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Some New Inequalities on e-Convex Functions

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Abstract: In this paper, we investigate *e*-convex functions, presenting Jensen-type inequalities and integral bounds. We give some properties of *e*-convex functions, and exhibit their controlled growth through inequalities, offering new tools for mathematical analysis.

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1 Introduction

Convex functions and their generalizations have played a pivotal role in the formulation and refinement of inequalities across diverse areas of mathematics. We call a function $\phi : \mathscr{I} \to \mathbb{R}$ convex [2, 10] whenever it meets the inequality condition

$$\phi(\alpha u + (1-\alpha)v) \leq \alpha\phi(u) + (1-\alpha)\phi(v) \, \forall u, v \in \mathscr{I} \text{ and } \alpha \in [0,1],$$

where $\mathscr{I} \neq \emptyset$ represents an interval in $\mathbb{R}$. If this inequality reverses, then ϕ is said to be concave.

Classical convexity, known for its analytical simplicity and geometric intuition, underpins a wide array of foundational inequalities such as Jensen's inequality and the Hermite-Hadamard inequality. These results not only provide bounds for functions but also find applications in approximation theory, optimization, numerical integration, and beyond. In particular, Jensen's inequality has been extended to various frameworks including affine combinations and multi-variable settings [9], while the Hermite-Hadamard inequality remains a central result in convex analysis [3–7, 13].

In recent years, several generalizations of convexity have emerged, including h-convexity [15], harmonic convexity [14], geometric convexity [12], and strong convexity [16]. These frameworks have enabled researchers to establish sharper inequalities and derive more refined bounds. For example, radical convexity has been employed to obtain improved matrix inequalities in functional analysis [11], and geometrically convex functions have been used to extend operator inequalities involving unitarily invariant norms [12]. Furthermore, integral inequalities involving the product of geometrically convex functions have yielded enhanced estimates based on classical inequalities such as Hölder's and Young's inequalities [1].

Inspired by these developments, this paper explores *e*-convex functions, which are defined by exponential weights in their defining inequality.

2 New Inequalities for e-Convex Functions: Foundations

The notion of *e*-convexity, first introduced in [8], extends the classical concept of convexity. In this section, we further investigate *e*-convex functions by establishing several new results and properties. These developments aim to deepen the understanding of *e*-convex and *e*-concave functions and highlight their potential applications.

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Definition 1. *Let $\phi : \mathscr{I} \to \mathbb{R}$ be any non-negative function. Then ϕ is called an e-convex function if for any $u, v \in \mathscr{I}$ and $\alpha \in [0,1]$, we have*

$$\phi(\alpha u + (1-\alpha)v) \leq e^{\alpha}\phi(u) + e^{1-\alpha}\phi(v).$$

We now provide an example of a function that is *e*-convex but not convex.

Example 1. [8] Let $f : [0,1] \to \mathbb{R}$ be a real-valued function defined by $f(x) = \sqrt{x}$. Then f is *e*-convex, but not convex.

Remark.(Lemma 3.2, [8]) Every positive convex function is *e*-convex.

Next, we explore some properties of *e*-convex functions.

Proposition 1. *Let $f : \mathscr{I} \to \mathbb{R}$ be a function exhibiting e-convexity. Then, for any real number $m \geq 1$, the function $f^{1/m}$ defined as*

$$f^{1/m}(x) := (f(x))^{1/m}, \quad \text{for all } x \in \mathscr{I},$$

also retains the property of e-convexity.

Proof. For any $x, y \in I$ and $\alpha \in [0,1]$, we have

$$\begin{aligned}
(f^{1/m})(\alpha x + (1-\alpha)y) &\leq (f(\alpha x + (1-\alpha)y))^{1/m} \\
&\leq \left(e^{\alpha} f(x) + e^{1-\alpha} f(y)\right)^{1/m} \\
&\leq (e^{\alpha} f(x))^{1/m} + \left(e^{1-\alpha} f(y)\right)^{1/m} \\
&= e^{\alpha/m}(f(x))^{1/m} + e^{(1-\alpha)/m}(f(y))^{1/m} \\
&\leq e^{\alpha} f^{1/m}(x) + e^{1-\alpha} f^{1/m}(y).
\end{aligned}$$

Thus, $f^{1/m}$ is *e*-convex.

Proposition 2. *Let $f : [a,b] \to \mathbb{R}$ be an e-convex function. Then f is bounded on $[a,b]$.*

Proof. For $x \in [a,b]$, there exists $\theta \in [0,1]$ such that $x = \theta b + (1-\theta)a$. Using the *e*-convexity of f, we have

$$f(x) = f(\theta b + (1-\theta)a) \leq e^{\theta} f(b) + e^{1-\theta} f(a).$$

Since $e^{\theta} \leq e$ for $0 \leq \theta \leq 1$ and $e^{1-\theta} \leq e$ for $0 \leq 1-\theta \leq 1$, we have $f(x) \leq e f(b) + e f(a)$. Let $L = \max\{f(a), f(b)\}$. Then, $f(x) \leq eL + eL = 2eL$.
Setting $M = 2eL$, we obtain

$$f(x) \leq M, \quad \forall x \in [a,b].$$

Thus, f is bounded.

Definition 2. *[15] A mapping $\phi : \mathscr{I} \to \mathbb{R}$ is called submultiplicative if*

$$\phi(uv) \leq \phi(u)\phi(v), \quad \forall u, v \in \mathscr{I}.$$

This property captures a kind of controlled growth behavior of the function with respect to multiplication. In particular, it implies that the value of the function at the product of two inputs is no greater than the product of the values at those inputs individually.

Lemma 1. *The exponential function is a submultiplicative function on $[0,1]$.*

Proof. Let $u, v \in [0,1]$. Then, $u + v - uv = u(1-v) + v \geq 0$.

Thus $e^{uv} \leq e^{u+v} = e^{u} e^{v}$.

Hence the result follows.

We now present Jensen-type inequalities for e-convex functions.

Theorem 1([8]). *Let $\phi : \mathscr{I} \to \mathbb{R}$ be an e-convex function, then*

$$\phi\left(\sum_{i=1}^{n} \alpha_i x_i\right) \le \sum_{i=1}^{n} e^{n-i}(e^{\alpha_i})^{1/(\alpha_1+\alpha_2)}\phi(x_i),$$

where $\alpha_i > 0$ such that $\sum_{i=1}^{n} \alpha_i = 1$, and $x_i \in \mathscr{I}$ for $i = 1,2,\ldots,n,\ n \in \mathbb{N}\setminus\{1\}$.

Theorem 2. *Let $\phi : \mathscr{I} \to \mathbb{R}$ be an e-convex function, then*

$$\phi\left(\sum_{i=1}^{n} \alpha_i x_i\right) \le \sum_{i=1}^{n} e^{n-i}(e^{\alpha_i})^{\left(\sum\limits_{k=1}^{n-1} \alpha_k\right)^{-1}}\phi(x_i),$$

where $\alpha_i > 0$ such that $\sum_{i=1}^{n} \alpha_i = 1$, and $x_i \in \mathscr{I}$ for $i = 1,2,\ldots,n,\ n \in \mathbb{N}\setminus\{1\}$.

Proof. Let us prove this result by the principle of mathematical induction.

For $n = 2$, we have

$$\begin{aligned}
\phi\left(\sum_{i=1}^{2} \alpha_i x_i\right) &\le e^{\alpha_1}\phi(x_1) + e^{\alpha_2}\phi(x_2) \\
&\le e\cdot e^{\alpha_1}\phi(x_1) + e^{\alpha_2}\phi(x_2) \\
&\le e\cdot (e^{\alpha_1})^{1/\alpha_1}\phi(x_1) + (e^{\alpha_2})^{1/\alpha_1}\phi(x_2) \\
&= \sum_{i=1}^{2} e^{2-i}(e^{\alpha_i})^{(\alpha_1)^{-1}}\phi(x_i).
\end{aligned}$$

Let us now suppose that the result holds true for $n = p$, i.e.,

$$\phi\left(\sum_{i=1}^{p} \alpha_i x_i\right) \le \sum_{i=1}^{p} e^{p-i}(e^{\alpha_i})^{\left(\sum\limits_{k=1}^{p-1} \alpha_k\right)^{-1}}\phi(x_i).$$

Now, let us compute

$$\begin{aligned}
\phi\left(\sum_{i=1}^{p+1} \alpha_i x_i\right) &\le e^{1-\alpha_{p+1}}\phi\left(\sum_{i=1}^{p} \frac{\alpha_i}{1-\alpha_{p+1}} x_i\right) + e^{\alpha_{p+1}}\phi(x_{p+1}) \\
&\le e\phi\left(\sum_{i=1}^{p} \frac{\alpha_i}{1-\alpha_{p+1}} x_i\right) + e^{\alpha_{p+1}}\phi(x_{p+1}) \\
&\le e\sum_{i=1}^{p} e^{p-i}\left(e^{\frac{\alpha_i}{1-\alpha_{p+1}}}\right)^{\frac{1}{\frac{\alpha_1}{1-\alpha_{p+1}}+\frac{\alpha_2}{1-\alpha_{p+1}}+\cdots+\frac{\alpha_{p-1}}{1-\alpha_{p+1}}}}\phi(x_i) + e^{\alpha_{p+1}}\phi(x_{p+1}) \\
&= \sum_{i=1}^{p} e^{p+1-i}(e^{\alpha_i})^{\left(\sum\limits_{k=1}^{p} \alpha_k\right)^{-1}}\phi(x_i) + e^{\alpha_{p+1}}\phi(x_{p+1}) \\
&\le \sum_{i=1}^{p+1} e^{p+1-i}(e^{\alpha_i})^{\left(\sum\limits_{k=1}^{p} \alpha_k\right)^{-1}}\phi(x_i).
\end{aligned}$$

Hence, by the principle of mathematical induction, we have

$$\phi\left(\sum_{i=1}^{n} \alpha_i x_i\right) \le \sum_{i=1}^{n} e^{n-i}(e^{\alpha_i})^{\left(\sum\limits_{k=1}^{n-1} \alpha_k\right)^{-1}}\phi(x_i).$$

From Theorems 1 and 2, we deduce that, for $n \geq 4$, the bound in Theorem 2 is sharper than that in Theorem 1 for e-convex functions.

Corollary 1. *Let $\phi : \mathscr{I} \to \mathbb{R}$ be an e-convex function, then*

$$\phi\left(\frac{1}{n}\sum_{i=1}^{n} u_i\right) \leq \sum_{i=1}^{n} e^{n-i}\, e^{1/n}\, \phi(u_i),$$

for $u_i \in \mathscr{I}, 1 \leq i \leq n$ and $n \in \mathbb{N}$.

The following theorem provides upper and lower bounds for the average value of an e-convex function evaluated at two complementary weighted combinations of two points.

Theorem 3. *Let $\phi : \mathscr{I} \to \mathbb{R}$ be an e-convex function, then for all $u, v \in \mathscr{I}$, the mapping $\psi_{u,v} : [0,1] \to \mathbb{R}$ defined by*

$$\psi_{u,v}(t) := \frac{\phi(tu + (1-t)v) + \phi((1-t)u + tv)}{2}$$

is also e-convex on [0,1]. In addition, we have the inequality

$$\phi\left(\frac{u+v}{2}\right) \leq e^{1/2}\psi_{u,v}(t) \leq e^{1/2}\left(e^t + e^{1-t}\right)\left(\frac{\phi(u)+\phi(v)}{2}\right).$$

Proof. Let $t_1, t_2 \in [0,1]$ and $\alpha_1, \alpha_2 \in [0,1]$ be such that $\alpha_1 + \alpha_2 = 1$. Then

$$\begin{aligned}
&\psi_{u,v}(\alpha t_1 + \beta t_2) \\
&= \frac{\phi((\alpha_1 t_1 + \alpha_2 t_2)u + (1 - \alpha_1 t_1 - \alpha_2 t_2)v) + \phi((1 - \alpha_1 t_1 - \alpha_2 t_2)u + (\alpha_1 t_1 + \alpha_2 t_2)v)}{2} \\
&= \frac{1}{2}\left[\phi\left(\alpha_1(t_1 u + (1-t_1)v) + \alpha_2(t_2 u + (1-t_2)v)\right) + \phi\left(\alpha_1((1-t_1)u + t_1 v) + \alpha_2((1-t_2)u + t_2 v)\right)\right] \\
&\leq \frac{1}{2}\left[e^{\alpha_1}\left(\phi(t_1 u + (1-t_1)v) + \phi((1-t_1)u + t_1 v)\right) + e^{\alpha_2}\left(\phi(t_2 u + (1-t_2)v) + \phi((1-t_2)u + t_2 v)\right)\right] \\
&\leq e^{\alpha_1}\psi_{u,v}(t_1) + e^{\alpha_2}\psi_{u,v}(t_2)
\end{aligned}$$

$\Rightarrow \quad \psi_{u,v}$ is e-convex on $[0,1]$.

Now, we have

$$\begin{aligned}
\phi\left(\frac{u+v}{2}\right) &= \phi\left(\frac{tu + (1-t)v + (1-t)u + tv}{2}\right) \\
&\leq e^{1/2}\left[\phi(tu + (1-t)v) + \phi((1-t)u + tv)\right].
\end{aligned}$$

Thus

$$\phi\left(\frac{u+v}{2}\right) \leq e^{1/2}\,\psi_{u,v}(t). \tag{1}$$

Also

$$\begin{aligned}
\psi_{u,v}(t) &= \frac{1}{2}\left[\phi(tu + (1-t)v) + \phi((1-t)u + tv)\right] \\
&\leq \frac{1}{2}\left[\left(e^t + e^{1-t}\right)\phi(u) + \left(e^t + e^{1-t}\right)\phi(v)\right] \\
&= \frac{\left(e^t + e^{1-t}\right)}{2}\left(\phi(u) + \phi(v)\right)
\end{aligned}$$

$$\Rightarrow e^{1/2}\,\psi_{u,v}(t) \leq e^{1/2}\left(\frac{e^t + e^{1-t}}{2}\right)\left(\phi(u) + \phi(v)\right). \tag{2}$$

Thus by (1) and (2), we get

$$\phi\left(\frac{u+v}{2}\right) \leq e^{1/2}\,\psi_{u,v}(t) \leq e^{1/2}\left(e^t + e^{1-t}\right)\left(\frac{\phi(u)+\phi(v)}{2}\right).$$

The next two results provide integral inequalities that leverage the properties of an e-convex function to establish useful upper bounds.

Theorem 4([8]). *If $\phi : \mathscr{I} \to [0,\infty)$ is an increasing e-convex function, and $\psi : \mathscr{J} \to [0,\infty)$ is a convex function such that $\psi(\mathscr{J}) \subseteq \mathscr{I}$, then the composition $\phi \circ \psi$ is an e-convex function on $\mathscr{J}$.*

Proposition 3. *Let $\alpha_1, \alpha_2 \geq 0$ and $r \geq 1$. Then*

$$\int_0^1 |1-2t| (t\alpha_1 + (1-t)\alpha_2)^r \, dt \leq \frac{1}{2}(\alpha_1^r + \alpha_2^r) + (e-2)\left(\frac{\alpha_1+\alpha_2}{2}\right)^r.$$

Proof. Firstly, we shall show that the function

$$L_1(t) = \big(t\alpha_1 + (1-t)\alpha_2\big)^r$$

is e-convex on $[0,1]$.

Define

$$\psi_1 : [0,1] \longrightarrow [\alpha_1, \alpha_2] \quad \text{as} \quad \psi_1(t) = \alpha_1 t + (1-t)\alpha_2.$$

Then ψ_1 is convex and $\psi_1([0,1]) \subseteq [\alpha_1, \alpha_2]$.

Next, consider

$$\phi_1 : [\alpha_1, \alpha_2] \longrightarrow [0,\infty) \quad \text{as} \quad \phi_1(x) = x^r.$$

Since $\alpha_1, \alpha_2 \geq 0$, it follows that ϕ_1 is a positive function and a convex function (because $r \geq 1$). Therefore, in view of Lemma 3.2 in [8], ϕ_1 is e-convex on $[\alpha_1, \alpha_2]$.

Thus ϕ is e-convex on $[\alpha_1, \alpha_2]$ and increasing.

Therefore, in view of Theorem 4, we have that

$$L_1(t) = \phi_1 \circ \psi_1(t) = \big(\alpha_1 t + (1-t)\alpha_2\big)^r$$

is e-convex on $[0,1]$.

Secondly, by proceeding analogously, it can be shown that the function

$$L_2(t) = \phi_2 \circ \psi_2(t) = \big(t\alpha_2 + (1-t)\alpha_1\big)^r$$

is e-convex on $[0,1]$, where $\psi_2 : [0,1] \longrightarrow [\alpha_1, \alpha_2]$ is defined as $\psi_2(t) = \alpha_2 t + (1-t)\alpha_1$ and $\phi_2 : [\alpha_1, \alpha_2] \longrightarrow [0,\infty)$ is defined as $\phi_2(x) = x^r$.

If we denote

$$L(t) = L_1(t) + L_2(t), \quad t \in [0,1],$$

then clearly $L(t)$ is e-convex on $[0,1]$. Consequently, for $0, 1/2 \in [0,1]$ and $t \in [0,1]$, we obtain

$$\begin{aligned} L\left(\frac{1-t}{2}\right) &\leq e^t L(0) + e^{1-t} L\left(\frac{1}{2}\right) \\ &\leq e^t(\alpha_1^r + \alpha_2^r) + 2e^{1-t}\left(\frac{\alpha_1+\alpha_2}{2}\right)^r. \end{aligned} \tag{3}$$

Now,

$$\int_0^1 |1-2t| (t\alpha_1 + (1-t)\alpha_2)^r \, dt = \int_0^{1/2} (1-2t) L(t) dt.$$

This gives

$$\begin{aligned} \int_0^{1/2} (1-2t)L(t)dt &= \frac{1}{2}\int_0^1 tL\left(\frac{1-t}{2}\right) dt \\ &\leq \frac{1}{2}\int_0^1 t\left(e^t(\alpha_1^r + \alpha_2^r) + 2e^{1-t}\left(\frac{\alpha_1+\alpha_2}{2}\right)^r\right) dt \quad \text{(using (3))} \\ &= \frac{1}{2}(\alpha_1^r + \alpha_2^r) + (e-2)\left(\frac{\alpha_1+\alpha_2}{2}\right)^r. \end{aligned}$$

Lemma 2. *Let* $P(u) = \begin{cases} -u, & \text{if } 0 \le u < \frac{1}{2}; \\ 1-u & \text{if } \frac{1}{2} \le u \le 1 \end{cases}$, $\alpha_1, \alpha_2 \ge 0$, *and* $r \ge 1$. *Then*

$$\int_0^1 |P(u)|\,(u\alpha_1 + (1-u)\alpha_2)^r\,du \le \frac{1}{4}\left[(e-2)\,(\alpha_1^r + \alpha_2^r) + 2\left(\frac{\alpha_1+\alpha_2}{2}\right)^r\right].$$

Proof. We have

$$\begin{aligned}
&\int_0^1 |P(u)|\,(u\alpha_1 + (1-u)\alpha_2)^r\,du \\
&\quad = \int_0^{1/2} |P(u)|\,(u\alpha_1 + (1-u)\alpha_2)^r\,du + \int_{1/2}^1 |P(u)|\,(u\alpha_1 + (1-u)\alpha_2)^r\,du \\
&\quad = \int_0^{1/2} u\,(u\alpha_1 + (1-u)\alpha_2)^r\,du + \int_0^{1/2} u\,((1-u)\alpha_1 + u\alpha_2)^r\,du \\
&\quad = \int_0^{1/2} [u\,(u\alpha_1 + (1-u)\alpha_2)^r + ((1-u)\alpha_1 + u\alpha_2)^r]\,du \\
&\quad = \int_0^{1/2} u \cdot L(u)\,du,
\end{aligned}$$

where $L(u) = (u\alpha_1 + (1-u)\alpha_2)^r + (u\alpha_2 + (1-u)\alpha_1)^r$.

This gives

$$\begin{aligned}
&\int_0^1 |P(u)|\,(u\alpha_1 + (1-u)\alpha_2)^r\,du \\
&\quad = \int_0^{1/2} u \cdot L\left(\frac{1-(1-2u)}{2}\right)du \\
&\quad \le \int_0^{1/2} u\left[e^{1-2u}(\alpha_1^r + \alpha_2^r) + 2e^{2u}\left(\frac{\alpha_1+\alpha_2}{2}\right)^r\right]du \\
&\quad = (\alpha_1^r + \alpha_2^r)\int_0^{1/2} ue^{1-2u}du + 2\left(\frac{\alpha_1+\alpha_2}{2}\right)^r\int_0^{1/2} ue^{2u}du \\
&\quad = (\alpha_1^r + \alpha_2^r)\left(\frac{1}{4}(e-2)\right) + 2\left(\frac{\xi_1+\xi_2}{2}\right)^r \cdot \frac{1}{4} \\
&\quad = \frac{1}{4}\left[(e-2)(\alpha_1^r + \alpha_2^r) + 2\left(\frac{\alpha_1+\alpha_2}{2}\right)^r\right].
\end{aligned}$$

Conclusion

In this work, we advanced the study of *e*-convex functions by establishing new structural properties, Jensen-type inequalities, and integral bounds. These results extend the classical framework of convexity through exponential weights, thereby offering a broader perspective on inequalities in mathematical analysis.

Beyond theoretical interest, the inequalities derived here have potential applications in several areas. For example, the framework of *e*-convexity can be adapted to investigate operator and matrix inequalities, where generalized convexity has already yielded notable improvements. Similarly, our results may be employed in approximation theory, providing sharper bounds for interpolation, quadrature, and error estimation. Another promising direction lies in the development of numerical schemes, where *e*-convexity-based inequalities could guide the construction of stable and efficient algorithms for differential and integral equations.

Future research may also explore connections between *e*-convexity and other generalized convexities, as well as fractional-order models and applications in optimization. These directions would not only deepen the theoretical landscape of convex analysis but also strengthen its applicability in computational mathematics and applied sciences.

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