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Impact of Inclined Magnetic Field on the Dynamics of Two Immiscible Viscous Fluids Flow in an Inclined Channel with Variable Permeability Porous Medium: A Finite Difference Approach

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Abstract: This paper utilizes the finite difference method to numerically analyze the fluid flow in an inclined channel filled with a porous medium and containing two immiscible, viscous, incompressible, and electrically conducting fluids. These fluids have different viscosities and are arranged in two equal-width separate layers within the channel. The channel experiences a magnetic field that is inclined, and the permeability of the porous material changes across the width of the channel. The upper layer's fluid has lower viscosity than the lower layer's fluid. The Brinkman equation is employed to characterize the flow within the porous material. The paper presents numerical expressions for velocity and volumetric flow rate using the finite difference method, incorporating no-slip boundary conditions at the top and bottom plates and continuity of velocity with shear stress continuity at the interface. The effects of key parameters, such as the Hartmann number, Gravitational parameter and the permeability parameter, etc on the velocity distribution and volumetric flow rate are investigated. The results are graphically represented and thoroughly discussed.

Keywords: Brinkman equation; Variable Permeability; Hartmann number; viscosity ratio parameter; Finite difference method.

2020 Mathematics Subject Classification. 76S05; 76W05; 76M20.

1 Introduction

The study of the flow of multiple immiscible fluids through pipes or channels filled with porous material is highly significant due to its broad range of applications across various fields. The study of the movement of non-miscible fluids in the porous material has garnered significant attention in the research community and finds numerous applications across many important fields such as biological activities, groundwater runoff, filtration processes, oil and gas refining, oil extraction from soil, blood circulation through veins and arteries, and more. Several alternative models for fluid flow in porous media are discussed in the well-known book Convection in Porous Media by Nield and Bejan [1]. In 1856, Henry Darcy introduced an equation to describe fluid behavior in a porous medium, commonly known as Darcy's equation. However, in 1949, Brinkman [2] made an adjustment to Darcy's equation by incorporating an additional term known as the Brinkman term. This modified equation is referred to as the Brinkman equation. So, investigating the flow of viscous fluids that do not mix within a channel which contains a porous material, the Brinkman equations are applicable within the fluid motion region.

Berman [3] conducted the pioneering investigation of laminar flow within a rectangular cross-sectional channel which contains a porous material using the perturbation method in 1953. His work marked the initial exploration of this particular research domain. Further in 1964, Kapur and Shukla [4] extended the study by examining a system involving n incompressible and immiscible fluids arranged in n layers. They demonstrated that irrespective of the number of fluids

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and the width of the layers, a single maximum velocity exists. Blottner [5] applied finite difference approach to solve the boundary-layer equation, in 1970. In 1998, Ford and Hamdan [6] utilized a finite difference scheme to achieve a numerical approximation for the interconnected parallel flow within composite porous layers. They employed two distinct fluid models, namely the Darcy-Lapwood-Brinkman (DLB) model and the Darcy-Lapwood-Forchheimer-Brinkman (DFB) model, within the porous region. Chamkha [7] conducted research on the continuous and smooth movement of two immiscible, incompressible, viscous, and electrically conductive fluids within channels containing a porous substance. The study focused on understanding how different factors influence the distribution of velocity within the flow. Bali and Awasthi [8] investigated the impact of an external magnetic field on blood flow in a stenotic artery. They specifically considered the magnetic field's influence in the transverse direction to the blood flow, and the blood viscosity was considered dependent on radial coordinates. To solve the nonlinear pressure equation, they employed the series method along with appropriate boundary conditions.

Siyyam and Hamdan [9] formulated analytical expressions for velocity and shear stress in the context of Poiseuille flow through a conduit enclosed by bonded porous layers. Further in 2010, Verma and Datta [10] examined the magnetohydrodynamic flow in a channel subjected to a transverse applied magnetic field with varying viscosity. They considered viscosity as a function of the distance from the mid-section of the channel. The study included the evaluation of velocity under two conditions: the first involved constant viscosity with a non-zero magnetic field, and the second considered zero magnetic field with non-constant viscosity. Varshney et al. [11] established a mathematical model to describe blood flow in a stenosed artery incorporating the impact of a perpendicular magnetic field. They applied the finite difference technique to analyze the characteristics of laminar, incompressible, fully developed, and non-Newtonian blood flow within an artery containing multiple stenosis. Beg et al. [12] conducted a numerical analysis to assess solutions for the flow of viscous, incompressible, and magnetohydrodynamic fluids in a rotating channel containing a Darcian porous medium, all subjected to a consistent pressure gradient. Sarojini et al. [13] conducted research on the magnetohydrodynamic (MHD) flow of a couple stress fluid through a parallel channel featuring a porous medium under an inclined magnetic field. They derived an exact solution for the velocity within the porous material and subsequently explored the impact of different parameters on the velocity.

In 2012, Murthy and Nagaraju [14] derived mathematical expressions to describe the magnetohydrodynamic (MHD) flow of a micropolar fluid between two concentric rotating cylinders with a porous lining. They computed values for slip velocity, azimuthal velocity, and micro-rotation component, and illustrated the impacts of different parameters through graphical representations. Manyonge et al. [15] investigated the behavior of a two-dimensional, steady flow of an incompressible, electrically conducting, viscous fluid between two infinitely parallel porous plates. The investigation examined how both a perpendicular magnetic field and a consistent pressure gradient affect the fluid's movement. In 2013, Deo and Srivastava [16] explored the impact of a transverse magnetic field on the flow of a viscous fluid with variable permeability within a channel in a porous medium. They employed numerical techniques to calculate the velocity and volumetric flow rate expressions for both Poiseuille and Couette-Poiseuille flows. Srivastava et al. [17] utilized the implicit finite-difference method to assess numerical solutions for a one-dimensional coupled nonlinear Burgers' equation. They conducted a graphical comparison between the obtained numerical results and analytical solutions. Srivastva et al. [18] analysed the numerical analysis of an one-dimensional coupled nonlinear Burgers' equation by an implicit logarithmic finite-difference method. In this method they provides a system of nonlinear difference equation which linearised by Newton's method and they solved the linear system by Gauss elimination method .The obtained numerical solutions are compared with analytic solution. In 2016, Zaidi et al. [19] Explored the fluid dynamics within an inclined channel holding two fluids that do not mix, affected by a magnetic field. They evaluated the solution to this problem using a perturbation method. Chutia [20] examined the Poiseuille flow of an electrically conductive fluid between two parallel plates that are inclined. In the year 2017, Ansari and Deo [21] investigated how a magnetic field affects the flow of two non-miscible fluids within a channel containing a porous material. They derived expressions for velocity and flow rate while also examining the effects of the magnetic field and other parameters on both flow rate and velocity profile.

Jaiswal and Yadav [22] considered the non-Newtonian fluid between horizontal porous channel. The porous channel is divided into 3 layers with different permeabilities. They evaluated the expression for the velocity, volumetric flow rate and shearing stresses in 2020. Jaiswal and Yadav [23] conducted a study on how a magnetic field influences the Poiseuille flow of immiscible Newtonian fluids within a highly porous medium. Their investigation involved considering distinct viscosities and different permeabilities in each regions. The researchers explored the impact of parameters such as permeability, porosity of the porous region, and magnetic number on both velocity profiles and flow rates. Raza et al. [24] employed numerical techniques, specifically the finite-difference method and the Haar wavelet approach, to solve the second-order diffusion equation and the third-order dispersive equation. In 2021, Yadav and Verma [25] conducted an analysis of the flow characteristics of two immiscible fluids with Newtonian and micropolar properties in inclined channels. They derived expressions for linear velocity, microrotation velocity, flow rate, and stresses. The study also investigated the impact of various parameters, such as viscosity ratio and gravitational parameters, on both velocity, flow rate, and stresses. One year later, Yadav and Verma [26] repeated their analysis, this time focusing on the flow characteristics of two immiscible electrically conducting micropolar fluid in an inclined porous channel. Currently, Maurya and Deo [27] explained the effect of magnetic field on the flow of Newtonian and Jeffery fluids through composite porous channel. They have considered Jeffery fluid between two porous channel of the Newtonian fluids and obtained the expressions for volumetric flow rate, velocity profile and shear stress. Further the effects of viscosity ratio parameter, Hartmann number, Jeffery parameter and permeability parameter on the flow rate and fluid velocity are presented graphically and discussed by them.The investigation of two-phase flows within an inclined channel is explored by the ref. [28,29,30,31]. McCartney and Job [32] carried out a finite element study of plane Poiseuille nanofluid flow and heat transfer using the time-dependent Buongiorno model equations. Patel et al. [33] studied the flow of three distinct fluid layers, and analyzed the velocity profiles and volumetric flow rates for each layer. Jaber [34] investigated the velocity profile in MHD nanofluid flow over a permeable stretching sheet, incorporating heat radiation and viscous dissipation, along with variable fluid properties and the influence of partial slip.

Numerous researchers have previously focused on multiphase flow problems specifically in horizontal channel configurations, solving them through analytical approaches. However, real-world scenarios often involve channels that are not exclusively horizontal, and analytical methods may not be universally applicable to all problems. This observation serves as the inspiration for our current investigation, where we choose to solve the present problem using numerical methods.The novelty of this particular issue lies in employing the finite difference approach to address a two-phase fluid flow scenario within an inclined channel, wherein the permeability of the porous material experiences nonlinear fluctuations. Such issues have wide-ranging applications in various fields such as soil mechanics and groundwater hydrology etc.

This paper aims to investigate how fluid flows in a sloped channel that is filled with a porous substance with variable permeability. The porous substance consists of two layers, each with the same width. The fluid involved is immiscible, incompressible, electrically conducting, and possesses different viscosities in each layer. An inclined magnetic field influences the system. The permeability of the porous material varies in the transverse direction. We specifically consider the scenario where the viscosity of the fluid in the upper layer is lower than that in the lower layer. Utilizing the finite difference method, we calculate the velocity of the fluid in both layers and determine the volumetric flow rate. The study explores the impact of various parameters on the velocity profile and flow rate through graphical representations generated using Matlab software, and the findings are discussed.

2 Mathematical Formulation

Let's analyze the stable and well-established motion of two immiscible, incompressible, electrically conductive viscous fluids with uniform velocity U^* and varying viscosity within an inclined channel which contains a porous material in two layers of equal width. The vertical coordinate is defined as $0 \leq$ y $\leq 2H^*$ (refer to Fig 1.). An inclined magnetic field, denoted as B*, is applied at an angle θ with respect to the x^*-axis. The Brinkman equation is employed to describe fluid flow within both the porous layers , and the flow is directed along the x^*-axis. The upper layer of the fluid has viscosity μ_1^*, while the lower layer has viscosity μ_2^*. Importantly, the fluid in the upper layer exhibits lower viscosity compared to the fluid in the lower layer. Regions I and II represent the upper and lower layers, respectively. The permeability of the porous material of the channel varies in the transverse direction. Both the top and bottom plates remain fixed in place, and the flow is propelled by a consistent pressure gradient. An inclined magnetic field B^* of uniform intensity is applied. The governing Brinkman equation for this system is expressed as

$$\mu_e^* \nabla^2 \mathbf{u}^* - \frac{\mu^*}{k^*(y)} \mathbf{u}^* + \mathbf{J}^* \times \mathbf{B}^* + \rho \mathbf{g} = \nabla \mathbf{p}^*, \quad (1)$$

where $k^*(y)$ is the permeability of the porous material which is variable with the transverse direction and μ^* is the viscosity of the fluid and μ_e^* is the effective viscosity of the fluid in porous medium, here we assume that $\mu^* = \mu_e^*$ and $\mathbf{J}^*$ is the electric current density and $\mathbf{B}^*$ is the magnetic induction vector of the applied uniform magnetic filed which makes angle θ with the x^* axis and $\rho \mathbf{g}$ is the external gravitational force on the fluid, and ρ is density of the fluid. Further,we assume that there is no external electric field and internal field due to separation of charges or polarization do not give rise to induced electric field, therefore $\mathbf{J}^* = \sigma^*(\mathbf{u}^* \times \mathbf{B}^*)$ where σ^* is the electrical conductivity of the fluid. Therefore it is conclude that the Lorentz force $\mathbf{F}^* = \mathbf{J}^* \times \mathbf{B}^*$ and the velocity $\mathbf{u}^*$ are collinear and in opposite in directions, hence $\mathbf{F}^* = -\sigma^* {B_0^*}^2 \mathbf{u}^*$ where $B_0^* = \mid \mathbf{B}^* \mid$ since inclination of magnetic field with the x^* axis is θ so $B_0^* sin\theta$ is magnitude of magnetic field in the

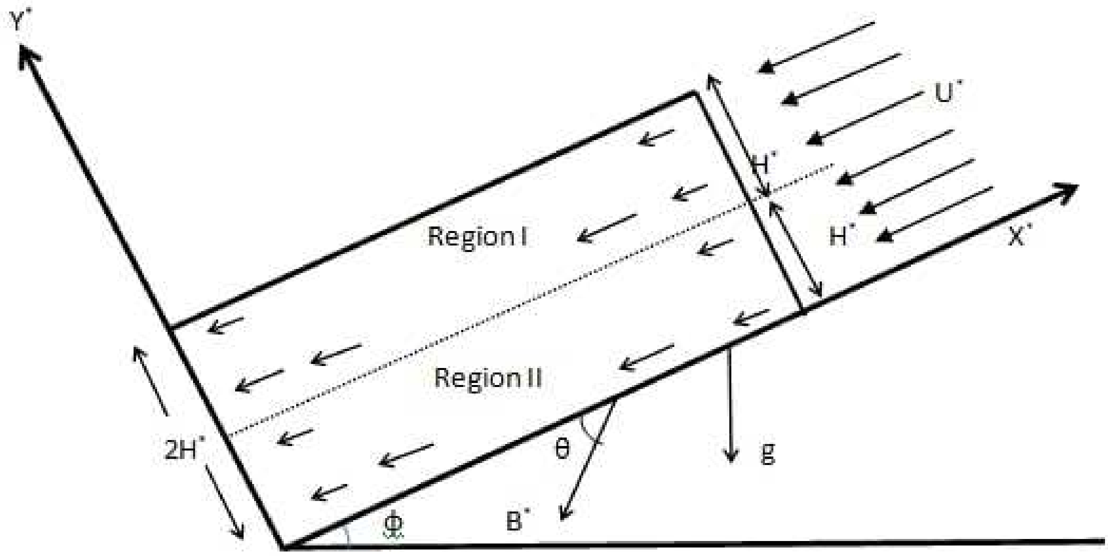

Fig. 1: Physical sketch of the problem

transverse direction. Since channel is inclined and inclination ϕ with the horizontal so $\rho g sin\phi$ is a force on the fluid in the direction of velocity. Therefore the governing Brinkman equation along x^* axis

$$\frac{d^2u^*}{dy^{*2}} - \frac{u^*}{k^*(y)} - \frac{\sigma^* B_0^{*2}\sin^2\theta u^*}{\mu^*} + \frac{\rho g sin\phi}{\mu^*} = \frac{1}{\mu^*}\frac{\partial p^*}{\partial x^*}. \tag{2}$$

Now equation (2) becomes as, for Upper layer or region I

$$\frac{d^2u_1^*}{dy^{*2}} - \frac{u_1^*}{k^*(y)} - \frac{\sigma_1^* B_0^{*2}\sin^2\theta u_1^*}{\mu_1^*} + \frac{\rho_1 g sin\phi}{\mu_1^*} = \frac{1}{\mu_1^*}\frac{\partial p^*}{\partial x^*}, \tag{3}$$

where, u_1^*, σ_1^* are the velocity and electrical conductivity of the fluid in the region I respectively.
And for the lower layer or region II

$$\frac{d^2u_2^*}{dy^{*2}} - \frac{u_2^*}{k^*(y)} - \frac{\sigma_2^* B_0^{*2}\sin^2\theta u_2^*}{\mu_2^*} + \frac{\rho_2 g sin\phi}{\mu_2^*} = \frac{1}{\mu_2^*}\frac{\partial p^*}{\partial x^*}, \tag{4}$$

where, u_2^*, σ_2^* are the velocity and electrical conductivity of the fluid in the region II respectively.
Permeability of the porous material is taken as $k^*(y) = k_0^*(1-\varepsilon y)^6$, where k_0^* is characteristic permeability of the medium of constant permeability and ε lies between 0 to 1. Introducing following dimensionless variables

$$u_1 = \frac{u_1^*}{U^*}, u_2 = \frac{u_2^*}{U^*}, x = \frac{x^*}{H^*}, y = \frac{y^*}{H^*}, p = \frac{p^*}{p_0^*}, p_0^* = \frac{\mu_1^* U^*}{H^*}, \gamma = \frac{\sigma_1^*}{\sigma_2^*}, \alpha = \frac{\mu_1^*}{\mu_2^*}, \rho = \frac{\rho_1^*}{\rho_2^*}. \tag{5}$$

The equation (3) and (4) becomes

for upper layer or region I

$$(1-\varepsilon y)^6\frac{d^2u_1}{dy^2} - [n^2 + (1-\varepsilon y)^6 M_1^2\sin^2\theta]u_1 = (1-\varepsilon y)^6(-P - G sin\phi). \tag{6}$$

and for lower or region II

$$(1-\varepsilon y)^6\frac{d^2u_2}{dy^2}-[n^2+(1-\varepsilon y)^6M_2^2\sin^2\theta]u_2=(1-\varepsilon y)^6(-\alpha P-\frac{\alpha G sin\phi}{\rho}). \tag{7}$$

where $n=\frac{H^*}{\sqrt{k_0^*}}$ is the permeability parameter, $M_1=\sqrt{\frac{\sigma_1^*B_0^{*2}H^{*2}}{\mu_1^*}}$ and $M_2=M_1\sqrt{\frac{\alpha}{\gamma}}$ are the Hartmann numbers; $G=\frac{\rho_1 gH^{*2}}{\mu_1^*U^*}$ is the gravitational parameter and pressure gradient $\frac{\partial p^*}{\partial x^*}=-P$. Further, $\gamma=\frac{\sigma_1^*}{\sigma_2^*}$ denotes the ratio of electrical conductivity, and $\alpha=\frac{\mu_1^*}{\mu_2^*}$ denotes viscosity ratio parameter.

2.1 Boundary conditions

The boundary condition for the fluid flow are as follows: The upper and lower plates are stationary and continuity of flow velocity and shear stress continuity at the interface, i.e.,

1. No slip condition at upper plate:
   $u_1=0$, at $y=2\quad(y^*=2H^*)$.
2. No slip condition at lower plate:
   $u_2=0$, at $y=0\quad(y^*=0)$.
3. Continuity of velocity at the interface:
   $u_1=u_2$, at $y=1$.
4. Shear stress continuity at the interface:
   $\alpha\frac{du_1}{dy}=\frac{du_2}{dy}$.

3 Numerical Solution of the Problem

Solution scheme of the problem: The finite difference method is a numerical strategy utilized to estimate solutions to differential equations by discretizing them into a finite set of points. Instead of handling continuous functions directly, this method subdivides the domain into a grid or mesh, and derivatives are approximated by computing the differences between values at adjacent points. This discretization transforms the original differential equations into algebraic equations, enabling the resolution of complex mathematical problems that may lack analytical solutions. Essentially, the finite difference method entails replacing derivatives in differential equations with finite difference approximations, facilitating the computation of numerical solutions. This approach finds widespread application across diverse fields such as physics, engineering, finance, and other disciplines where dynamic systems are described by differential equations.
To solve the differential equations (6) and (7) using finite difference method with boundary conditions. The symmetric derivative formulae at the j^{th} node

$$\frac{du}{dy}=\frac{u_{j+1}-u_j}{h}. \tag{8}$$

$$\frac{d^2u}{dy^2}=\frac{u_{j+1}-2u_j+u_{j-1}}{h^2}. \tag{9}$$

Utilizing this method of difference approximation in conjunction with the provided boundary conditions. For upper layer or region I

$$\frac{u_{1j+1}-2u_{1j}+u_{1j-1}}{h^2}-[\frac{n^2}{(1-\varepsilon y_j)^6}+M_1^2\sin^2\theta]u_{1j}=(-P-Gsin\phi). \tag{10}$$

Where, $y_j=0+(j-1)h$.
Rearranging the term of equation (10) it becomes,

$$\frac{u_{1j-1}}{h^2}-[\frac{2}{h^2}+\frac{n^2}{(1-\varepsilon y_j)^6}+M_1^2\sin^2\theta]u_{1j}+\frac{u_{1j+1}}{h^2}=(-P-Gsin\phi). \tag{11}$$

And for the lower layer or region II

$$\frac{u_{2j+1}-2u_{2j}+u_{2j-1}}{h^2}-[\frac{n^2}{(1-\varepsilon y_j)^6}+M_2^2\sin^2\theta]u_{2j}=(-\alpha P-\frac{\alpha G sin\phi}{\rho}). \quad (12)$$

Rearranging the term of equation (12) it becomes,

$$\frac{u_{2j-1}}{h^2}-[\frac{2}{h^2}+\frac{n^2}{(1-\varepsilon y_j)^6}+M_2^2\sin^2\theta]u_{2j}+\frac{u_{2j+1}}{h^2}=(-\alpha P-\frac{\alpha G sin\phi}{\rho}). \quad (13)$$

With discretize boundary conditions :

$$\left.\begin{array}{l} u_{1j}=0,\ \ at\ \ j=n+1 \\ u_{1j}=u_{2j},\ \ at\ \ j=\frac{n}{2} \\ u_{2j}=0,\ \ at\ \ j=1 \\ \alpha(u_{1j+1}-u_{1j})=u_{2j+1}-u_{2j},\ \ at\ \ j=\frac{n}{2} \end{array}\right\}.$$

Now, by substituting $j=2,3,\ldots,n/2$ in the equation (13),and $j=(n/2)+1,(n/2)+2,\ldots,n.$ in the equation (11), we can establish a system of equations. This system can be solved using MATLAB programming to determine the fluid velocity at various locations within the domain.
The volumetric flow rate Q for the channel is defined as:

$$Q=\int_0^2 udy=\int_0^1 u_{2j}dy+\int_1^2 u_{1j}dy. \quad (14)$$

By substituting the fluid velocity values into various intervals, the volumetric flow rate can be calculated.

4 Result and Discussion

In this paper, we investigate the flow of two immiscible viscous fluids through a channel containing a porous medium with variable permeability under the influence of an inclined magnetic field. The Brinkman equation is utilized to model the flow within the porous material. Numerical solutions for the fluid velocities in both layers, as well as the volumetric flow rates, are obtained using the finite difference method. No-slip conditions are applied at the channel walls, while continuity of velocity and shear stress are enforced at the interface as boundary conditions. For the numerical computation, y is taken as in the domain $0 \leq y \leq 1$ for the lower region and $1 \leq y \leq 2$ for upper region with a grid spacing of h=0.025, i.e., the whole computational domain is equally divided into 40 grids and, we calculated u_{2j} for j=2,3,...,20. and u_{1j} for j=21,22,...,40 and all numerical results are shown in the forms of graphs.

4.1 Analysis of fluid velocity with different parameters

In this segment, we will explore the influence of different parameters on the fluid velocity.
Impact of Hartmann number: Figure 2 illustrates how the velocity of the fluid in both regions is influenced by the Hartmann number. The plots distinctly show that as the applied magnetic field, represented by the Hartmann number M_1, decreases, the velocities u_1 and u_2 increase. This trend is explained by the counteractive effect of the magnetic field on the fluid's motion. At the walls' boundaries, where the fluid interacts with solid surfaces, both velocities u_1 and u_2 are zero, adhering to the no-slip boundary condition.
Impact of the viscosity ratio parameter: Figure 3 displays the impact of varying the viscosity ratio parameter α, with values set at 0.4, 0.6, and 0.8 while keeping other parameters constant. The observed graphs indicate that as the viscosity ratio parameter α increases, both velocities u_1 and u_2 also increase. At the boundaries of the walls, both velocities u_1 and u_2 become zero, adhering to the no-slip condition at the interfaces between solid and fluid.

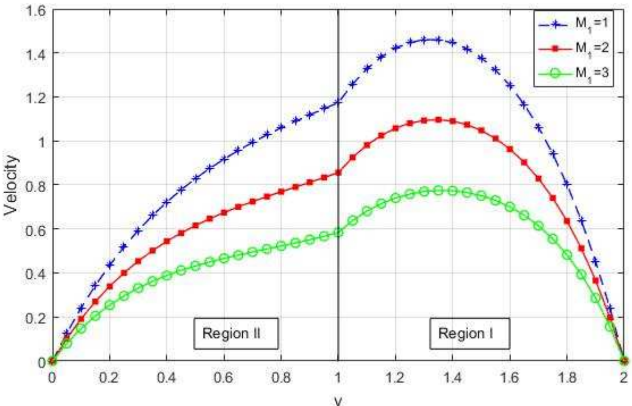

Fig. 2: Influence of the Hartmann number M_1 on the velocity profile where the parameters are fixed at $P = 10, n = 2, G = 3, \rho = 0.6, \alpha = 0.4, \gamma = 0.5, \varepsilon = 0, \phi = \pi/3$, and $\theta = \pi/2$.

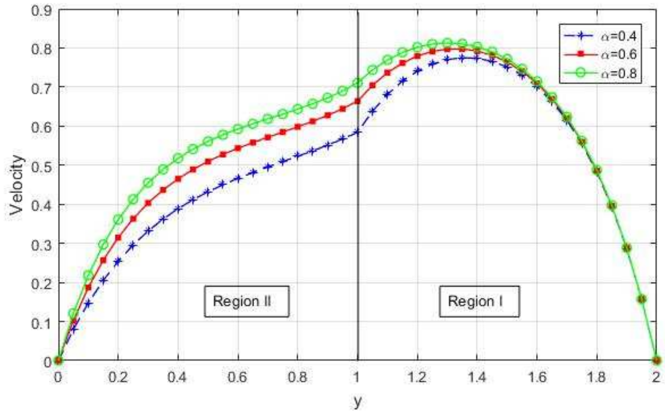

Fig. 3: Influence of the viscosity ratio parameter α on the velocity profile where the parameters are fixed at $P = 10, n = 2$, $M_1 = 3$, $G = 3, \rho = 0.6, \gamma = 0.5, \varepsilon = 0, \phi = \pi/3$, and $\theta = \pi/2$.

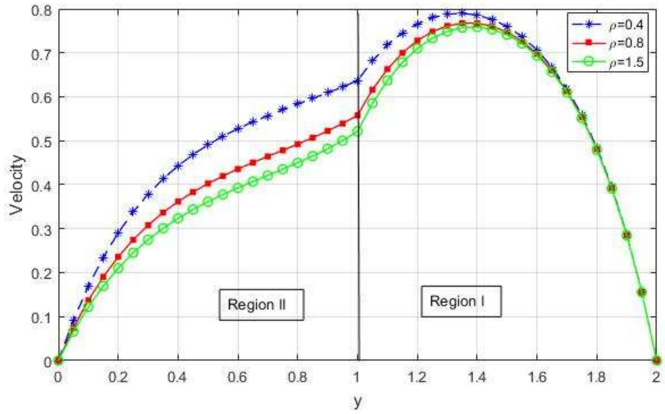

Fig. 4: Influence of the density ratio parameter ρ on the velocity profile where the parameters are fixed at $P = 10, n = 2$, $M_1 = 3$, $G = 3, \alpha = 0.4, \gamma = 0.5, \varepsilon = 0, \phi = \pi/3$, and $\theta = \pi/2$.

Impact of the density ratio parameter: Figure 4 displays the impact of varying the density ratio parameter ρ, with values set at 0.4, 0.8, and 1.5 while keeping other parameters constant. The depicted graphs suggest that as the density ratio parameter rises, both velocities u_1 and u_2 show a decline.

Impact of the permeability: Figure 5 illustrates the influence of permeability by changing the values of ε to 0.1, 0.2, and 0.3, while maintaining other parameters constant. The graphs demonstrate that as ε increases, the fluid velocity in the region decreases. This is attributed to the fact that an increase in ε leads to a decrease in the permeability of the porous material.

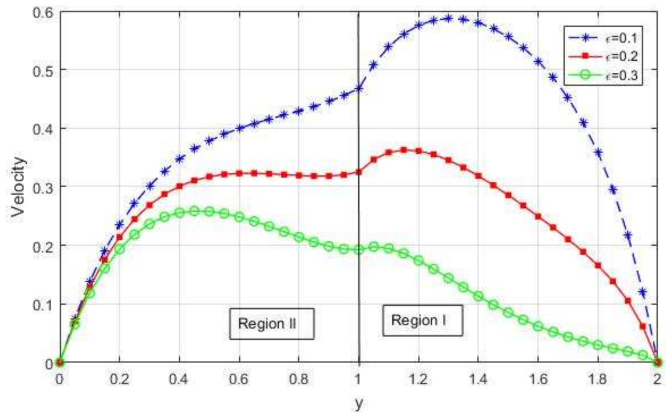

Fig. 5: Influence of ε on the velocity profile where the parameters are fixed at $P = 10, n = 2$, $M_1 = 3$, $G = 3, \rho = 0.6, \alpha = 0.4, \gamma = 0.5, \phi = \pi/3$, and $\theta = \pi/2$.

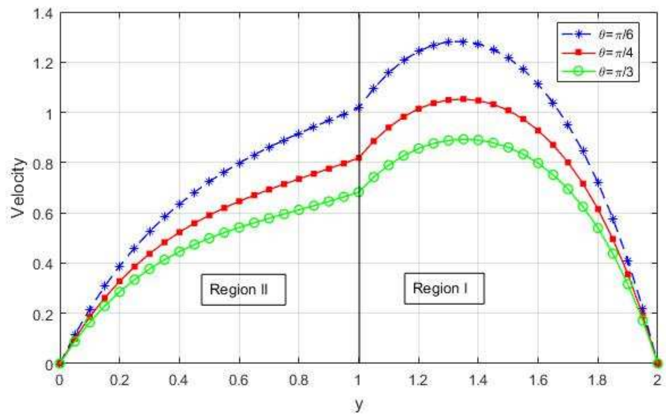

Fig. 6: Influence of the inclination of magnetic field θ on the velocity profile where the parameters are fixed at $P = 10, n = 2$, $M_1 = 3$, $G = 3, \rho = 0.6, \alpha = 0.4, \gamma = 0.5, \varepsilon = 0$, and $\phi = \pi/3$.

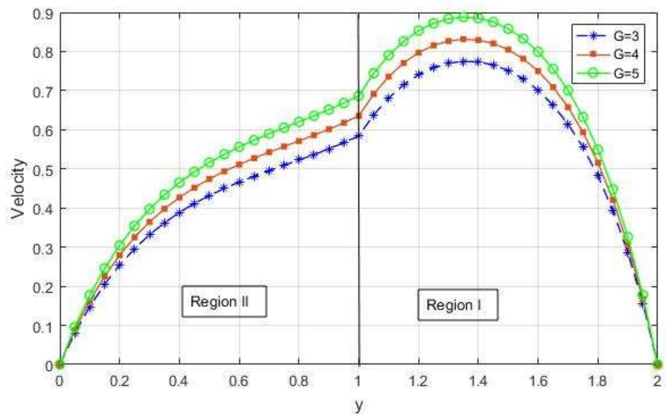

Fig. 7: Influence of the gravitational parameter G on the velocity profile where the parameters are fixed at $P = 10, n = 2$, $M_1 = 3$, $\rho = 0.6, \alpha = 0.4, \gamma = 0.5, \varepsilon = 0, \phi = \pi/3$, and $\theta = \pi/2$.

Impact of the inclination of the magnetic field: Figure 6 demonstrates the influence of the inclination of the magnetic field, represented by θ with respect to the x-axis, by considering inclinations of $\pi/6, \pi/4$, and $\pi/3$, while keeping other parameters constant. The graphs obtained indicate that as θ increases from θ to $\pi/2$, the fluid velocities in both regions decrease. This reduction is attributed to the diminishing impact of the magnetic field inclination.

Impact of the gravitational parameter: Figure 7 depicts the impact of gravitational parameter G by varying the values to 3, 4, and 5, while keeping other parameters constant. The graphs indicate that with an increase in G, there is a corresponding increase in fluid velocity within the region. This phenomenon is explained by the fact that an elevation in

G results in a greater gravitational force acting on the fluids.
Impact of the inclination of the channel: Figure 8 depicts the impact of channel inclination by varying the values of ϕ to $\pi/6, \pi/4$, and $\pi/3$, while keeping other parameters constant. The graphs show that as ϕ increases from 0 to $\pi/2$, there is a corresponding increase in fluid velocity within the region. This observed trend is linked to the fact that an increase in ϕ results in a greater gravitational force acting on the fluids.

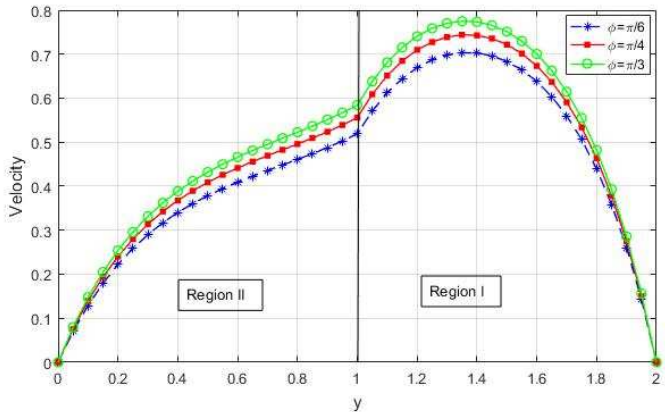

Fig. 8: Influence of the inclination of the channel ϕ on the velocity profile where the parameters are fixed at $P = 10, n = 2$, $M_1 = 3$, $G = 3, \rho = 0.6, \alpha = 0.4, \gamma = 0.5, \varepsilon = 0$, and $\theta = \pi/2$.

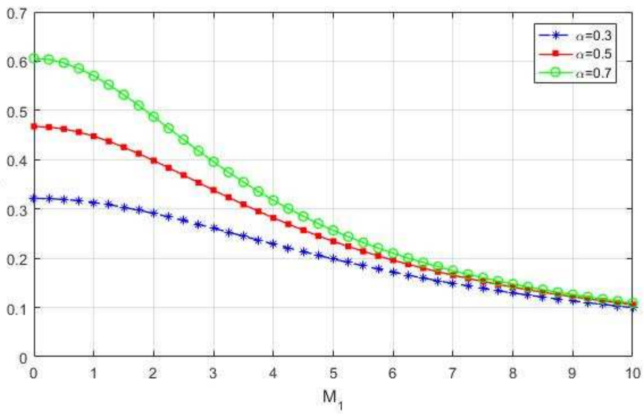

Fig. 9: Influence of the viscosity ratio α with Hartmann number M_1 on volumetric flow rate when the parameter are $P = 10, n = 2$, $G = 3$, $\rho = 0.6, \gamma = 0.5, \varepsilon = 0.3, \phi = \pi/3$, and $\theta = \pi/2$.

4.2 Analysis of volumetric flow rate with different parameters

In this segment, we will explore the influence of different parameters on the volumetric flow rate. Now for volumetric flow rate Q describe the figures from 9 to 14.
Figure 9 illustrates the influence of the viscosity ratio parameter α on the flow rate Q with Hartmann number M_1. The observation indicates that an increase in the viscosity ratio α amplifies the volumetric flow rate, whereas the Hartmann number M_1 has a suppressive effect on the volumetric flow rate.
Figure 10 depicts the impact of permeability, varies with ε, on the flow rate Q with Hartmann number M_1. The findings reveal that as the ε increases, there is a corresponding decrease in the volumetric flow rate.
Figure 11 depicts the impact of inclination of the channel ϕ, on the flow rate Q with Hartmann number M_1. The observation indicates that an increase in the inclination of the channel ϕ amplifies the volumetric flow rate, whereas the

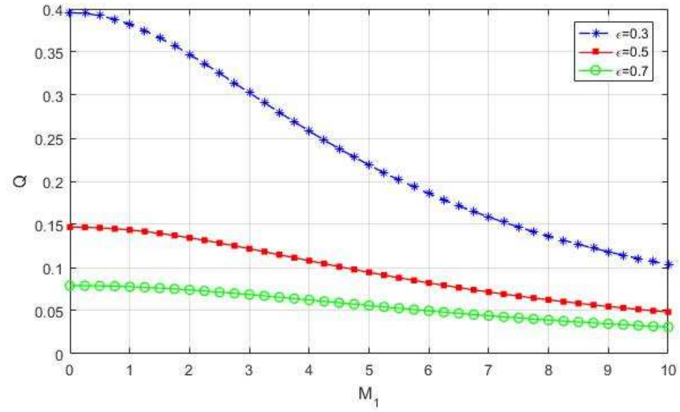

Fig. 10: Influence of the ε with Hartmann number M_1 on volumetric flow rate when the parameter are $P = 10, n = 2$, $G = 3$, $\rho = 0.6, \gamma = 0.5, \alpha = 0.4, \phi = \pi/3$, and $\theta = \pi/2$.

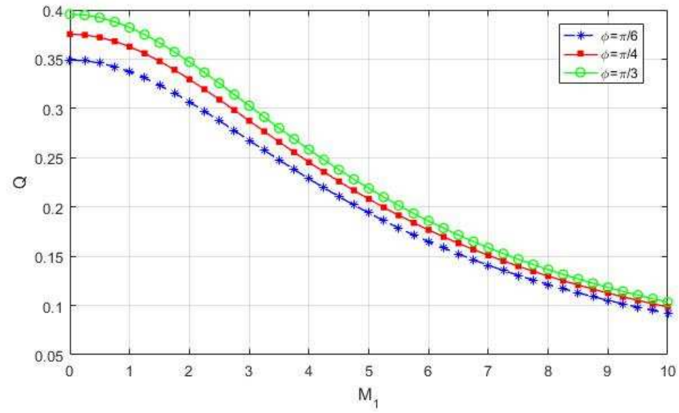

Fig. 11: Influence of the inclination of the channel ϕ with Hartmann number M_1 on volumetric flow rate when the parameter are $P = 10, n = 2$, $G = 3$, $\rho = 0.6, \gamma = 0.5, \varepsilon = 0.3, \alpha = 0.4$, and $\theta = \pi/2$.

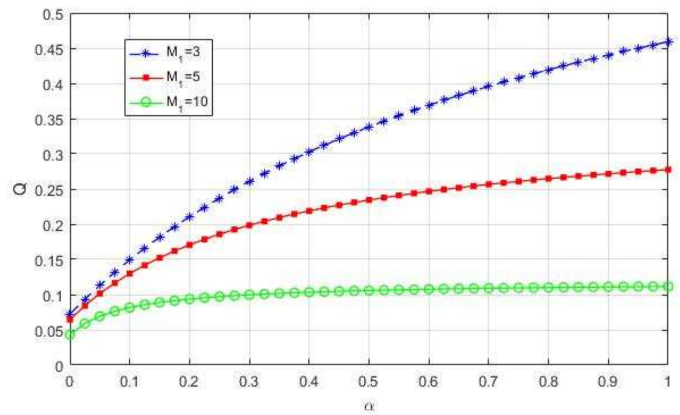

Fig. 12: Influence of the Hartmann number M_1 with viscosity ratio parameter α on volumetric flow rate when the parameter are $P = 10, n = 2$, $G = 3$, $\rho = 0.6, \gamma = 0.5, \varepsilon = 0.3, \phi = \pi/3$, and $\theta = \pi/2$

Hartmann number M_1 has a suppressive effect on the volumetric flow rate.

Figure 12 depicts the impact of Hartmann number M_1, on the flow rate Q with viscosity ratio parameter α. The findings reveal that as the Hartmann number M_1 increases, there is a corresponding decrease in the volumetric flow rate. Additionally, the viscosity ratio α amplifies the volumetric flow rate.

Figure 13 depicts the impact of Hartmann number M_1, on the flow rate Q with density ratio ρ.The findings reveal that as the Hartmann number M_1 increases, there is a corresponding decrease in the volumetric flow rate. Additionally, the density ratio ρ has a suppressive effect on the volumetric flow rate.

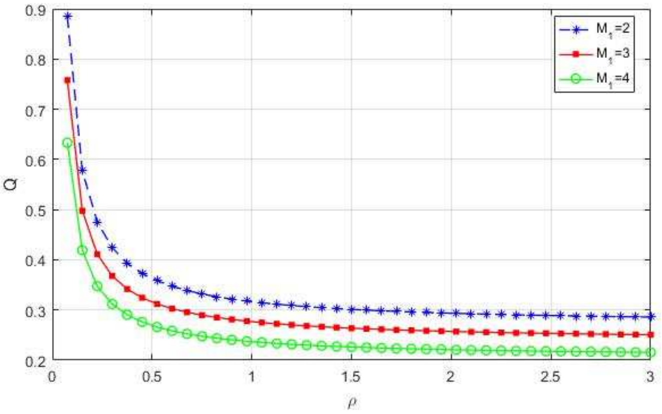

Fig. 13: Influence of the Hartmann number M_1 with density ratio ρ on volumetric flow rate when the parameter are $P = 10, n = 2$, $G = 3$, $\alpha = 0.4, \gamma = 0.5, \varepsilon = 0.3, \phi = \pi/3$, and $\theta = \pi/2$.

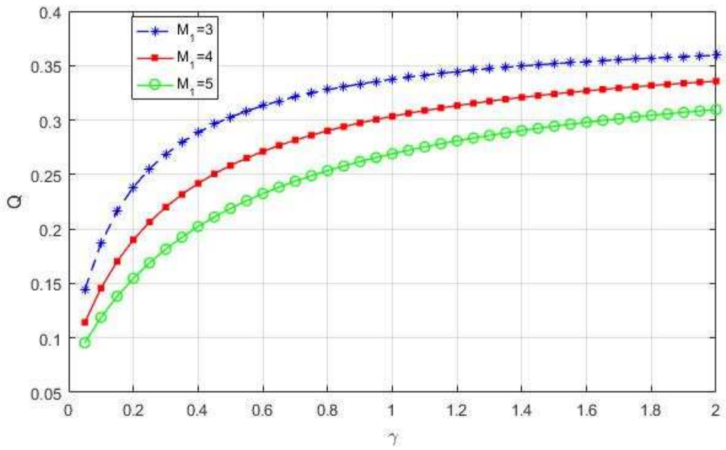

Fig. 14: Influence of the Hartmann number M_1 with the ratio of electric conductivity γ on volumetric flow rate when parameters $P = 10, n = 2$, $G = 3$, $\alpha = 0.4$, $\rho = 0.6, \varepsilon = 0.3, \phi = \pi/3$, and $\theta = \pi/2$.

Figure 14 depicts the impact of Hartmann number M_1, on the flow rate Q with ratio of electrical conductivity γ. The findings reveal that as the Hartmann number M_1 increases, there is a corresponding decrease in the volumetric flow rate. Additionally, ratio of electrical conductivity γ amplifies the volumetric flow rate.

5 Conclusion

This study presents a numerical investigation of the flow of two immiscible viscous fluids through an inclined channel containing a porous medium with variable permeability, influenced by an inclined magnetic field. Numerical solutions for velocity profiles and volumetric flow rates are obtained. Graphical analyses illustrate the effects of various parameters on fluid velocities within each layer and on overall flow rates. The results enhance the understanding of multiphase flow behavior. This study has potential applications in reducing power consumption during the extraction of one fluid from a channel by introducing another fluid with different viscosity. Additional applications include oil transportation processes and groundwater hydrology,fields such as soil mechanics, filtration system, biomedical devices, oil recovery among others. Based on our analysis, we have drawn the following key conclusions:

–The fluid velocity in both regions is enhanced by the gravitational parameter G, viscosity ratio parameter α and the channel inclination ϕ.
–The fluid velocity in both regions is suppressed by the Hartmann number M_1, density ratio ρ, and the magnetic field inclination θ.
–The Hartmann number M_1 and density ratio ρ suppressed the volumetric flow rate.
–The viscosity ratio α, inclination of channel ϕ and electrical conductivity ratio γ enhanced the volumetric flow rate.

Declarations

Conflicts of Interest: The authors state that they have no conflicts of interest.
Authors' contributions: Angad Prasad: Conceptualization, Methodology, Software, Validation, Formal Analysis, Resources, Original Draft Writing Preparation, Review and Editing.
P.K. Singh: Visualization, and Supervision.
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