = (\kappa, \mathscr{D})$;*
(4) *for any two filters $\mathscr{K}, \mathscr{L}$ of $\mathscr{R}$, $\mathscr{K} \cap \mathscr{L} \subseteq \mathscr{D} \Leftrightarrow \mathscr{K} \subseteq \mathscr{L}^{\blacklozenge}$;*
(5) *for $\kappa, \upsilon \in \mathscr{R}, \kappa \vee \upsilon \in \mathscr{D} \Rightarrow \kappa^{**} \vee \upsilon^{**} = 0^*$;*
(6) *for $\kappa \in \mathscr{R}$, $(\kappa)^{\blacklozenge\blacklozenge} = (\kappa^*)^{\blacklozenge}$.*

Proof.(1) $\Rightarrow$ (2): Assume (1). Clearly we have $\mathscr{K}^{\blacklozenge} \subseteq (\mathscr{K},\mathscr{D})$, for any filter $\mathscr{K}$ of $\mathscr{R}$. Let $\theta \in (\mathscr{K},\mathscr{D})$. Then $\theta \vee \psi \in \mathscr{D}$ for all $\psi \in \mathscr{K}$. Since $\mathscr{R}$ is stone, $\theta^{**} \vee \psi^{**} = (\theta \vee \psi)^{**} = 0^*$ for all $\psi \in \mathscr{K}$. Hence $\theta \in \mathscr{K}^{\blacklozenge}$. Therefore, $(\mathscr{K},\mathscr{D}) \subseteq \mathscr{K}^{\blacklozenge}$.
(2) $\Rightarrow$ (3): It's clear.
(3) $\Rightarrow$ (4): Assume (3). Let $\mathscr{K},\mathscr{L}$ be two filters of $\mathscr{R}$. Assume $\mathscr{K} \cap \mathscr{L} \subseteq \mathscr{D}$. Let $\theta \in \mathscr{K}$. For any $\psi \in \mathscr{L}$, we have $\theta \vee \psi \in \mathscr{K} \cap \mathscr{L} \subseteq \mathscr{D}$. Hence $\theta \vee \psi \in \mathscr{D}$. Now for every $y \in \mathscr{L}'$, $\theta \vee \psi \in \mathscr{D} \Rightarrow \theta \in (\psi,\mathscr{D})$ for all $\psi \in \mathscr{L} \Rightarrow \theta \in (\psi)^{\blacklozenge}$ for all $\psi \in \mathscr{L} \Rightarrow \theta^{**} \vee \psi^{**} = 0^*$, for all $\psi \in \mathscr{L}$, which yields that $\theta \in \mathscr{L}^{\blacklozenge}$. Conversely, suppose $\mathscr{K} \subseteq \mathscr{L}^{\blacklozenge}$. Let $\theta \in \mathscr{L} \cap \mathscr{K}$. Then $\theta \in \mathscr{L}$ and $\theta \in \mathscr{K} \subseteq \mathscr{L}^{\blacklozenge}$. Hence $\theta \in \mathscr{L} \cap \mathscr{L}^{\blacklozenge} \subseteq \mathscr{D}$. Therefore, $\mathscr{K} \cap \mathscr{L} \subseteq \mathscr{D}$.
(4) $\Rightarrow$ (5): Assume (4). Let $\kappa, \upsilon \in \mathscr{R}$ be such that $\kappa \vee \upsilon \in \mathscr{D}$. Then $\kappa \vee \upsilon \in \mathscr{D} \Rightarrow [\kappa) \cap [\upsilon) \subseteq \mathscr{D} \Rightarrow [\kappa) \subseteq [\upsilon)^{\blacklozenge} \Rightarrow \kappa \in [\upsilon)^{\blacklozenge} \Rightarrow \kappa^{**} \vee \upsilon^{**} = 0^*$.
(5) $\Rightarrow$ (6): Assume (5). Let $\kappa \in \mathscr{R}$. Clearly $\kappa \vee \kappa^* \in \mathscr{D}$. By (5), we get $\kappa^{**} \vee \kappa^{***} = 0^*$. Hence $\kappa^* \in (\kappa)^{\blacklozenge}$. Thus $(\kappa)^{\blacklozenge\blacklozenge} \subseteq (\kappa^*)^{\blacklozenge}$. Let $\theta \in (\kappa^*)^{\blacklozenge}$ and $t \in (\kappa)^{\blacklozenge}$. Since $t \in (\kappa)^{\blacklozenge}$, we have $\kappa^{**} \vee t^{**} = 0^*$. Which gives that $\kappa^* \wedge t^* = 0$. Therefore $\kappa^{**} \wedge t^* = t^*$. Now $\theta \in (\kappa^*)^{\blacklozenge} \Rightarrow \kappa^* \vee \theta^{**} = 0^* \Rightarrow \kappa^{**} \wedge \theta^* = 0 \Rightarrow t^* \wedge \theta^* = 0,$ since $\kappa^{**} \wedge t^* = t^* \Rightarrow t \vee \theta \in \mathscr{D} \Rightarrow t^{**} \vee \theta^{**} = 0^*$, which gives for all $t \in (\kappa)^{\blacklozenge}$. Hence $\theta \in (\kappa)^{\blacklozenge\blacklozenge}$. Therefore, $(\kappa^*)^{\blacklozenge} \subseteq (\kappa)^{\blacklozenge\blacklozenge}$.
(6) $\Rightarrow$ (1): Assume (6). Let $\kappa \in \mathscr{R}$. Since $\kappa \in (\kappa)^{\blacklozenge\blacklozenge} = (\kappa^*)^{\blacklozenge}$, we have $\kappa^{***} \vee \kappa^{**} = 0^*$ and hence $\kappa^* \vee \kappa^{**} = 0^*$. Therefore, $\mathscr{R}$ is stone.

Now, the notion of coherent filters is presented in an ADL.

Definition 5. *A filter $\mathscr{K}$ of $\mathscr{R}$ is called coherent if for all $\theta, \psi \in \mathscr{R}, (\theta)^{\blacklozenge} = (\psi)^{\blacklozenge}$ and $\theta \in \mathscr{K}$ imply $\psi \in \mathscr{K}$.*

Lemma 2. *In a ADL, we have:*

(1) *for any $\kappa \in \mathscr{R}$, $(\kappa)^{\blacklozenge}$ is coherent;*
(1) *for any element κ of a filter $\mathscr{K}$ with $(\kappa)^{\blacklozenge\blacklozenge} \subseteq \mathscr{K}$, $\mathscr{K}$ is coherent of $\mathscr{R}$.*

Theorem 4. *The following assertions are equivalent in an ADL $\mathscr{R}$:*

(1) *$\mathscr{R}$ is a Boolean algebra;*
(2) *every element in $\mathscr{R}$ is closed;*
(3) *for any filter $\mathscr{K}$, $\theta^{**} \in \mathscr{K} \Rightarrow \theta \in \mathscr{K}$;*
(4) *every principal filter is a coherent filter;*
(5) *every filter is coherent;*
(6) *every prime filter is coherent;*
(7) *for $\kappa, \upsilon \in \mathscr{R}, (\kappa)^{\blacklozenge} = (\upsilon)^{\blacklozenge} \Rightarrow [\kappa) = [\upsilon)$;*
(8) *for $\kappa, \upsilon \in \mathscr{R}, \kappa^* = \upsilon^* \Rightarrow [\kappa) = [\upsilon)$.*

Proof.(1) $\Rightarrow$ (2): It's clear.
(2) $\Rightarrow$ (3): It's clear.
(3) $\Rightarrow$ (4): Assume that every element of $\mathscr{R}$ is closed. Let $[\theta)$ be a principal filter of $\mathscr{R}$. Since $\theta \vee \theta^* \in \mathscr{D}$, we get $(\theta \vee \theta^*)^{**} = 0^* \in [0^*)$. By (3), we get $\theta \vee \theta^* \in [0^*)$, which gives $\theta \vee \theta^* = 0^*$. Let $\kappa, \upsilon \in \mathscr{R}$ with $(\kappa)^{\blacklozenge} = (\upsilon)^{\blacklozenge}$, $\kappa \in [\theta)$. Then, we get $\theta \vee \theta^* = 0^* \Rightarrow \kappa \vee \theta^* = 0^*,$ since $\kappa \in [\theta) \Rightarrow \kappa^{**} \vee \theta^{***} = 0^* \Rightarrow \theta^* \in (\kappa)^{\blacklozenge} = (\upsilon)^{\blacklozenge} \Rightarrow \upsilon^{**} \vee \theta^* = 0^* \Rightarrow (\upsilon^{**} \vee \theta^*)^* = 0 \Rightarrow \upsilon^* \wedge \theta^{**} = 0 \Rightarrow \upsilon^* \wedge \theta = 0,$ since $\theta \leq \theta^{**} \Rightarrow \theta \leq \upsilon^{**}$, which yields $\upsilon^{**} \in [\theta)$. By (3), we get $\upsilon \in [\theta)$. Therefore, $[\theta)$ is a coherent filter of $\mathscr{R}$.
(4) $\Rightarrow$ (5): Assume (4). Let $\mathscr{K}$ be a filter of $\mathscr{R}$. Choose $\kappa, \upsilon \in \mathscr{R}$. Suppose $(\kappa)^{\blacklozenge} = (\upsilon)^{\blacklozenge}$ and $\kappa \in \mathscr{K}$. Then clearly $[\kappa) \subseteq \mathscr{K}$. Since $(\kappa)^{\blacklozenge} = (\upsilon)^{\blacklozenge}$ and $[\kappa)$ is a coherent filter, we get that $\upsilon \in [\kappa) \subseteq \mathscr{K}$. Therefore, $\mathscr{K}$ is coherent.
(5) $\Rightarrow$ (6): It is clear.
(6) $\Rightarrow$ (7): Assume (6). Let $\kappa, \upsilon \in \mathscr{R}$ with $(\kappa)^{\blacklozenge} = (\upsilon)^{\blacklozenge}$. Suppose $\kappa \neq \upsilon$. Then there is a prime filter $\mathscr{U}$ such that $\kappa \in \mathscr{U}$ and $\upsilon \notin \mathscr{U}$. From the hypothesis, $\mathscr{U}$ is a coherent filter of $\mathscr{R}$. Since $(\kappa)^{\blacklozenge} = (\upsilon)^{\blacklozenge}$ and $\kappa \in \mathscr{U}$, we get $\upsilon \in \mathscr{U}$, which is a contradiction. Therefore, $\kappa = \upsilon$.
(7) $\Rightarrow$ (8): It is clear.
(8) $\Rightarrow$ (1): It is Clear.

Definition 6. *For any filter $\mathscr{K}$ of $\mathscr{R}$, define $\chi(\mathscr{K})$ as follows:*

$$\chi(\mathscr{K}) = \{\theta \in \mathscr{R} \mid (\theta)^{\blacklozenge} \vee \mathscr{K} = \mathscr{R}\}$$

The definition above immediately leads to the following result.

Lemma 3. *Let $\mathscr{K}, \mathscr{L}$ be any two filters of $\mathscr{R}$. Then:*

(1) *$\mathscr{K} \subseteq \mathscr{L}$ implies $\chi(\mathscr{K}) \subseteq \chi(\mathscr{L})$;*
(2) *$\chi(\mathscr{K} \cap \mathscr{L}) = \chi(\mathscr{K}) \cap \chi(\mathscr{L})$.*

Proposition 3. *For any filter $\mathscr{K}$ of $\mathscr{R}$, $\chi(\mathscr{K})$ is a $\mathscr{D}$-filter of $\mathscr{R}$.*

Proof. Clearly $\mathscr{D} \subseteq \chi(\mathscr{K})$. Let $\theta, \psi \in \chi(\mathscr{K})$. Then $(\theta)^{\blacklozenge} \vee \mathscr{K} = (\psi)^{\blacklozenge} \vee \mathscr{K} = \mathscr{R}$. Hence $(\theta \wedge \psi)^{\blacklozenge} \vee \mathscr{K} = \{(\theta)^{\blacklozenge} \cap (\psi)^{\blacklozenge}\} \vee \mathscr{K} = \{(\theta)^{\blacklozenge} \vee \mathscr{K}\} \cap \{(\psi)^{\blacklozenge} \vee \mathscr{K}\} = \mathscr{R}$. Hence $\theta \wedge \psi \in \chi(\mathscr{K})$. Let $\theta \in \chi(\mathscr{K})$. Then $\mathscr{R} = (\theta)^{\blacklozenge} \vee \mathscr{K}$. Let $\psi \in \mathscr{R}$. Since $\theta \leq \theta \vee \psi$, we get $(\theta)^{\blacklozenge} \subseteq (\theta \vee \psi)^{\blacklozenge} = (\psi \vee \theta)^{\blacklozenge}$ and hence $\mathscr{R} = (\theta)^{\blacklozenge} \vee \mathscr{K} \subseteq (\psi \vee \theta)^{\blacklozenge} \vee \mathscr{K}$. Which gives $(\psi \vee \theta)^{\blacklozenge} \vee \mathscr{K} = \mathscr{R}$. Thus $\psi \vee \theta \in \chi(\mathscr{K})$. Therefore, $\chi(\mathscr{K})$ is a $\mathscr{D}$-filter in $\mathscr{R}$.

Lemma 4. *Let $\mathscr{K}$ be any filter of $\mathscr{R}$. Then $\mathscr{K}$ is a $\mathscr{D}$-filter iff $\chi(\mathscr{K}) \subseteq \mathscr{K}$.*

Proof. Assume that $\mathscr{K}$ is a $\mathscr{D}$-filter of $\mathscr{R}$. Let $\theta \in \chi(\mathscr{K})$. Then $(\theta)^{\blacklozenge} \vee \mathscr{K} = \mathscr{R}$. Hence $\theta = \kappa \wedge \upsilon$ for some $\kappa \in (\theta)^{\blacklozenge} \subseteq (\theta, \mathscr{D})$ and $\upsilon \in \mathscr{K}$. Then $\theta \vee \kappa \in \mathscr{D} \subseteq \mathscr{K}$ and $\theta \vee \upsilon \in \mathscr{K}$. Thus $\theta = \theta \vee \theta = \theta \vee (\kappa \wedge \upsilon) = (\theta \vee \kappa) \wedge (\theta \vee \upsilon) \in \mathscr{K}$. Therefore, $\chi(\mathscr{K}) \subseteq \mathscr{K}$. The converse is clear because of $\mathscr{D} \subseteq \chi(\mathscr{K}) \subseteq \mathscr{K}$.

Definition 7. *A filter $\mathscr{K}$ of $\mathscr{R}$ is said to be strongly coherent if $\mathscr{K} = \chi(\mathscr{K})$.*

Proposition 4. *Every strongly coherent filter of $\mathscr{R}$ is coherent.*

Proof. Let $\mathscr{K}$ be a strongly coherent filter of $\mathscr{R}$. Clearly $\mathscr{K}$ is a $\mathscr{D}$-filter of $\mathscr{R}$. Let $\theta, \psi \in \mathscr{R}$ with $(\theta)^{\blacklozenge} = (\psi)^{\blacklozenge}$ and $\theta \in \mathscr{K} = \chi(\mathscr{K})$. Then $(\theta)^{\blacklozenge} \vee \mathscr{K} = \mathscr{R}$. Hence $(\psi)^{\blacklozenge} \vee \mathscr{K} = \mathscr{R}$ and so $\psi \in \chi(\mathscr{K}) = \mathscr{K}$. Therefore, $\mathscr{K}$ is a coherent filter of $\mathscr{R}$.

For any filter $\mathscr{K}$ of $\mathscr{R}$, observe that $\mathscr{K} \subseteq \mathscr{D}$ iff $\mathscr{K}^{\blacklozenge\blacklozenge} = \mathscr{D}$. A $\mathscr{D}$-filter $\mathscr{K}$ of an ADL $\mathscr{R}$ is called a $\blacklozenge$-closed if $\mathscr{K} = \mathscr{K}^{\blacklozenge\blacklozenge}$. Clearly, $\mathscr{D}$ is the smallest $\blacklozenge$-closed filter and $\mathscr{R}$ is the largest $\blacklozenge$-closed filter of an ADL $\mathscr{R}$.

Proposition 5. *Every $\blacklozenge$-closed filter of an ADL is a coherent filter.*

Proof. Let $\mathscr{K}$ be a $\blacklozenge$-closed filter of $\mathscr{R}$. Let $\theta, \psi \in \mathscr{R}$ with $(\theta)^{\blacklozenge} = (\psi)^{\blacklozenge}$. Suppose $\theta \in \mathscr{K}$. Then $\psi \in (\psi)^{\blacklozenge\blacklozenge} = (\theta)^{\blacklozenge\blacklozenge} \subseteq \mathscr{K}^{\blacklozenge\blacklozenge} = \mathscr{K}$. Therefore, $\mathscr{K}$ is coherent.

Definition 8. *An ADL $\mathscr{R}$ is said to be semi stone if $(\theta)^{\blacklozenge} \vee (\theta)^{\blacklozenge\blacklozenge} = \mathscr{R}$ for all $\theta \in \mathscr{R}$.*

Theorem 5. *Every stone ADL is semi stone.*

Proof. Let $\mathscr{R}$ be a stone ADL. Let $\theta \in \mathscr{R}$. Suppose $(\theta)^{\blacklozenge} \vee (\theta)^{\blacklozenge\blacklozenge} \neq \mathscr{R}$. Then $(\theta)^{\blacklozenge} \vee (\theta)^{\blacklozenge\blacklozenge} \subseteq \mathscr{N}$, for some a maximal filter $\mathscr{N}$ of $\mathscr{R}$. Then $(\theta)^{\blacklozenge} \subseteq \mathscr{N}$ and $\theta \in (\theta)^{\blacklozenge\blacklozenge} \subseteq \mathscr{N}$. Since $\mathscr{N}$ is maximal, we get $\theta^* \notin \mathscr{N}$. Since $\mathscr{R}$ is stone, we get $\theta^* \vee \theta^{**} = 0^*$. Hence $\theta^* \in (\theta)^{\blacklozenge}$. Thus $(\theta)^{\blacklozenge} \nsubseteq \mathscr{N}$, which is a contradiction. Hence $\mathscr{R} = (\theta)^{\blacklozenge} \vee (\theta)^{\blacklozenge\blacklozenge}$. Therefore, $\mathscr{R}$ is semi stone.

The converse of the above theorem is false. For, consider

Example 2. Let $\mathscr{R} = \{0, \kappa, \upsilon, \sigma, \mu, \rho\}$ and define $\vee$, $\wedge$ on $\mathscr{R}$ as follows:

$\wedge$	0	κ	υ	σ	μ	ρ
0	0	0	0	0	0	0
κ	0	κ	υ	σ	μ	ρ
υ	0	κ	υ	σ	μ	ρ
σ	0	σ	σ	σ	0	σ
μ	0	μ	μ	0	μ	μ
ρ	0	ρ	ρ	σ	μ	ρ

$\vee$	0	κ	υ	σ	μ	ρ
0	0	κ	υ	σ	μ	ρ
κ	κ	κ	κ	κ	κ	κ
υ	υ	υ	υ	υ	υ	υ
σ	σ	κ	υ	σ	ρ	ρ
μ	μ	κ	υ	ρ	μ	ρ
ρ	ρ	κ	υ	ρ	ρ	ρ

Then $(\mathscr{R}, \vee, \wedge)$ is an ADL. Define $0^* = \kappa, \kappa^* = \upsilon^* = \rho^* = 0, \sigma^* = \mu, \mu^* = \sigma$. Clearly, it is pseudo-complemented ADL but not a stone ADL since $\sigma^* \vee \sigma^{**} = \mu \vee \sigma = \rho \neq \kappa = 0^*$. Clearly, we have $(\theta)^{\blacklozenge} \vee (\theta)^{\blacklozenge\blacklozenge} = \mathscr{R}$, for all $\theta \in \mathscr{R}$. Hence, $\mathscr{R}$ is semi stone.

Theorem 6. *The following statements are equivalent in an ADL $\mathscr{R}$ are as follows:*

(1) *$\mathscr{R}$ is semi stone;*
(2) *every $\blacklozenge$-closed filter is strongly coherent;*
(3) *for each $\theta \in \mathscr{R}, (\theta)^{\blacklozenge\blacklozenge}$ is strongly coherent.*

Proof.(1) $\Rightarrow$ (2): Assume (1). Let $\mathscr{K}$ be a $\blacklozenge$-closed filter of $\mathscr{R}$. Then $\mathscr{K}$ is a $\mathscr{D}$-filter with $\mathscr{K}^{\blacklozenge\blacklozenge} = \mathscr{K}$. Clearly $\chi(\mathscr{K}) \subseteq \mathscr{K}$. Conversely, let $\theta \in \mathscr{K}$. It can be easily verified that $(\theta)^{\blacklozenge\blacklozenge} \subseteq \mathscr{K}^{\blacklozenge\blacklozenge}$. Hence $\mathscr{R} = (\theta)^{\blacklozenge} \vee (\theta)^{\blacklozenge\blacklozenge} \subseteq (\theta)^{\blacklozenge} \vee \mathscr{K}^{\blacklozenge\blacklozenge} = (\theta)^{\blacklozenge} \vee \mathscr{K}$. Thus $\theta \in \chi(\mathscr{K})$. Therefore, $\mathscr{K}$ is strongly coherent.
(2) $\Rightarrow$ (3): It is obvious.
(3) $\Rightarrow$ (1): Assume (3). Let $\theta \in \mathscr{R}$. Therefore $\chi((\theta)^{\blacklozenge\blacklozenge}) = (\theta)^{\blacklozenge\blacklozenge}$. Since $\theta \in (\theta)^{\blacklozenge\blacklozenge}$, we get $(\theta)^{\blacklozenge} \vee (\theta)^{\blacklozenge\blacklozenge} = \mathscr{R}$. Therefore, $\mathscr{R}$ is a semi stone ADL.

3 Median filters

In this section, the notion of a median filter is introduced in pseudo-complemented ADLs. Characterization theorems of median filters are derived for every prime $\mathscr{D}$-filter to become median and every maximal filter to become median.

Proposition 6. *Let $\mathscr{U}$ be a prime filter of $\mathscr{R}$. Then the following assertions are equivalent:*

(1) $\mathscr{D} \subseteq \mathscr{U}$;
(2) $\theta \in \mathscr{U} \Leftrightarrow \theta^* \notin \mathscr{U}$, *for all* $\theta \in \mathscr{R}$;
(3) $\theta^{**} \in \mathscr{U} \Leftrightarrow \theta \in \mathscr{U}$, *for all* $\theta \in \mathscr{R}$;
(4) *for any* $\theta, \psi \in \mathscr{R}$ $\theta^* = \psi^*$ *and* $\theta \in \mathscr{U} \Rightarrow \psi \in \mathscr{U}$.

Proof.(1) $\Rightarrow$ (2): Assume that $\mathscr{D} \subseteq \mathscr{U}$. Suppose $\theta \in \mathscr{U}$. If $\theta^* \in \mathscr{U}$, then $0 = \theta \wedge \theta^* \in \mathscr{U}$, which is a contradiction. Hence $\theta^* \notin \mathscr{U}$. Conversely, suppose that $\theta^* \notin \mathscr{U}$. Clearly, $\theta \vee \theta^* \in \mathscr{D} \subseteq \mathscr{U}$. Since $\mathscr{U}$ is prime and $\theta^* \notin \mathscr{U}$, we get $\theta \in \mathscr{U}$.
(2) $\Rightarrow$ (3): It is clear.
(3) $\Rightarrow$ (4): It is clear.
(4) $\Rightarrow$ (1): Assume (4). Suppose $\mathscr{D} \cap (\mathscr{R} \setminus \mathscr{U}) \neq \emptyset$. Then there is $\theta \in \mathscr{R}$ such that $\theta \in \mathscr{D} \cap (\mathscr{R} \setminus \mathscr{U})$. Therefore $\theta \notin \mathscr{U}$ and $\theta^* = 0$. Since $0^* \in \mathscr{U}$, by (4), we get $\theta \in \mathscr{U}$ which is a contradiction. Hence $\mathscr{D} \cap (\mathscr{R} \setminus \mathscr{U}) = \emptyset$. Therefore, $\mathscr{D} \subseteq \mathscr{U}$.

Theorem 7. *The following assertions are equivalent for any proper filter $\mathscr{N}$ in $\mathscr{R}$:*

(1) *$\mathscr{N}$ is maximal;*
(2) *$\mathscr{N}$ is prime $\mathscr{D}$-filter;*
(3) $\theta \notin \mathscr{N}$ *implies* $\theta^* \in \mathscr{N}$.

Proof.(1) $\Rightarrow$ (2): Assume (1). Clearly $\mathscr{N}$ is prime. Let $\theta \in \mathscr{D}$. Then $\theta^* = 0$. Suppose $\theta \notin \mathscr{N}$. Then $\mathscr{N} \vee [\theta) = \mathscr{R}$. Hence $0 = \eta \wedge \theta$, for some $0 \neq \eta \in \mathscr{N}$. Then $\eta = \theta^* \wedge n = 0$, which leads a contradiction. So that $\theta \in \mathscr{N}$. Thus $\mathscr{D} \subseteq \mathscr{N}$. Therefore, $\mathscr{N}$ is a prime $\mathscr{D}$-filter.
(2) $\Rightarrow$ (3): Assume that $\mathscr{N}$ is a prime $\mathscr{D}$-filter of $\mathscr{R}$. Suppose $\theta \notin \mathscr{N}$. Clearly $\theta \vee \theta^* \in \mathscr{D} \subseteq \mathscr{N}$. Since $\mathscr{N}$ is prime and $\theta \notin \mathscr{N}$, we must have $\theta^* \in \mathscr{N}$.
(3) $\Rightarrow$ (1): Assume condition (3). Suppose $\mathscr{N}$ is not maximal. Let $\mathscr{V}$ be a proper filter such that $\mathscr{N} \subsetneq \mathscr{V}$. Choose $\theta \in \mathscr{V} \setminus \mathscr{N}$. Since $\theta \notin \mathscr{N}$, by (3), we get $\theta^* \in \mathscr{N} \subsetneq \mathscr{V}$. Hence $0 = \theta \wedge \theta^* \in \mathscr{V}$, which gives a contradiction. Thus, $\mathscr{N}$ is maximal.

In Theorem 7, it's observed that the collection of all maximal filters and the set of all prime $\mathscr{D}$-filters coincide. Given that every prime $\mathscr{D}$-filter is maximal, it follows that they are also minimal. Consequently, in a pseudo-complemented ADL, maximal filters, prime $\mathscr{D}$-filters, and minimal prime $\mathscr{D}$-filters are the same.
Now, we present the idea of median filters in a pseudo-complemented ADL.

Definition 9. *A maximal filter $\mathscr{N}$ of $\mathscr{R}$ is called a median filter if for every $\theta \in \mathscr{N}$, there is a $\psi \notin \mathscr{N}$ such that $\theta^{**} \vee \psi^{**} = 0^*$.*

From Example 2, we note that every maximal filter of a pseudo-complemented ADL need not be median, For take the maximal filter $M = \{\kappa, \upsilon, \mu, \rho\}$ of $\mathscr{R}$. Observe that for $a \in M$, there is no $x \notin M$ such that $a^{**} \vee x^{**} = 0^*$. Thus, M is not median.

Lemma 5. *Let $\mathscr{N}$ be a maximal filter of $\mathscr{R}$. To each $\theta \in \mathscr{R}$, we have*

$$\theta \notin \mathscr{N} \quad \Rightarrow \quad (\theta)^{\blacklozenge} \subseteq \mathscr{N}$$

Proof. Suppose $\theta \notin \mathscr{N}$. Let $\kappa \in (\theta)^{\blacklozenge}$. Then $\kappa^{**} \vee \theta^{**} = 0^*$. Hence $(\kappa \vee \theta)^{**} = 0^*$ and so $\kappa \vee \theta \in \mathscr{D} \subseteq \mathscr{N}$. Since $\theta \notin \mathscr{N}$, we must have $\kappa \in \mathscr{N}$. Therefore, $(\theta)^{\blacklozenge} \subseteq \mathscr{N}$.

Lemma 6. *Let $\mathscr{N}$ be a median filter in $\mathscr{R}$. For any $\theta \in \mathscr{R}$, we have*

$$\theta \in \mathscr{N} \Leftrightarrow (\theta)^{\blacklozenge\blacklozenge} \subseteq \mathscr{N}$$

Proof. Suppose $\theta \in \mathscr{N}$. Let $\kappa \in (\theta)^{\blacklozenge\blacklozenge}$. Then $(\theta)^{\blacklozenge} \subseteq (\kappa)^{\blacklozenge}$. Since $\theta \in \mathscr{N}$ and $\mathscr{N}$ is median, there exists $\psi \notin \mathscr{N}$ such that $\theta^{**} \vee \psi^{**} = 0^*$. Then $\psi \in (\theta)^{\blacklozenge} \subseteq (\kappa)^{\blacklozenge}$. Since $\psi \notin \mathscr{N}$, we get $(\psi)^{\blacklozenge} \subseteq \mathscr{N}$. Hence $\kappa \in (\kappa)^{\blacklozenge\blacklozenge} \subseteq (\psi)^{\blacklozenge} \subseteq \mathscr{N}$. Therefore, $(\theta)^{\blacklozenge\blacklozenge} \subseteq \mathscr{N}$.

In the following result, we construct a characterization statement of median filters.

Theorem 8. *The following assertions are equivalent for any maximal filter $\mathscr{N}$ of $\mathscr{R}$ and $\theta \in \mathscr{R}$:*

(1) *$\mathscr{N}$ is median;*
(2) *$\theta \notin \mathscr{N}$ iff $(\theta)^{\blacklozenge} \subseteq \mathscr{N}$;*
(3) *$\theta^{**} \in \mathscr{N}$ implies $(\theta)^{\blacklozenge} \nsubseteq \mathscr{N}$.*

Proof. (1) $\Rightarrow$ (2): Assume that $\mathscr{N}$ is a median filter of $\mathscr{R}$ and $\theta \in \mathscr{R}$. Suppose $\theta \notin \mathscr{N}$. By Lemma 5, we have $(\theta)^{\blacklozenge} \subseteq \mathscr{N}$. Conversely, assume $(\theta)^{\blacklozenge} \subseteq \mathscr{N}$. Suppose $\theta \in \mathscr{N}$. Since $\mathscr{N}$ is median, there is a $\psi \notin \mathscr{N}$ such that $\theta^{**} \vee \psi^{**} = 0^*$. Hence $\psi \in (\theta)^{\blacklozenge} \subseteq \mathscr{N}$, which leads a contradiction. Therefore, $\theta \notin \mathscr{N}$.
(2) $\Rightarrow$ (3): Assume (2). Let $\theta \in \mathscr{R}$. Suppose $\theta^{**} \in \mathscr{N}$. By Proposition 6, we get $\theta \in \mathscr{N}$. By (2), we get $(\theta)^{\blacklozenge} \nsubseteq \mathscr{N}$.
(3) $\Rightarrow$ (1): Assume (3). Suppose $\theta \in \mathscr{N}$. Clearly $\theta^{**} \in \mathscr{N}$. By the assumed condition, we have $(\theta)^{\blacklozenge} \nsubseteq \mathscr{N}$. Then there is a $\psi \in (\theta)^{\blacklozenge}$ such that $\psi \notin \mathscr{N}$. Hence $\theta^{**} \vee \psi^{**} = 0^*$, where $\psi \notin \mathscr{N}$. Therefore, $\mathscr{N}$ is median.

Theorem 9. *Every median filter of an ADL $\mathscr{R}$ is a coherent filter.*

Proof. Let $\mathscr{N}$ be a median filter of $\mathscr{R}$. Suppose $\theta, \psi \in \mathscr{R}$ are such that $(\theta)^{\blacklozenge} = (\psi)^{\blacklozenge}$ and $\theta \in \mathscr{N}$. Since $\mathscr{N}$ is median, there exists $\kappa \notin \mathscr{N}$ such that $\theta^{**} \vee \kappa^{**} = 0^*$. Thus $\kappa \in (\theta)^{\blacklozenge} = (\psi)^{\blacklozenge}$. As $\psi^{**} \vee \kappa^{**} \leq (\psi \vee \kappa)^{**}$, we have $(\psi \vee \kappa)^{**} = 0^*$, which gives that $(\psi \vee \kappa)^* = 0$. Thus $\psi \vee \kappa \in \mathscr{D} \subseteq \mathscr{N}$. Since $\kappa \notin \mathscr{N}$ and $\mathscr{N}$ is prime, it leads that $\psi \in \mathscr{N}$. Thus, $\mathscr{N}$ is coherent.

In the following result, we construct a set of equivalent condition for a stone ADL to transform into a Boolean algebra with the help of maximal and median filters.

Theorem 10. *The following assertions are equivalent in a stone ADL $\mathscr{R}$:*

(1) *$\mathscr{R}$ is a Boolean algebra;*
(2) *every prime filter is maximal;*
(3) *every prime filter is median;*
(4) *every prime filter is a $\mathscr{D}$-filter.*

Proof. (1) $\Rightarrow$ (2): Assume that $\mathscr{R}$ is a Boolean algebra. Let $\mathscr{U}$ be a prime filters of $\mathscr{R}$. Suppose there exists a proper filter $\mathscr{V}$ such that $\mathscr{U} \subset \mathscr{V}$. Choose $\theta \in \mathscr{V} \setminus \mathscr{U}$. Since $\mathscr{R}$ is a Boolean algebra, we have $\theta \vee \theta^* = 0^* \in \mathscr{U}$. Since $\theta \notin \mathscr{U}$, we must have $\theta^* \in \mathscr{U} \subset \mathscr{V}$. Hence $0 = \theta \wedge \theta^* \in \mathscr{V}$, which gives a contradiction. Thus, $\mathscr{U}$ is maximal.
(2) $\Rightarrow$ (3): It is clear.
(3) $\Rightarrow$ (4): Since every median filter is a $\mathscr{D}$-filter, it is clear.
(4) $\Rightarrow$ (1): Assume (3). Then $\mathscr{D} \subseteq \bigcap\{\mathscr{U} \mid \mathscr{U}$ is a prime filter$\} = \mathscr{M}_{Max.elts}$. Hence, $D = \mathscr{M}_{Max.elts}$.

Definition 10. *For any maximal filter $\mathscr{N}$ of $\mathscr{R}$, define*

$$\Omega(\mathscr{N}) = \{\theta \in \mathscr{R} \mid (\theta)^{\blacklozenge} \nsubseteq \mathscr{N}\}.$$

Lemma 7. *For every maximal filter $\mathscr{N}$, $\Omega(\mathscr{N})$ is a $\mathscr{D}$-filter contained in $\mathscr{N}$.*

Proof. Clearly $\mathscr{D} \subseteq \Omega(\mathscr{N})$. Let $\theta, \psi \in \Omega(\mathscr{N})$. Then $(\theta)^{\blacklozenge} \nsubseteq \mathscr{N}$, $(\psi)^{\blacklozenge} \nsubseteq \mathscr{N}$. As $\mathscr{N}$ is prime, we have $(\theta \wedge \psi)^{\blacklozenge} = (\theta)^{\blacklozenge} \cap (\psi)^{\blacklozenge} \nsubseteq \mathscr{N}$. Hence $\theta \wedge \psi \in \Omega(\mathscr{N})$. Let $\theta \in \Omega(\mathscr{N})$ and $\psi \in \mathscr{R}$. Then $(\theta)^{\blacklozenge} \nsubseteq \mathscr{N}$ and $(\theta)^{\blacklozenge} \subseteq (\psi \vee \theta)^{\blacklozenge}$. Since $(\theta)^{\blacklozenge} \nsubseteq \mathscr{N}$, we get $(\psi \vee \theta)^{\blacklozenge} \nsubseteq \mathscr{N}$. Thus $\psi \vee \theta \in \Omega(\mathscr{N})$. Therefore $\Omega(\mathscr{N})$ is a $\mathscr{D}$-filter of $\mathscr{R}$. Now, let $\theta \in \Omega(\mathscr{N})$. Then, we get $(\theta)^{\blacklozenge} \nsubseteq \mathscr{N}$. Hence there is a $\kappa \in (\theta)^{\blacklozenge}$ such that $\kappa \notin \mathscr{N}$. Since $\kappa \in (\theta)^{\blacklozenge}$, we get $\kappa^{**} \vee \theta^{**} = 0^*$ and hence $(\kappa \vee \theta)^{**} = 0^*$. Thus $\kappa \vee \theta \in \mathscr{D} \subseteq \mathscr{N}$. Since $\kappa \notin \mathscr{N}$, we have $\theta \in \mathscr{N}$. Therefore, $\Omega(\mathscr{N})$ is a subset of $\mathscr{N}$.

Let's indicate that $Max_{\mathscr{F}}\mathscr{R}$ is the set of all maximal filters of $\mathscr{R}$. For every $\kappa \in \mathscr{R}$, we also define that $\mathscr{N}_{\kappa^*} = \{\mathscr{N} \in Max_{\mathscr{F}}\mathscr{R} \mid \kappa^* \in \mathscr{N}\}$.

Theorem 11. *For any $\kappa \in \mathscr{R}$, $(\kappa)^{\blacklozenge} \subseteq \bigcap_{\mathscr{N} \in \mathscr{N}_{\kappa^*}} \Omega(\mathscr{N})$.*

Proof. Let $\theta \in (\kappa)^{\blacklozenge}$ and $\mathscr{N} \in \mathscr{N}_{\kappa^*}$. Then $\theta^{**} \vee \kappa^{**} = 0^*$ and $\kappa^* \in \mathscr{N}$. Suppose $a \in \mathscr{N}$. Then $0 = \kappa \wedge \kappa^* \in \mathscr{N}$, which gives a contradiction. Therefore $\kappa \notin \mathscr{N}$. Hence $\kappa \in (\theta)^{\blacklozenge}$ such that $\kappa \notin \mathscr{N}$. Thus $(\theta)^{\blacklozenge} \nsubseteq \mathscr{N}$. Hence $\theta \in \Omega(\mathscr{N})$. Thus $(\kappa)^{\blacklozenge} \subseteq \Omega(\mathscr{N})$ which is true for every $\mathscr{N} \in \mathscr{N}_{\kappa^*}$. Therefore, $(\kappa)^{\blacklozenge} \subseteq \bigcap_{\mathscr{N} \in \mathscr{N}_{\kappa^*}} \Omega(\mathscr{N})$.

Corollary 2. *For any $\kappa \in \mathscr{R}$, $\kappa^* \in \mathscr{N}$ implies $(\kappa)^{\blacklozenge} \subseteq \Omega(\mathscr{N})$.*

In Example 2, consider $\mathscr{U} = \{\kappa, \upsilon, \sigma, \rho\}$. Clearly it is observed that $\mathscr{D} = \{\kappa, \upsilon, \rho\}$ and $\mathscr{U}$ is a prime $\mathscr{D}$-filter. For any element $x \in \mathscr{U}$, there is no $y \notin \mathscr{U}$ such that $x^{**} \vee y^{**} = 0^*$. Hence $\mathscr{U}$ is not median. Furthermore, the following result establishes several equivalent conditions under which any prime $\mathscr{D}$-filter of a pseudo-complemented ADL become a median filter.

Theorem 12. *The following conditions are equivalent in an ADL $\mathscr{R}$:*

(1) *$\mathscr{R}$ is a stone ADL;*
(2) *every $\mathscr{D}$-filter is strongly coherent;*
(3) *every maximal filter is strongly coherent;*
(4) *every maximal filter is median;*
(5) *for any $\mathscr{N} \in Max_{\mathscr{F}}\mathscr{R}$, $\Omega(\mathscr{N})$ is median;*
(6) *for any $\kappa, \upsilon \in \mathscr{R}$, $\kappa \vee \upsilon \in \mathscr{D}$ implies $(\kappa)^{\blacklozenge} \vee (\upsilon)^{\blacklozenge} = \mathscr{R}$;*
(7) *for any $\kappa \in \mathscr{R}$, $(\kappa)^{\blacklozenge} \vee (\kappa^*)^{\blacklozenge} = \mathscr{R}$.*

Proof. (1) $\Rightarrow$ (2): Assume that $\mathscr{R}$ is a stone ADL. Let $\mathscr{K}$ be a $\mathscr{D}$-filter of $\mathscr{R}$. Clearly $\chi(\mathscr{K}) \subseteq \mathscr{K}$. Let $\theta \in \mathscr{K}$. By (1), we get $\theta^* \vee \theta^{**} = 0^*$. Suppose $(\theta)^{\blacklozenge} \vee \mathscr{K} \neq \mathscr{R}$. Then there exists a maximal filter $\mathscr{N}$ of $\mathscr{R}$ such that $(\theta)^{\blacklozenge} \vee \mathscr{K} \subseteq \mathscr{N}$. Hence $(\theta)^{\blacklozenge} \subseteq \mathscr{N}$ and $\theta \in \mathscr{K} \subseteq \mathscr{N}$. Since $\mathscr{N}$ is a prime, we get $\theta^* \notin \mathscr{N}$. Since $\theta^{***} \vee \theta^{**} = 0^*$, we get $\theta^* \in (\theta)^{\blacklozenge} \subseteq \mathscr{N}$ which is a contradiction. Thus $(\theta)^{\blacklozenge} \vee \mathscr{K} = \mathscr{R}$. Hence, $\mathscr{K}$ is strongly coherent.
(2) $\Rightarrow$ (3): It is obvious.
(3) $\Rightarrow$ (4): Assume that every maximal filter is strongly coherent. Let $\mathscr{N}$ be a maximal filter of $\mathscr{R}$. Then by our supposition, $\mathscr{N} = \chi(\mathscr{N})$. Let $\theta \in \mathscr{N}$. Then $(\theta)^{\blacklozenge} \vee \mathscr{N} = \mathscr{R}$. Hence $\kappa \wedge \upsilon = 0$ for some $\kappa \in (\theta)^{\blacklozenge}$ and $\upsilon \in \mathscr{N}$. We get $\kappa^{**} \vee \theta^{**} = 0^*$, since $\kappa \in (\theta)^{\blacklozenge}$. Suppose $\kappa \in \mathscr{N}$. Then $0 = \kappa \wedge \upsilon \in \mathscr{N}$, which gives a contradiction. Hence $\kappa \notin \mathscr{N}$. Thus, $\mathscr{N}$ is median.
(4) $\Rightarrow$ (5): Assume (4). Let $\mathscr{N} \in Max_{\mathscr{F}}\mathscr{R}$. Clearly $\Omega(\mathscr{N}) \subseteq \mathscr{N}$. Let $\theta \in \mathscr{N}$. Since $\mathscr{N}$ is median, there is $\psi \notin \mathscr{N}$ such that $\theta^{**} \vee \psi^{**} = 0^*$. Hence $(\theta)^{\blacklozenge} \nsubseteq \mathscr{N}$. Thus $\theta \in \Omega(\mathscr{N})$. Therefore, $\Omega(\mathscr{N}) = \mathscr{N}$ is median.
(5) $\Rightarrow$ (6) : Assume condition (5). Let $\kappa, \upsilon \in \mathscr{R}$ with $\kappa \vee \upsilon \in \mathscr{D}$. Suppose $(\kappa)^{\blacklozenge} \vee (\upsilon)^{\blacklozenge} \neq \mathscr{R}$. Then there is a maximal filter $\mathscr{N}$ such that $(\kappa)^{\blacklozenge} \vee (\upsilon)^{\blacklozenge} \subseteq \mathscr{N}$. Since $\Omega(\mathscr{N})$ is median, by Theorem 8, we get $(\kappa)^{\blacklozenge} \vee (\upsilon)^{\blacklozenge} \subseteq \mathscr{N} \Rightarrow (\kappa)^{\blacklozenge} \subseteq \mathscr{N}$ and $(\upsilon)^{\blacklozenge} \subseteq \mathscr{N} \Rightarrow (\kappa)^{\blacklozenge} \subseteq \Omega(\mathscr{N})$ and $(\upsilon)^{\blacklozenge} \subseteq \Omega(\mathscr{N}) \Rightarrow \kappa \notin \Omega(\mathscr{N})$ and $\upsilon \notin \Omega(\mathscr{N})$, since $\Omega(\mathscr{N})$ is median $\Rightarrow \kappa \vee \upsilon \notin \mathscr{N}$, which leads a contradiction to that $\kappa \vee \upsilon \in \mathscr{D} \subseteq \mathscr{N}$. Thus, $(\kappa)^{\blacklozenge} \vee (\upsilon)^{\blacklozenge} = \mathscr{R}$.
(6) $\Rightarrow$ (7): Let $\kappa \in \mathscr{R}$. Since $\kappa \vee \kappa^* \in \mathscr{D}$, by (6), we are through.
(7) $\Rightarrow$ (1): Assume condition (7). Let $\theta \in \mathscr{R}$. Then by (7), we have $(\theta)^{\blacklozenge} \vee (\theta^*)^{\blacklozenge} = \mathscr{R}$. Hence $0 \in (\theta)^{\blacklozenge} \vee (\theta^*)^{\blacklozenge}$. Then $0 = \kappa \wedge \upsilon$ for some $\kappa \in (\theta)^{\blacklozenge}$ and $\upsilon \in (\theta^*)^{\blacklozenge}$. Therefore $\kappa^* \wedge \upsilon = \upsilon$ and hence $\kappa^{**} \vee \upsilon^* = \upsilon^*$. Thus $0^* = \theta^{**} \vee \upsilon^*$. As $\upsilon \in (\theta^*)^{\blacklozenge}$, we get $\upsilon^{**} \vee \theta^* = 0^*$, and so $\upsilon^* \wedge \theta^{**} = 0$. Thus $\theta^* \wedge \upsilon^* = \upsilon^*$. Let γ be any element of $\mathscr{R}$. Now, $(\theta^* \vee \theta^{**}) \wedge \gamma = (0^* \wedge (\theta^* \vee \theta^{**})) \wedge \gamma = ((\theta^{**} \vee \upsilon^*) \wedge (\theta^{**} \vee \theta^*)) \wedge \gamma = (\theta^{**} \vee (\upsilon^* \wedge \theta^*)) \wedge \gamma = (\theta^{**} \vee (\theta^* \wedge \upsilon^*)) \wedge \gamma = (\theta^{**} \vee \upsilon^*) \wedge \gamma = 0^* \wedge \gamma = \gamma$. Therefore, $\theta^* \vee \theta^{**}$ is maximal. Therefore, $\mathscr{R}$ is stone.

For any filter $\mathscr{K}$ of $\mathscr{R}$, we denote $\mathscr{N}_{\mathscr{K}} = \{\mathscr{N} \in Max_{\mathscr{F}}\mathscr{R} \mid \mathscr{K} \subseteq \mathscr{N}\}$.

Theorem 13. *For any filter $\mathscr{K}$ of $\mathscr{R}$, $\chi(\mathscr{K}) = \bigcap_{\mathscr{N} \in \mathscr{N}_{\mathscr{K}}} \Omega(\mathscr{N})$.*

Proof. Let $\theta \in \chi(\mathscr{K})$ and $\mathscr{K} \subseteq \mathscr{N}$ where $\mathscr{N} \in Max_{\mathscr{F}}\mathscr{R}$. Then $\mathscr{R} = (\theta)^{\blacklozenge} \vee \mathscr{K} \subseteq (\theta)^{\blacklozenge} \vee \mathscr{N}$. Suppose $(\theta)^{\blacklozenge} \subseteq \mathscr{N}$, then $\mathscr{N} = \mathscr{R}$, we get a contradiction. So that $(\theta)^{\blacklozenge} \nsubseteq \mathscr{N}$. Thus $\theta \in \Omega(\mathscr{N})$ for all $\mathscr{N} \in \mathscr{N}_{\mathscr{K}}$. Therefore $\chi(\mathscr{K}) \subseteq$

$\bigcap_{\mathscr{N}\in\mathscr{N}_{\mathscr{K}}} \Omega(\mathscr{N})$. Let $\theta \in \bigcap_{\mathscr{N}\in\mathscr{N}_{\mathscr{K}}} \Omega(\mathscr{N})$. Then, we get $\theta \in \Omega(\mathscr{N})$ for all $\mathscr{N} \in \mathscr{N}_{\mathscr{K}}$. Suppose $(\theta)^{\blacklozenge} \vee \mathscr{K} \neq \mathscr{R}$. Then there is a maximal filter $\mathscr{N}_0$ such that $(\theta)^{\blacklozenge} \vee \mathscr{K} \subseteq \mathscr{N}_0$. Hence $(\theta)^{\blacklozenge} \subseteq \mathscr{N}_0$ and $\mathscr{K} \subseteq \mathscr{N}_0$. Since $\mathscr{K} \subseteq \mathscr{N}_0$, by hypothesis, we get $\theta \in \Omega(\mathscr{N}_0)$. Thus $(\theta)^{\blacklozenge} \nsubseteq \mathscr{N}_0$, which gives a contradiction. So that $(\theta)^{\blacklozenge} \vee \mathscr{K} = \mathscr{R}$. Thus $\theta \in \chi(\mathscr{K})$. Therefore, $\bigcap_{\mathscr{N}\in\mathscr{N}_{\mathscr{K}}} \Omega(\mathscr{N}) \subseteq \chi(\mathscr{K})$.

Theorem 14. *In $\mathscr{R}$, the following statements are equivalent:*

(1) *$\mathscr{R}$ is a stone ADL;*
(2) *for any $\mathscr{N} \in \mathscr{N}$, $\Omega(\mathscr{N})$ is maximal;*
(3) *for any filters $\mathscr{K}, \mathscr{L}$, $\mathscr{K} \vee \mathscr{L} = \mathscr{R}$ implies $\chi(\mathscr{K}) \vee \chi(\mathscr{L}) = \mathscr{R}$;*
(4) *for any filters $\mathscr{K}, \mathscr{L}$, $\chi(\mathscr{K}) \vee \chi(\mathscr{L}) = \chi(\mathscr{K} \vee \mathscr{L})$;*
(5) *for any two distinct maximal filters $\mathscr{N}, \mathscr{M}$, $\Omega(\mathscr{N}) \vee \Omega(\mathscr{M}) = \mathscr{R}$;*
(6) *for any $\mathscr{N} \in Max_{\mathscr{F}}\mathscr{R}$, $\mathscr{N}$ is the unique member of μ such that $\Omega(\mathscr{N}) \subseteq \mathscr{N}$.*

Proof. $(1) \Rightarrow (2)$: Assume (1). Let $\mathscr{N} \in Max_{\mathscr{F}}\mathscr{R}$. Clearly, we get $\Omega(\mathscr{N}) \subseteq \mathscr{N}$. Let $\theta \in \mathscr{N}$. Since $\mathscr{R}$ is stone, By Theorem 12, we get that $\mathscr{N}$ is median. Then there is $\psi \notin \mathscr{N}$ such that $\theta^{**} \vee \psi^{**} = 0^*$. Hence $\psi \in (\theta)^{\blacklozenge}$ and $\psi \notin \mathscr{N}$. Hence $(\theta)^{\blacklozenge} \nsubseteq \Omega(\mathscr{N})$. Hence $\theta \in \Omega(\mathscr{N})$. Therefore, $\Omega(\mathscr{N}) = \mathscr{N}$ is maximal.

$(2) \Rightarrow (3)$: Assume condition (2). Clearly $\Omega(\mathscr{N}) = \mathscr{N}$ for all $\mathscr{N} \in Max_{\mathscr{F}}\mathscr{R}$. Let $\mathscr{K}, \mathscr{L}$ be filters of $\mathscr{R}$ with $\mathscr{K} \vee \mathscr{L} = \mathscr{R}$. If $\chi(\mathscr{K}) \vee \chi(\mathscr{L}) \neq \mathscr{R}$, then there is $\mathscr{N} \in Max_{\mathscr{F}}\mathscr{R}$ such that $\chi(\mathscr{K}) \vee \chi(\mathscr{L}) \subseteq \mathscr{N}$. Hence $\chi(\mathscr{K}) \subseteq \mathscr{N}$ and $\chi(\mathscr{L}) \subseteq \mathscr{N}$. Now $\chi(F) \subseteq \mathscr{N} \Rightarrow \bigcap_{\mathscr{N}\in\mathscr{N}_{\mathscr{K}}} \Omega(\mathscr{N}) \subseteq \mathscr{N} \Rightarrow \Omega(\mathscr{N}_i) \subseteq \mathscr{N}$, for some $\mathscr{N}_i \in \mathscr{N}_{\mathscr{K}} \Rightarrow \mathscr{N}_i \subseteq \mathscr{N}$ By condition (2) $\Rightarrow \mathscr{K} \subseteq \mathscr{N}$. Similarly, we can get $\mathscr{L} \subseteq \mathscr{N}$. Hence $\mathscr{R} = \mathscr{K} \vee \mathscr{L} \subseteq \mathscr{N}$, we get a contradiction. Thus, $\chi(\mathscr{K}) \vee \chi(\mathscr{L}) = \mathscr{R}$.

$(3) \Rightarrow (4)$: Assume (3). Let $\mathscr{K}, \mathscr{L} \in \mathscr{K}(\mathscr{R})$. Clearly, we have $\chi(\mathscr{K}) \vee \chi(\mathscr{L}) \subseteq \chi(\mathscr{K} \vee \mathscr{L})$. Let $\theta \in \chi(\mathscr{K} \vee \mathscr{L})$. Then $((\theta)^{\blacklozenge} \vee \mathscr{K}) \vee ((\theta)^{\blacklozenge} \vee \mathscr{L}) = (\theta)^{\blacklozenge} \vee \mathscr{K} \vee \mathscr{L} = \mathscr{R}$. By (3), we get that $\chi((\theta)^{\blacklozenge} \vee \mathscr{K}) \vee \chi((\theta)^{\blacklozenge} \vee \mathscr{L}) = \mathscr{R}$. Thus $\theta \in \chi((\theta)^{\blacklozenge} \vee \mathscr{K}) \vee \chi((\theta)^{\blacklozenge} \vee \mathscr{L})$. Hence $\theta = \nu \wedge \vartheta$ for some $\nu \in \chi((\theta)^{\blacklozenge} \vee \mathscr{K})$ and $\vartheta \in \chi((\theta)^{\blacklozenge} \vee \mathscr{L})$. Now $\nu \in \chi((\theta)^{\blacklozenge} \vee \mathscr{K}) \Rightarrow (\nu)^{\blacklozenge} \vee (\theta)^{\blacklozenge} \vee \mathscr{K} = \mathscr{R} \Rightarrow \mathscr{R} = ((\nu)^{\blacklozenge} \vee (\theta)^{\blacklozenge}) \vee \mathscr{K} \subseteq (\nu \vee \theta)^{\blacklozenge} \vee \mathscr{K} \Rightarrow (\nu \vee \theta)^{\blacklozenge} \vee \mathscr{K} = \mathscr{R} \Rightarrow \nu \vee \theta \in \chi(\mathscr{K})$. Similarly, we can get $\vartheta \vee \theta \in \chi(\mathscr{L})$. Hence $\theta = \theta \vee \theta = \theta \vee (\nu \wedge \vartheta) = (\theta \vee \nu) \wedge (\theta \vee \vartheta) \in \chi(\mathscr{K}) \vee \chi(\mathscr{L})$. Hence $\chi(\mathscr{K} \vee \mathscr{L}) \subseteq \chi(\mathscr{K}) \vee \chi(\mathscr{L})$. Therefore, $\chi(\mathscr{K}) \vee \chi(\mathscr{L}) = \chi(\mathscr{K} \vee \mathscr{L})$.

$(4) \Rightarrow (5)$: Assume (4). Let $\mathscr{N}, \mathscr{M} \in Max_{\mathscr{F}}\mathscr{R}$ with $\mathscr{N} \neq \mathscr{M}$. Choose $\theta \in \mathscr{N} \setminus \mathscr{M}$ and $\psi \in \mathscr{M} \setminus \mathscr{N}$. Since $\theta \notin \mathscr{M}$ and $\psi \notin \mathscr{N}$, we get $\theta^* \in \mathscr{M}$ and $\psi^* \in \mathscr{N}$. Hence $(\theta \wedge \psi^*) \wedge (\psi \wedge \theta^*) = (\theta \wedge \theta^*) \wedge (\psi \wedge \psi^*) = 0$. Then $\mathscr{R} = \chi(\mathscr{R}) = \chi([0)) = \chi([(\theta \wedge \psi^*) \wedge (\psi \wedge \theta^*))) = \chi([\theta \wedge \psi^*) \vee [\psi \wedge \theta^*)) = \chi([\theta \wedge \psi^*)) \vee \chi([\psi \wedge \theta^*)) \subseteq \Omega(\mathscr{N}) \vee \Omega(\mathscr{M})$. Therefore, $\Omega(\mathscr{N}) \vee \Omega(\mathscr{M}) = \mathscr{R}$.

$(5) \Rightarrow (6)$: Assume (5). Let $\mathscr{N} \in Max_{\mathscr{F}}\mathscr{R}$. Clearly $\Omega(\mathscr{N}) \subseteq \mathscr{N}$. Suppose $\mathscr{M} \in \mathscr{N}$ such that $\mathscr{M} \neq \mathscr{N}$ and $\Omega(\mathscr{N}) \subseteq \mathscr{M}$. Since $\Omega(\mathscr{M}) \subseteq \mathscr{M}$, we get that $\mathscr{R} = \Omega(\mathscr{N}) \vee \Omega(\mathscr{M}) \subseteq \mathscr{M}$, which gives a contradiction. Therefore, $\mathscr{N}$ is the unique maximal filter of $\mathscr{R}$ such that $\Omega(\mathscr{N})$ is contained in $\mathscr{N}$.

$(6) \Rightarrow (1)$: Let $\mathscr{N} \in Max_{\mathscr{F}}\mathscr{R}$. If $\Omega(\mathscr{N}) \neq \mathscr{N}$, then there is $\mathscr{N}_0 \in Max_{\mathscr{F}}\mathscr{R}$ such that $\Omega(\mathscr{N}) \subseteq \mathscr{N}_0$, which contradicts uniqueness of $\mathscr{N}$. Hence $\Omega(\mathscr{N}) = \mathscr{N}$. Let $\theta \in \mathscr{N} = \Omega(\mathscr{N})$. Hence there is $\psi \notin \mathscr{N}$ such that $\theta^{**} \vee \psi^{**}$ is maximal. Therefore $\mathscr{N}$ is median. By Theorem 12, $\mathscr{R}$ is stone.

Declarations

Acknowledgments: We would like to thank the editor and the referees for their valuable comments and suggestions.

References

[1] G. Birkhoff, *Lattice theory*, Amer. Math. Soc. Colloq. XXV, Providence, U.S.A., (1967).
[2] G.C. Rao, *Almost Distributive Lattices*, Doctoral Thesis, Dept. of Mathematics, Andhra University, Visakhapatnam, (1980).
[3] G.C. Rao and S. Ravi Kumar, Minimal prime ideals in an ADL, *Int. J. Contemp. Sciences*, **4** (2009), 475-484.
[4] G. Gratzer, *General lattice theory*, Academic press, New york, San Francisco, U.S.A., 1978.
[5] N. Rafi, Ravi Kumar Bandaru, S. N. Rao, $\delta-$ideals in pseudo-complemented almost distributive lattices, *Palestine Journal of Mathematics*, **5(2)** (2016), 240-248.
[6] N. Rafi, P. Vijaya Saradhi and M. Balaiah, The Space of Minimal Prime $D-$filters of Almost Distributive Lattices, *Accepted for publication in Discussiones Mathematicae - General Algebra and Applications*.

[7] U.M. Swamy and G.C. Rao, Almost distributive lattices, *Jour. Aust. Math. Soc. (Series A)*, **31** (1981), 77-91.
[8] U.M. Swamy, G.C. Rao and G.N. Rao, Pseudo-complementation on almost distributive lattices, *Southeast Asian Bull. Math.*, **24** (2000), 95-104.
[9] U.M. Swamy, G.C. Rao and G.N. Rao, Stone almost distributive lattices, *Southeast Asian Bull. Math.*, **27(6)** (2003), 513-526.