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A Computational Study Of Fuzzy Fractional Sharma-Tasso-Olver Equation Via Analytical Method

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Abstract: In this article, we obtain the numerical solution of the fuzzy form of the fractional Sharma-Tasso-Olver equation using the Homotopy Analysis Transform Method. This method combines two powerful and well-known methods: the Homotopy Analysis Method and the Laplace Transform Method. Two approximate solutions of the fuzzy fractional Sharma-Tasso-Olver equation are obtained using this approach. Comparisons are made between the results obtained by the proposed method and the exact solution available in the open literature. All the obtained numerical computations justify that the proposed method is highly reliable, simple, efficient, and effective for handling fuzzy fractional-order equations like the Sharma-Tasso-Olver equation.

Keywords: Laplace transform method; Caputo derivative; Homotopy analysis transform method; Fractional Sharma-Tasso-Olver equation.

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1 Introduction

Fractional calculus (FC) is an elegant extension of classical calculus that explores derivatives and integrals of arbitrary order. In recent years, FC has gained considerable importance due to its wide range of practical applications and its ability to describe various real-life physical phenomena. Comprehensive details on FC theory can be found in [10,12]. Extensive research has been dedicated to solving fractional ordinary differential equations, fractional partial differential equations, and fractional integral equations, benefiting both physicists and engineers [9,11]. Since most nonlinear fractional equations lack exact solutions, analytical techniques have been developed and implemented to obtain approximate solutions to these problems [2,3,4,20,21,22].

Fractional differential equations (FDEs) have been extensively studied due to their wide-ranging applications in various fields of engineering and science. The concept of fuzzy FDEs (FFDEs) was first introduced by Agarwal et al. [17]. Following this pioneering work, several authors have applied different analytical techniques to solve fuzzy FDEs [1,13].

In 1965, Zadeh introduced set theory with fuzziness, which was later expanded by Chang and Zadeh through the development of fuzzy mapping and control [6,14]. Building on these foundations, many researchers further generalized the concept to establish elementary fuzzy calculus [7,15]. Over the past few decades, FFDEs have attracted significant interest among researchers. When dealing with imprecise and vague information, parameters are expressed using fuzzy numbers instead of crisp values. For further literature on fuzzy environments, see [13,16,23].

In this work, we employ a widely used analytical technique, the Homotopy Analysis Transform Method (HATM) [19], within a fuzzy environment. The application of the Homotopy Analysis Method (HAM) to fuzzy differential equations was

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first introduced in 2012. Later, in 2016, FFDEs with fuzzy initial conditions were successfully solved using HAM [8]. The HAM was originally proposed by Liao [18]. It is grounded in homotopy, a fundamental concept in differential geometry and topology. This method provides flexibility in selecting an auxiliary linear operator and an initial approximation. Within HAM, the initial approximation gradually transforms into the exact solution of the problem. To regulate the convergence region of the approximate solution series, an auxiliary parameter h is introduced. When HAM is combined with the Laplace Transform (LT), it results in HATM. A key advantage of HATM is its ability to generate rapidly convergent series solutions for FFDEs.

The objective of this article is to apply the Fractional HATM (FHATM) in a fuzzy environment to derive and examine the approximate solution of the time-fractional Sharma-Tasso-Olver (STO) equation [5], expressed as:

$$\frac{\partial^{\gamma}\tilde{s}(x,t)}{\partial t^{\gamma}}+3c(\frac{\partial\tilde{s}(x,t)}{\partial x})^{2}+3c\tilde{s}^{2}\frac{\partial\tilde{s}(x,t)}{\partial x}+3c\tilde{s}\frac{\partial^{2}\tilde{s}(x,t)}{\partial x^{2}}+c\frac{\partial^{3}\tilde{s}(x,t)}{\partial x^{3}}=0, \tag{1.1}$$

where $t>0,\quad \gamma\in(0,1]$.
Here, c is an arbitrary constant, and γ is the order of the time-fractional derivative.
The STO equation (1.1) plays an important role in describing nonlinear wave phenomena. Recently, equation (1.1) has been studied using the Homotopy Perturbation Method, Variational Iteration Method, Residual Power Series Method, and Adomian Decomposition Method. However, to the best of our knowledge, this is the first attempt to find an approximate solution of equation (1.1) using FHATM in a fuzzy environment.

The paper is organized as follows: First, we discuss the basic definitions related to fractional derivatives and fuzzy calculus. The algorithm for FHATM is presented in Section 3. In Section 4, we discuss the application of this method to the fractional STO equation. The obtained results are analyzed in Section 5. Finally, Section 6 concludes the paper.

2 Basic definitions related to fuzzy system and fractional derivatives

Definition 1.[6,8] A fuzzy number $\tilde{s}$ is an ordered pair of functions $(\underline{s}(\omega),\bar{s}(\omega)), 0\leq\omega\leq 1$, which satisfy the following conditions*:*

1.$\underline{s}(\omega)$ is bounded, left continuous and non-decreasing function on [0,1].
2.$\bar{s}(\omega)$ is bounded, left continuous and non-increasing function on [0,1].
3.$\underline{s}(\omega)\leq\bar{s}(\omega),\qquad\forall\omega\in[0,1]$.

Definition 2.[6,8] A mapping $\tilde{s}:P\to E^{1},\;\; P\subseteq E^{1},$ is said to be fuzzy function with non-fuzzy variable when the ω set is defined as:
$[\tilde{s}(t)]_{\omega}=[\underline{s}(t,\omega),\bar{s}(t,\omega)],\qquad$ where $t\in P$ and $0\leq\omega\leq 1$.

Definition 3. [5,10,19] The Caputo fractional derivative of order γ of fuzzy function $\tilde{s}(x,\omega)$ is defined as:
$_{C}D_{\gamma}^{x_0}\tilde{s}(x,\omega)=[_{C}D_{\gamma}^{x_0}\underline{s}(x,\omega),_{C}D_{\gamma}^{x_0}\bar{s}(x,\omega)]$, where

$$\begin{aligned}
_{C}D_{\gamma}^{x_0}\underline{s}(x) &= \tfrac{1}{\Gamma(q-\gamma)}\int_{x}^{x_0}(x-t)^{q-\gamma-1}\underline{s}^{q}(t)dt, && q-1<\gamma<q\\
&= \tfrac{d^{q}\underline{s}}{dt^{q}}, && \gamma=q.\\
_{C}D_{\gamma}^{x_0}\bar{s}(x) &= \tfrac{1}{\Gamma(q-\gamma)}\int_{x}^{x_0}(x-t)^{q-\gamma-1}\bar{s}^{q}(t)dt, && q-1<\gamma<q\\
&= \tfrac{d^{q}\bar{s}}{dt^{q}}, && \gamma=q.
\end{aligned}$$

Definition 4.[5,10,19] The LT of the fuzzy valued function $\tilde{s}(x,\omega)$ is denoted by $\tilde{S}(x,\omega)$ and defined as:

$$\begin{aligned}
&\tilde{S}(x,\omega)=[\underline{S}(x,\omega),\bar{S}(x,\omega)]\\
&\underline{S}(x,\omega)=\mathscr{L}[\underline{s}(x)]=\textstyle\int_{0}^{\infty}exp^{-xt}\underline{s}(t)dt, && t>0\\
&\bar{S}(x,\omega)=\mathscr{L}[\bar{s}(x)]=\textstyle\int_{0}^{\infty}exp^{-xt}\bar{s}(t)dt, && t>0.
\end{aligned}$$

3 Algorithm of fractional homotopy analysis transform method in fuzzy environment

Let us consider the fractional Sharma-Tasso-Olver equation in a fuzzy environment, which is of the form:

$$\frac{\partial^{\gamma}\tilde{s}(x,t)}{\partial t^{\gamma}}+3c\left(\frac{\partial\tilde{s}(x,t)}{\partial x}\right)^{2}+3c\tilde{s}^{2}\frac{\partial\tilde{s}(x,t)}{\partial x}+3c\tilde{s}\frac{\partial^{2}\tilde{s}(x,t)}{\partial x^{2}}+c\frac{\partial^{3}\tilde{s}(x,t)}{\partial x^{3}}=0, \tag{3.1}$$

$t>0, \quad \gamma\in(0,1]$, where $\frac{\partial^{\gamma}\tilde{s}(x,t)}{\partial t^{\gamma}}$ is Caputo fractional derivative.
Applying the LT to both sides of the equation above, we obtain

$$\mathscr{L}[\frac{\partial^{\gamma}\tilde{s}(x,t)}{\partial t^{\gamma}}]+\mathscr{L}[3c(\frac{\partial\tilde{s}(x,t)}{\partial x})^2+3c\tilde{s}^2\frac{\partial\tilde{s}(x,t)}{\partial x}+3c\tilde{s}\frac{\partial^2\tilde{s}(x,t)}{\partial x^2}+c\frac{\partial^3\tilde{s}(x,t)}{\partial x^3}]=0. \tag{3.2}$$

Using the LT of the Caputo fractional derivative, we have

$$\begin{aligned} m^{\gamma}\mathscr{L}[\tilde{s}(x,t)]-\sum_{n=0}^{l-1}m^{\gamma-n-1}\tilde{s}^{(n)}(0,t)+\mathscr{L}[3c(\frac{\partial\tilde{s}(x,t)}{\partial x})^2+3c\tilde{s}^2\frac{\partial\tilde{s}(x,t)}{\partial x}+\\ 3c\tilde{s}\frac{\partial^2\tilde{s}(x,t)}{\partial x^2}+c\frac{\partial^3\tilde{s}(x,t)}{\partial x^3}]=0,\end{aligned} \tag{3.3}$$

or,

$$\begin{aligned} \mathscr{L}[\tilde{s}(x,t)]=m^{-\gamma}\sum_{n=0}^{l-1}m^{\gamma-n-1}\tilde{s}^{(n)}(0,t)+m^{-\gamma}\mathscr{L}[3c(\frac{\partial\tilde{s}(x,t)}{\partial x})^2+3c\tilde{s}^2\frac{\partial\tilde{s}(x,t)}{\partial x}+\\ 3c\tilde{s}\frac{\partial^2\tilde{s}(x,t)}{\partial x^2}+c\frac{\partial^3\tilde{s}(x,t)}{\partial x^3}]=0.\end{aligned} \tag{3.4}$$

The nonlinear operator is of the form

$$\begin{aligned} \mathscr{T}[\tilde{\psi}(x,t;v)]=\mathscr{L}[\tilde{\psi}(x,t;v)]-m^{-\gamma}\sum_{n=0}^{l-1}m^{\gamma-n-1}\tilde{s}^{(n)}(0,t)-m^{-\gamma}\mathscr{L}[3c(\frac{\partial\tilde{s}(x,t)}{\partial x})^2+\\ 3c\tilde{s}^2\frac{\partial\tilde{s}(x,t)}{\partial x}+3c\tilde{s}\frac{\partial^2\tilde{s}(x,t)}{\partial x^2}+c\frac{\partial^3\tilde{s}(x,t)}{\partial x^3}]=0,\end{aligned} \tag{3.5}$$

where $v\in[0,1]$, is an auxiliary parameter, $\tilde{\psi}(x,t;v)$ is real valued function of x,t and v.
The deformation equation of order zero for the fractional partial differential equation (3.1) in fuzziness is

$$(1-v)\mathscr{L}[\tilde{\psi}(x,t;v)-\tilde{s}_0(x,t)]=hvR(x,t)\mathscr{T}[\tilde{\psi}(x,t)], \tag{3.6}$$

where $R(x,t)\neq 0$ is an auxiliary function, $u\in[0,1]$, is an auxiliary parameter and nonzero auxiliary parameter h is a converging control parameter.
Here the unknown function $\tilde{\psi}(x,t;v)$ will be computed and the initial approximate of $\tilde{s}(x,t)$ is $\tilde{s}_0(x,t)$.
In FHATM, there has a freedom to choose auxiliary parameter and initial approximation.
Note that if $v=0$, we have $\tilde{\psi}(x,t;0)=\tilde{s}_0(x,t)$. Also, if $v=1$, then $\tilde{\psi}(x,t;1)=\tilde{s}(x,t)$.
As v varies from 0 to 1, solution $\tilde{\psi}(x,t;v)$ changes from initial guess approximation $\tilde{s}_0(x,t)$ to the solution $\tilde{s}(x,t)$.
Using Taylor series expansion on $\tilde{\psi}(x,t;v)$ with respect to v, we have

$$\tilde{\psi}(x,t;v)=\tilde{s}_0(x,t)+\sum_{l=1}^{+\infty}\tilde{s}_l(x,t)v^l, \tag{3.7}$$

where

$$\tilde{s}_l(x,t)=\left[\frac{1}{\Gamma(l+1)}\frac{\partial^l\tilde{\psi}(x,t;v)}{\partial v^l}\right]_{v=0}. \tag{3.8}$$

If initial guess, auxiliary function, auxiliary linear operator, and auxiliary parameter h are chosen properly, the series converges at $v=1$.

$$\tilde{\psi}(x,t)=\tilde{s}_0(x,t)+\sum_{l=1}^{+\infty}\tilde{s}_l(x,t). \tag{3.9}$$

Now, let us define vector $\vec{\tilde{s}}_i$ as

$$\vec{\tilde{s}}_i=\{\tilde{s}_0(x,t),\tilde{s}_1(x,t),....,\tilde{s}_i(x,t)\}. \tag{3.10}$$

Differentiating equation (3.6) l times with respect to parameter v and the setting $v=0$, then dividing by $\Gamma(l+1)$, the Lth order deformation equation is

$$\mathscr{L}[\tilde{s}_l(x,t)-\chi_l\tilde{s}_{l-1}(x,t)]=h\mathscr{T}(x,t)\rho_l(\vec{\tilde{s}}_{l-1},x,t), \tag{3.11}$$

where

$$\rho_l(\vec{\tilde{s}}_{l-1},x,t)=\frac{1}{\Gamma(l)}\left[\frac{\partial^{(l-1)}\mathscr{T}[\tilde{\psi}(x,t;v)]}{\partial v^{(l-1)}}\right]_{v=0}, \tag{3.12}$$

and $\chi_l=\begin{cases}0, & l\le 1\\ 1, & l>1.\end{cases}$

Applying the inverse LT to equation (3.11) in fuzziness, we get

$$\tilde{s}_l(x,t)=\chi_l\tilde{s}_{l-1}(x,t)]+\mathscr{L}^{-1}[h\mathscr{T}(x,t)\rho_l(\vec{\tilde{s}}_{l-1},x,t)]. \tag{3.13}$$

Thus for $l\geq 1$, we obtain $\tilde{s}_l(x,t)$.

For Lth order,

$$\tilde{s}(x,t)=\sum_{l=0}^{L}\tilde{s}_l(x,t). \tag{3.14}$$

We obtain an accurate approximate solution of the fractional partial differential equation as $L\rightarrow\infty$.

In fuzzy setting, there are two solutions.

The upper bound solution is given by

$$\bar{s}(x,t)=\sum_{l=0}^{\infty}\bar{s}_l(x,t), \tag{3.15}$$

and the lower bound solution is given by

$$\underline{s}(x,t)=\sum_{l=0}^{\infty}\underline{s}_l(x,t). \tag{3.16}$$

4 Application of the FHATM with fuzziness

Consider fuzzy fractional STO equation [5] of the form:

$$\frac{\partial^{\gamma}\tilde{s}(x,t)}{\partial t^{\gamma}}+3c(\frac{\partial\tilde{s}(x,t)}{\partial x})^2+3c\tilde{s}^2\frac{\partial\tilde{s}(x,t)}{\partial x}+3c\tilde{s}\frac{\partial^2\tilde{s}(x,t)}{\partial x^2}+c\frac{\partial^3\tilde{s}(x,t)}{\partial x^3}=0, \tag{4.1}$$

where $t>0,\quad \gamma\in(0,1]$.

Here, the initial condition is

$$\tilde{s}(x,0)=\tilde{r}(\omega)\frac{2k(u+\tanh kx)}{1+u\tanh kx},\, x\in[0,1],\, k,u\in C,$$

where $\tilde{r}(\omega)=(\underline{r}(\omega),\bar{r}(\omega))=(\omega-1,1-\omega)\ ,\quad \omega\in[0,1]$.

Note that the exact solution [5] is

$$\tilde{s}(x,t)=\tilde{r}(\omega)\frac{2k(u+\tanh k(x-4ck^2t))}{1+u\tanh k(x-4ck^2t)}.$$

Applying FHATM in fuzzy environment, we get

$$\begin{aligned}m^{\gamma}\mathscr{L}[\tilde{s}(x,t)]-\sum_{n=0}^{l-1}m^{\gamma-n-1}\tilde{s}^{(n)}(0,t)+\mathscr{L}[3c(\frac{\partial\tilde{s}(x,t)}{\partial x})^2+3c\tilde{s}^2\frac{\partial\tilde{s}(x,t)}{\partial x}+\\ 3c\tilde{s}\frac{\partial^2\tilde{s}(x,t)}{\partial x^2}+c\frac{\partial^3\tilde{s}(x,t)}{\partial x^3}]=0.\end{aligned} \tag{4.2}$$

Then, we obtain the nonlinear operator as

$$\begin{aligned}\mathscr{T}[\tilde{\psi}(x,t;v)]=\mathscr{L}[\tilde{\psi}(x,t;v)]-m^{-\gamma}\sum_{n=0}^{l-1}m^{\gamma-n-1}\tilde{s}^{(n)}(0,t)-m^{-\gamma}\mathscr{L}[3c(\frac{\partial\tilde{s}(x,t)}{\partial x})^2+\\ 3c\tilde{s}^2\frac{\partial\tilde{s}(x,t)}{\partial x}+3c\tilde{s}\frac{\partial^2\tilde{s}(x,t)}{\partial x^2}+c\frac{\partial^3\tilde{s}(x,t)}{\partial x^3}]=0,\end{aligned} \tag{4.3}$$

Now, applying the FHATM with $R(x,t) = 1$, we obtain the approximations

$$\tilde{s}_0(x,t;\omega) = \tilde{s}(x,0) = \tilde{r}(\omega)\frac{2k(u+\tanh kx)}{1+u\tanh kx}, \tag{4.4}$$

$$\begin{aligned} \tilde{s}_1(x,t;\omega) = -\tilde{r}(\omega)^3 \frac{24chk^4t^\gamma u^4(sechkx)^2}{\Gamma[1+\gamma](1+u\tanh kx)^4} + \\ \tilde{r}(\omega)^3 \frac{36chk^4t^\gamma u^4(sechkx)^4}{\Gamma[1+\gamma](1+u\tanh kx)^4} - \\ \tilde{r}(\omega)^3 \frac{72chk^4t^\gamma u^3(sechkx)^2\tanh kx}{\Gamma[1+\gamma](1+u\tanh kx)^4}. \end{aligned} \tag{4.5}$$

The approximate solution of the fuzzy fractional STO equation (4.1) is:

$$\tilde{s}(x,t) = \sum_{l=0}^{\infty} \tilde{s}_l(x,t), \tag{4.6}$$

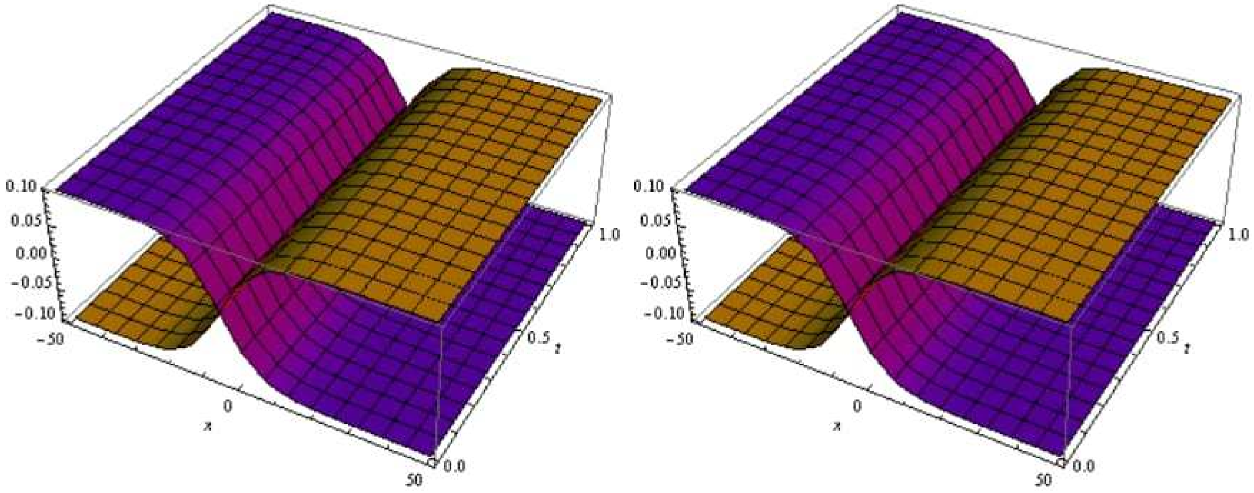
and correspondingly, we can obtain the upper and lower bound solutions of equation (4.1).

5 Results and discussion

In this section, we discuss and analyze the efficiency and accuracy of the obtained approximate solutions of the FDE (4.1) using FHATM in a fuzzy environment.

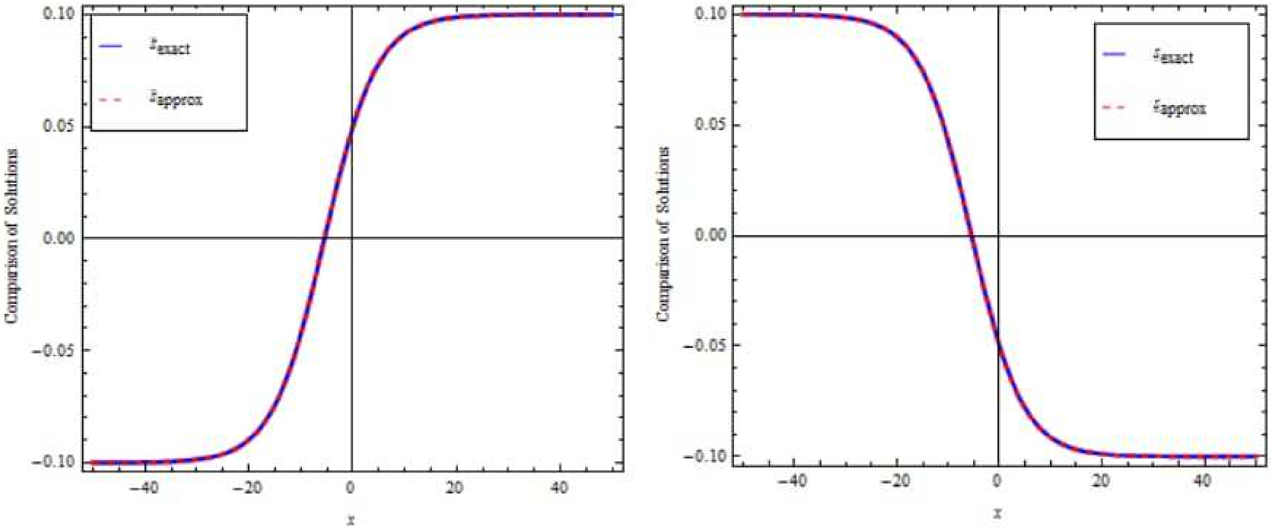
Figure 1 presents a graphical comparison between the surface graphs of the exact and approximate upper and lower bound solutions of equation (4.1) using FHATM. From Figure 1, it is observed that the upper and lower bound approximate solutions obtained by the proposed method closely match the known exact solutions.

A two-dimensional comparison is shown in Figure 2. It is evident from both figures that there is a strong agreement between the fuzzy exact solutions and the obtained fuzzy approximate solutions.



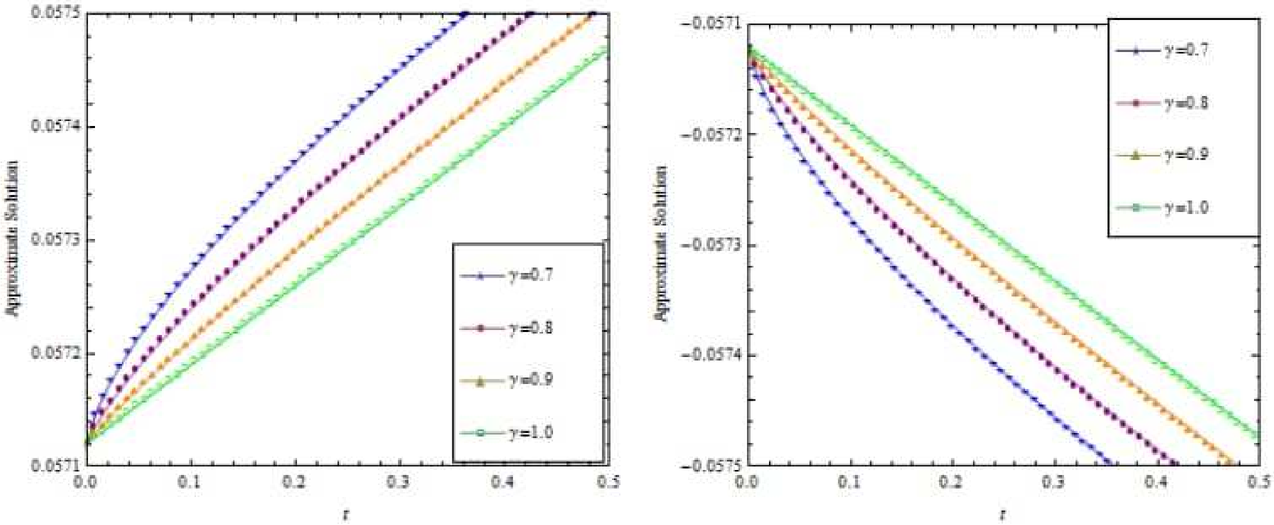
(A) (B)

Fig. 1: (A) The surface graph of the upper and lower bound approximate solution of the fuzzy FDE (4.1) when $\gamma = 1$. (B) The surface graph of the upper and lower bound exact solution of the fuzzy FDE (4.1).



(A) (B)

Fig. 2: (A) Comparison of upper-bound exact and upper-bound approximate solution of the fuzzy FDE (4.1). (B) Comparison of lower-bound exact and lower-bound approximate solution of the fuzzy FDE (4.1).



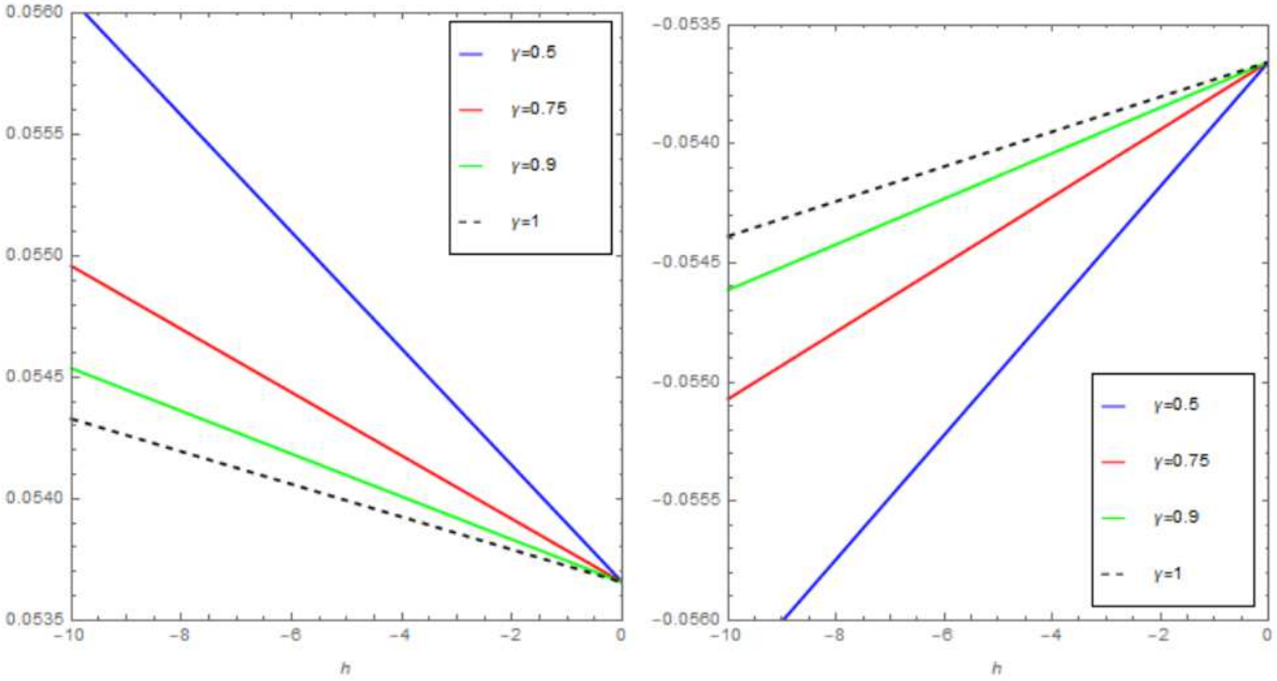
(A) (B)

Fig. 3: (A) Solution plots of different values of the fractional derivative γ for the upper bound fuzzy solution. (B) Solution plots of different values of the fractional derivative γ for the lower bound fuzzy solution.

Next, in Figure 3, we illustrate two-dimensional curves for the upper and lower bound fuzzy approximate solutions of equation (4.1) for different values of the fractional derivative γ.

Figure 4 represents the converging control parameter in HAM, displaying the h-cut curves of the upper and lower bound solutions. The blue, red, green, and black curves correspond to γ values of 0.5, 0.75, 0.9, and 1, respectively. According to h- cut curve in FHAM, there is flexibility in choosing the auxiliary converging control parameter h, with an acceptable

range of h is $-10 \leq h \leq 0$.
Next, Figure 5 shows the absolute error curves of the upper and lower bound approximate solutions of equation (4.1),



(A) (B)

Fig. 4: (A) Plot of h- cut curve for the upper-bound fuzzy solution of the equation (4.1) at $x = 0.5$ and $t = 0.1$. (B) Plot of h-cut curve for the lower-bound fuzzy solution of the equation (4.1) at $x = 0.5$ and $t = 0.1$.

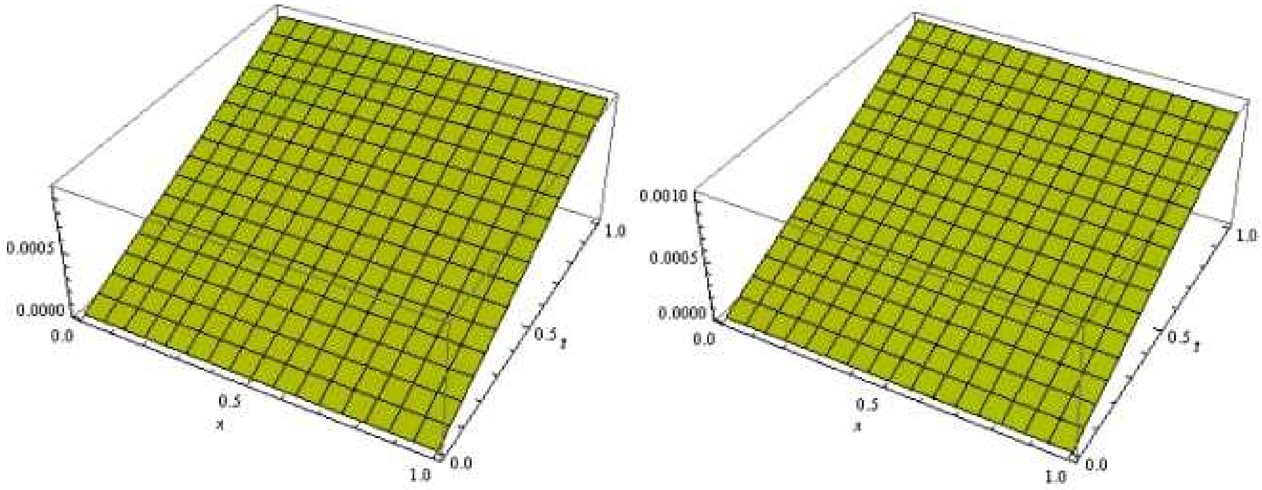
where the absolute error is obtained by calculating the absolute difference between the exact and approximate solutions. From Figure 5, it is evident that the approximate solution obtained using FHATM in a fuzzy setting converges rapidly to the exact solution.

Finally, Table 1 provides the numerical values of the upper and lower bound exact and approximate solutions for $\gamma = 1, \omega = 0.5, t = 0.001, k = 1, u = 0.5$, and $c = 1$, with varying values of x. It is observed that the obtained upper and lower bound approximate solutions are very close to the corresponding exact solutions.

Table 1

x	Uexact	Uapprox	Lexact	Lapprox
0	0.496994	0.499625	-0.496994	-0.50075
1	0.913008	0.914305	-0.913008	-0.912803
2	0.987767	0.98798	-0.987767	-0.987701
3	0.998336	0.998365	-0.998336	-0.998326
4	0.999775	0.999779	-0.999775	-0.999773
5	0.999969	0.999978	-0.999969	-0.999969

Numerical values of upper bound exact solution (Uexact), upper bound approximate solution (Uapprox), lower bound exact solution (Lexact), lower bound approximate solution(Lapprox) of the equation (4.1) when $\gamma = 1, \omega = 0.5, t = 0.001, k = 1, u = 0.5, c = 1$.



(A) (B)

Fig. 5: (A) Plot of absolute error curve of the upper-bound approximate solution. (B) Plot of absolute error curve of the lower-bound approximate solution.

6 Conclusion

In this paper, an effective iterative technique, known as the fractional homotopy analysis transform method (FHATM) in a fuzzy environment, is applied to determine the upper and lower bound approximate solutions of the time-fractional Sharma-Tasso-Olver equation. FHATM effectively solves both integer- and fractional-order differential equations within a fuzzy framework.

A fuzzy fractional differential equation represents a meaningful integration of a fractional-order differential equation with fuzziness. This topic remains one of the most significant and widely explored areas in contemporary research. To assess the accuracy and efficiency of the proposed FHATM in a fuzzy environment, various graphical representations have been constructed.

The absolute error values demonstrate a negligible difference between the exact and approximate solutions. Hence, based on our findings, we conclude that FHATM in a fuzzy setting is a highly effective and robust technique for obtaining both approximate and analytical solutions for fuzzy fractional physical models encountered in real-world applications.

Declarations

Competing interests: The authors declare that they have no competing interests.
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