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Some Common Fixed-Point Theorems of Kannan-type Contractions with CLR and EA Property on Fuzzy Metric Spaces

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Abstract: This manuscript explores generalized Kannan-type contractions in the framework of fuzzy metric spaces. We prove several common fixed-point results of these new contractive mappings via common limit in the range and El Moutawakil-Aamri properties. We provide examples to clarify our results. Our results improvise and generalize a few common fixed-point theorems established by earlier studies.

Keywords: Fuzzy metric spaces; Common fixed points; Kannan-type contraction; CLR property; EA property.

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1 Introduction

Fuzzy set is an idea proposed by Zadeh [35] in 1965. Since then, this concept has been widely acknowledged by researchers and utilized in diverse branches of mathematics as well as real life applications. At a later point, Kramosil and Michálek [18] presented fuzzy metric spaces as an extension for the probabilistic metric spaces with the perspective of fuzzy sets. Their notion was later modified by George and Veeramani [8] so that Hausdorff topology can be studied on this space. Grabiec [13] pioneered the investigation of fixed-point theory on fuzzy metric spaces. Consequently, researchers studied fixed-point theory intensively on this abstract spaces and its generalized spaces. A few fixed-point theories on these spaces may be seen in [10], [9], [22], [23], [25], [26], and [33].

In fixed-point theory, one of the well-known contraction mappings is Kannan-type contractive mapping introduced by Kannan [16, 17]. There are several thoughts about the important of Kannan-type contractions, especially under the scope of metric fixed-point theory. One of the reasons is the famous Banach contraction by Banach [3] requires continuous mapping, but Kannan-type contractive mapping needs not to be continuous. Another reason is the relationship between Kannan-type contractive mapping and the completeness of the metric spaces. Connell [7] gave an illustration of metric space that is not complete and yet any Banach contractive mapping assigned on it have fixed point. However, this is not the case for Kannan-type contraction mappings in metric spaces. Subrahmanyam [30] demonstrated that metric space is complete implies and is implied by all Kannan-type contractive mappings in this space contain fixed points. Recent works related to Kannan-type contractive mappings can found in [6], [11], [12], [20], and [36].

Aamri and El Moutawakil [1] proposed El Moutawakil-Aamri (E.A. for short) property for noncompatible self-mapping on metric space in 2002. This (E.A.) property allows one to acquire fixed point results without the completeness of the space. However, it requires a condition of closeness of range for fixed point to exist. Later, Sintunavarat and Kumam [28] proposed a novel property, dubbed “common limit in the range” (CLR for short) that is

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more versatile compared to (E.A.) property, as it no longer needs the condition of closeness of range. These two properties are studied extensively in different spaces (see [2], [4], [14], [19], [21], [29], [31] and [32]).

The objective of this research is to validate several common fixed-point theorems for generalized Kannan-type contractive mappings equipped with a common limits in the range or (E.A.) properties on fuzzy metric spaces. This manuscript is arranged into four main sections as follows: Section 1 presents introduction. Section 2 provides preliminary definitions and notions. Section 3 contains primary findings and their proofs. Section 4 is the conclusion and open problems.

2 Preliminaries

We recollect some terminologies from the fuzzy fixed point theory that will be employed in this manuscript.

Definition 1([16]). *Let (X,δ) denoted as metric space and $\mathscr{T}: E \to E$ be a self-mapping. Then, $\mathscr{T}$ is called a Kannan-type contractive mapping if there exist $k \in [0, \frac{1}{2})$ satisfy*

$$\delta(\mathscr{T}\xi, \mathscr{T}\chi) \leq k[\delta(\xi, \mathscr{T}\xi) + \delta(\chi, \mathscr{T}\chi)]$$

for all $\xi, \chi \in X$.

Definition 2([27]). *A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is referred to as continuous t-norm if the conditions below hold:*

1.$a * 1 = a$ for every a in $[0,1]$;
2.$*$ is associative and commutative;
3.$a * b \leq i * j$ provided $a \leq i$ and $b \leq j$, where $a,i,b,j \in [0,1]$;
4.$*$ is continuous.

Definition 3([8]). *Let E be a nonempty set, $*$ be a continuous t-norm and Γ be a fuzzy set defined on $E \times E \times (0,\infty)$ such that the following conditions hold:*

1.$0 < \Gamma(\varpi, \omega, \varkappa)$;
2.$\Gamma(\varpi, \omega, \varkappa) = 1 \iff \varpi = \omega$;
3.$\Gamma(\varpi, \omega, \varkappa) = \Gamma(\omega, \varpi, \varkappa)$;
4.$\Gamma(\varpi, \vartheta, \varkappa + \varsigma) \geq \Gamma(\varpi, \omega, \varkappa) * \Gamma(\omega, \vartheta, \varsigma)$;
5.$\Gamma(\varpi, \omega, \cdot) : (0,\infty) \to (0,1]$ is continuous,

for every $\varpi, \omega, \vartheta \in E$ and any $\varkappa, \varsigma > 0$. Then, an ordered triple $(E, \Gamma, *)$ is called a fuzzy metric space.

Lemma 1([13]). *If $(E,\Gamma,*)$ is a fuzzy metric space, then $\Gamma(\varpi, \omega, \varkappa)$ is increasing for any pair of ϖ, ω in E.*

Definition 4([8]). *Let $(E,\Gamma,*)$ be a fuzzy metric space and $\{\varpi_n\}$ be a sequence in E. Then,*

1.$\{\varpi_n\}$ is convergent provided there exists $x \in E$ satisfies $\lim_{n\to\infty} \Gamma(\varpi_n, x, \varkappa) = 1$ for any $\varkappa > 0$;
2.$\{\varpi_n\}$ is called Cauchy sequence provided that for any $0 < \varepsilon < 1$ and $\varkappa > 0$, there is $n_0 \in \mathbb{N}$ satisfies $\Gamma(\varpi_n, \varpi_m, \varkappa) > 1 - \varepsilon$ for every $n,m \geq n_0$;
3.$(E,\Gamma,*)$ is complete whenever each Cauchy sequence in E is convergent.

Consider $\mathscr{F}, \mathscr{G} : E \to E$ where E is a nonempty set and consider an element $\omega \in E$. We say that ω is a fixed point of $\mathscr{F}$ if it satisfies $\mathscr{F}\omega = \omega$. For the case where $\mathscr{F}\omega = \mathscr{G}\omega$, ω is called a coincidence point of $\mathscr{F}$ and $\mathscr{G}$. Moreover, if $\mathscr{F}\omega = \omega = \mathscr{G}\omega$, then ω is known as the common fixed point of $\mathscr{F}$ and $\mathscr{G}$.

Definition 5([15]). *Let E be a nonempty set. Two self-mappings $\mathscr{F}, \mathscr{G} : E \to E$ are weakly compatible if both $\mathscr{F}$ and $\mathscr{G}$ commute at the coincidence point of $\mathscr{F}$ and $\mathscr{G}$, for instance, $\mathscr{F}\omega = \mathscr{G}\omega$ for some ω in E implies that $\mathscr{F}\mathscr{G}\omega = \mathscr{G}\mathscr{F}\omega$.*

The following definitions are (E.A.) and CLR property defined on two and four self-mappings. It is notable that definitions below are written under the framework of fuzzy metric space instead of the space where they originally defined.

Definition 6([1]). *For a fuzzy metric space $(E,\Gamma,*)$, a pair $(\mathscr{F}, \mathscr{T})$ of self-mappings satisfy the (E.A.) property if there is a sequence $\{\varpi_n\} \subset E$ such that*

$$\lim_{n\to\infty} \Gamma(\mathscr{F}\varpi_n, z, \varkappa) = \lim_{n\to\infty} \Gamma(\mathscr{T}\varpi_n, z, \varkappa) = 1$$

for some $z \in E$ and for all $\varkappa > 0$.

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Definition 7([21]). *For a fuzzy metric space $(E,\Gamma,*)$, two pairs $(\mathscr{F},\mathscr{T})$ and $(\mathscr{G},\mathscr{S})$ of self-mappings satisfy the common (E.A.) property if there are two sequences $\{\varpi_n\},\{\omega_n\}\subset E$ such that*

$$\lim_{n\to\infty}\Gamma(\mathscr{F}\varpi_n,z,\varkappa)=\lim_{n\to\infty}\Gamma(\mathscr{T}\varpi_n,z,\varkappa)=\lim_{n\to\infty}\Gamma(\mathscr{G}\omega_n,z,\varkappa)=\lim_{n\to\infty}\Gamma(\mathscr{S}\omega_n,z,\varkappa)=1$$

for some $z\in E$ and for all $\varkappa>0$.

Definition 8([28]). *For a fuzzy metric space $(E,\Gamma,*)$, a pair $(\mathscr{F},\mathscr{T})$ of self-mappings satisfy the common limit in the range of $\mathscr{T}$ property, denoted by (CLR$_{\mathscr{T}}$) if there is a sequence $\{\varpi_n\}\subset E$ such that*

$$\lim_{n\to\infty}\Gamma(\mathscr{F}\varpi_n,z,\varkappa)=\lim_{n\to\infty}\Gamma(\mathscr{T}\varpi_n,z,\varkappa)=1$$

for some $z\in\mathscr{T}E$ and for all $\varkappa>0$.

Definition 9([34]). *For a fuzzy metric space $(E,\Gamma,*)$, two pairs $(\mathscr{F},\mathscr{T})$ and $(\mathscr{G},\mathscr{S})$ of self-mappings satisfy the common limit in the range of $\mathscr{T}$ and $\mathscr{S}$ property, denoted by (CLR$_{\mathscr{T}\mathscr{S}}$) if there are two sequences $\{\varpi_n\},\{\omega_n\}\subset E$ such that*

$$\lim_{n\to\infty}\Gamma(\mathscr{F}\varpi_n,z,\varkappa)=\lim_{n\to\infty}\Gamma(\mathscr{T}\varpi_n,z,\varkappa)=\lim_{n\to\infty}\Gamma(\mathscr{G}\omega_n,z,\varkappa)=\lim_{n\to\infty}\Gamma(\mathscr{S}\omega_n,z,\varkappa)=1$$

for some $z\in\mathscr{T}E\cap\mathscr{S}E$ and for all $\varkappa>0$.

Definition 10([24]). *For a fuzzy metric space $(E,\Gamma,*)$, assume $\mathscr{F},\mathscr{T}$, and $\mathscr{S}$ are three self-mappings of E. The pair $(\mathscr{F},\mathscr{T})$ satisfy the common limit in the range of $\mathscr{S}$ property, denoted by (CLR$_{(\mathscr{F},\mathscr{T}),\mathscr{S}}$), if there exists sequence $\{\varpi_n\}\subset E$ such that*

$$\lim_{n\to\infty}\Gamma(\mathscr{F}\varpi_n,z,\varkappa)=\lim_{n\to\infty}\Gamma(\mathscr{T}\varpi_n,z,\varkappa)=1$$

for some $z\in\mathscr{T}E\cap\mathscr{S}E$ and for all $\varkappa>0$.

*Remark.*Using condition (2) in Definition 3, Definition 6 can be expressed in a way similar to its metric counterpart, that is, the pair $(\mathscr{F},\mathscr{T})$ satisfies the (E.A.) property if there is a sequence $\{\varpi_n\}\subset E$ such that for some $z\in E$,

$$\lim_{n\to\infty}\mathscr{F}\varpi_n=\lim_{n\to\infty}\mathscr{T}\varpi_n=z.$$

This is applicable to Definitions 7, 8, 9, and 10 as well.

By setting $\mathscr{F}=\mathscr{G}$ and $\mathscr{T}=\mathscr{S}$ in Definition 7 and Definition 9, one can obtain Definition 6 and Definition 8, respectively. Moreover, we can see that Definition 9 implies Definition 10, but this is not the case for converse. This is shown in the examples below.

*Example 1.*Suppose $(E,\Gamma,*)$ is a fuzzy metric space where $E=[0,\infty)$, Γ is a fuzzy set on $E\times E\times(0,\infty)$ and $*$ is a continuous t-norm. In addition, consider $\mathscr{F},\mathscr{G},\mathscr{T},\mathscr{S}:E\to E$ expressed as:

$$\mathscr{F}(\varpi)=\frac{7\varpi}{8},$$
$$\mathscr{G}(\varpi)=\varpi^2,$$
$$\mathscr{T}(\varpi)=\frac{\varpi}{8},$$
$$\mathscr{S}(\varpi)=5\varpi^2.$$

We have $\mathscr{T}E\cap\mathscr{S}E=[0,\infty)$. Define sequences $\{\varpi_n\}=\{\frac{1}{n}\}$ and $\{\omega_n\}=\{\frac{1}{n^2}\}$ for every $n\in\mathbb{N}$. Considering that

$$\lim_{n\to\infty}\mathscr{F}\varpi_n=\lim_{n\to\infty}\mathscr{T}\varpi_n=\lim_{n\to\infty}\mathscr{G}\omega_n=\lim_{n\to\infty}\mathscr{S}\omega_n=0$$

and $0\in\mathscr{T}E\cap\mathscr{S}E$, both $(\mathscr{F},\mathscr{T})$ and $(\mathscr{G},\mathscr{S})$ satisfy the (CLR$_{\mathscr{T}\mathscr{S}}$) property. Moreover, $(\mathscr{F},\mathscr{T})$ satisfy (CLR$_{(\mathscr{F},\mathscr{T}),\mathscr{S}}$) property.

*Example 2.*Suppose $(E,\Gamma,*)$ is a fuzzy metric space where $E=[0,\infty)$, Γ is a fuzzy set on $E\times E\times(0,\infty)$ and $*$ is a continuous t-norm. Furthermore, consider $\mathscr{F},\mathscr{G},\mathscr{T},\mathscr{S}:E\to E$ expressed as:

$$\mathscr{F}(\varpi)=\varpi+2,$$
$$\mathscr{G}(\varpi)=\frac{\varpi+1}{2},$$
$$\mathscr{T}(\varpi)=3\varpi,$$
$$\mathscr{S}(\varpi)=\varpi+3.$$

We have $\mathscr{T}E=[0,\infty)$ and $\mathscr{S}E=[3,\infty)$ which implies $\mathscr{T}E\cap\mathscr{S}E=[3,\infty)$. Consider a sequence $\{\varpi_n\}=\{\frac{n+1}{n}\}$. It is clear that

$$\lim_{n\to\infty}\mathscr{F}\varpi_n=\lim_{n\to\infty}\mathscr{T}\varpi_n=3$$

and $3\in\mathscr{T}E\cap\mathscr{S}E$. Thus, the pair $(\mathscr{F},\mathscr{T})$ satisfy the (CLR$_{\mathscr{F},\mathscr{T}),\mathscr{S}}$) property.

If we let sequence $\{\omega_n\}=\{\frac{1}{n}\}$, we get

$$\lim_{n\to\infty}\mathscr{G}\omega_n=\frac{1}{2}\text{ and }\lim_{n\to\infty}\mathscr{S}\omega_n=3$$

which means that $\lim_{n\to\infty}\mathscr{G}\omega_n\neq\lim_{n\to\infty}\mathscr{S}\omega_n$. This concludes both $(\mathscr{F},\mathscr{T})$, $(\mathscr{G},\mathscr{S})$ do not satisfy (CLR$_{\mathscr{T}\mathscr{S}}$) property.

The function below will be utilized in our later results.

Definition 11.*A mapping $\psi:[0,1]\times[0,1]\to[0,1]$ is called as Ψ-function if:*

1. *$\psi(u,v)$ is monotonically nondecreasing in both u and v variables;*
2. *$\psi(u,v)$ is lower semicontinuous in both u and v variables;*
3. *$\psi(v,v)>v$ for every $v\in(0,1)$;*
4. *$\psi(1,1)=1$ and $\psi(0,0)=0$.*

Ψ_f is denoted as the collection of all Ψ-functions. Examples of Ψ-functions are $\psi(u,v)=\frac{k\sqrt{u}+l\sqrt{v}}{k+l}$ where $k,l\in\mathbb{R}^+$, $\psi(u,v)=\sqrt{uv}$, and $\psi(u,v)=\min\{u,v\}$ for all $u,v\in[0,1]$.

3 Main Results

Theorem 1.*Suppose that $(E,\Gamma,*)$ is a fuzzy metric space and $\mathscr{F},\mathscr{G},\mathscr{S},\mathscr{T}$ are self-mappings of E satisfying the following condition:*

$$\begin{aligned}&\Gamma(\mathscr{F}\varpi,\mathscr{G}\omega,\varkappa)+h(1-\max\{\Gamma(\mathscr{T}\varpi,\mathscr{G}\omega,\varkappa),\Gamma(\mathscr{S}\omega,\mathscr{F}\varpi,\varkappa),\Gamma(\mathscr{T}\varpi,\mathscr{S}\omega,\varkappa)\})\\&\geq\psi\left(\Gamma\left(\mathscr{T}\varpi,\mathscr{F}\varpi,\frac{\varkappa_1}{p}\right),\Gamma\left(\mathscr{S}\omega,\mathscr{G}\omega,\frac{\varkappa_2}{q}\right)\right)\end{aligned}\tag{1}$$

for any $\varpi,\omega\in E$ and $\varkappa>0$ where $h\geq0,\varkappa,\varkappa_1,\varkappa_2>0$ with $\varkappa=\varkappa_1+\varkappa_2$, $p,q>0$ with $p+q\in(0,1)$ and $\psi\in\Psi_f$. Assume that both pairs $(\mathscr{F},\mathscr{T})$ and $(\mathscr{G},\mathscr{S})$ satisfy the (CLR$_{\mathscr{T}\mathscr{S}}$) property, then the pairs $(\mathscr{F},\mathscr{T})$ and $(\mathscr{G},\mathscr{S})$ have a coincidence point in E.

*Proof.*Given that both pairs $(\mathscr{F},\mathscr{T})$, $(\mathscr{G},\mathscr{S})$ satisfy the (CLR$_{\mathscr{T}\mathscr{S}}$) property, there exist sequences $\{\varpi_n\}$ and $\{\omega_n\}$ in E such that for all $\varkappa>0$,

$$\lim_{n\to\infty}\Gamma(\mathscr{F}\varpi_n,z,\varkappa)=\lim_{n\to\infty}\Gamma(\mathscr{T}\varpi_n,z,\varkappa)=\lim_{n\to\infty}\Gamma(\mathscr{G}\omega_n,z,\varkappa)=\lim_{n\to\infty}\Gamma(\mathscr{S}\omega_n,z,\varkappa)=1$$

for some $z\in\mathscr{T}E\cap\mathscr{S}E$. This means that

$$\lim_{n\to\infty}\mathscr{F}\varpi_n=\lim_{n\to\infty}\mathscr{T}\varpi_n=\lim_{n\to\infty}\mathscr{G}\omega_n=\lim_{n\to\infty}\mathscr{S}\omega_n=z.$$

As $z\in\mathscr{T}E$, one can find an element $u\in E$ satisfy $z=\mathscr{T}u$. We will show that $\mathscr{F}u=\mathscr{T}u$. Assume $\mathscr{F}u\neq\mathscr{T}u$, which means, $0<\Gamma(\mathscr{F}u,\mathscr{T}u,\varkappa)<1$ for some $\varkappa>0$. Using inequality (1), for all $\varkappa>0$, we yield

$$\begin{aligned}&\Gamma(\mathscr{F}u,\mathscr{G}\omega_n,\varkappa)+h(1-\max\{\Gamma(\mathscr{T}u,\mathscr{G}\omega_n,\varkappa),\Gamma(\mathscr{S}\omega_n,\mathscr{F}u,\varkappa),\Gamma(\mathscr{T}u,\mathscr{S}\omega_n,\varkappa)\})\\&\geq\psi\left(\Gamma\left(\mathscr{T}u,\mathscr{F}u,\frac{\varkappa_1}{p}\right),\Gamma\left(\mathscr{S}\omega_n,\mathscr{G}\omega_n,\frac{\varkappa_2}{q}\right)\right).\end{aligned}\tag{2}$$

Let $\varkappa_1 = \frac{p\varkappa}{p+q}, \varkappa_2 = \frac{q\varkappa}{p+q}$ and $r = p+q$. Clearly, we have $\frac{\varkappa_1}{p} = \frac{\varkappa_2}{q} = \frac{\varkappa}{r}$ and $0 < r < 1$. Then, from (2) we can obtain the following:

$$\begin{aligned}&\Gamma(\mathscr{F}u,\mathscr{G}\omega_n,\varkappa)+h(1-\max\{\Gamma(\mathscr{T}u,\mathscr{G}\omega_n,\varkappa),\Gamma(\mathscr{S}\omega_n,\mathscr{F}u,\varkappa),\Gamma(\mathscr{T}u,\mathscr{S}\omega_n,\varkappa)\})\\ &\geq\psi\left(\Gamma\left(\mathscr{T}u,\mathscr{F}u,\frac{\varkappa}{r}\right),\Gamma\left(\mathscr{S}\omega_n,\mathscr{G}\omega_n,\frac{\varkappa}{r}\right)\right).\end{aligned}$$

By taking the limit as $n\to\infty$, we yield

$$\begin{aligned}&\Gamma(\mathscr{F}u,z,\varkappa)+h(1-\max\{\Gamma(\mathscr{T}u,z,\varkappa),\Gamma(z,\mathscr{F}u,\varkappa),\Gamma(\mathscr{T}u,z,\varkappa)\})\\ &\geq\psi\left(\Gamma\left(\mathscr{T}u,\mathscr{F}u,\frac{\varkappa}{r}\right),\Gamma\left(z,z,\frac{\varkappa}{r}\right)\right)\\ &=\psi\left(\Gamma\left(\mathscr{T}u,\mathscr{F}u,\frac{\varkappa}{r}\right),1\right).\end{aligned}$$

Since $z=\mathscr{T}u$, the inequality above can be rewritten as

$$\begin{aligned}\Gamma(\mathscr{F}u,\mathscr{T}u,\varkappa)+h(1-\max\{1,\Gamma(z,\mathscr{F}u,\varkappa),1\})&\geq\psi\left(\Gamma\left(\mathscr{T}u,\mathscr{F}u,\frac{\varkappa}{r}\right),1\right)\\ \Gamma(\mathscr{F}u,\mathscr{T}u,\varkappa)+h(1-1)&\geq\psi\left(\Gamma\left(\mathscr{T}u,\mathscr{F}u,\frac{\varkappa}{r}\right),1\right)\\ \Gamma(\mathscr{F}u,\mathscr{T}u,\varkappa)&\geq\psi\left(\Gamma\left(\mathscr{T}u,\mathscr{F}u,\frac{\varkappa}{r}\right),1\right).\end{aligned}$$

By Ψ-function's properties and Lemma 1, we yield

$$\begin{aligned}\Gamma(\mathscr{F}u,\mathscr{T}u,\varkappa)&\geq\psi\left(\Gamma\left(\mathscr{T}u,\mathscr{F}u,\frac{\varkappa}{r}\right),1\right)\\ &\geq\psi\left(\Gamma\left(\mathscr{T}u,\mathscr{F}u,\frac{\varkappa}{r}\right),\Gamma\left(\mathscr{T}u,\mathscr{F}u,\frac{\varkappa}{r}\right)\right)\\ &>\Gamma\left(\mathscr{T}u,\mathscr{F}u,\frac{\varkappa}{r}\right)\\ &>\Gamma(\mathscr{T}u,\mathscr{F}u,\varkappa)\\ &=\Gamma(\mathscr{F}u,\mathscr{T}u,\varkappa)\end{aligned}$$

which leads to a contradiction. As a result, $\Gamma(\mathscr{F}u,\mathscr{T}u,\varkappa)=1$ for each $\varkappa>0$. By the condition (2) from Definition 3, we yield $\mathscr{F}u=\mathscr{T}u=z$. This implies that point u is a coincidence point of the pair $(\mathscr{F},\mathscr{T})$.

Additionally, since $z\in\mathscr{S}E$, one can find an element $v\in E$ satisfy $z=\mathscr{S}v$. We will show that $\mathscr{G}v=\mathscr{S}v$. Assume $\mathscr{G}v\neq\mathscr{S}v$, which means, $0<\Gamma(\mathscr{G}v,\mathscr{S}v,\varkappa)<1$ for some $\varkappa>0$. Using inequality (1), for each $\varkappa>0$, it follows that

$$\begin{aligned}&\Gamma(\mathscr{F}\varpi_n,\mathscr{G}v,\varkappa)+h(1-\max\{\Gamma(\mathscr{T}\varpi_n,\mathscr{G}v,\varkappa),\Gamma(\mathscr{S}v,\mathscr{F}\varpi_n,\varkappa),\Gamma(\mathscr{T}\varpi_n,\mathscr{S}v,\varkappa)\})\\ &\geq\psi\left(\Gamma\left(\mathscr{T}\varpi_n,\mathscr{F}\varpi_n,\frac{\varkappa_1}{p}\right),\Gamma\left(\mathscr{S}v,\mathscr{G}v,\frac{\varkappa_2}{q}\right)\right).\end{aligned}\tag{3}$$

Again let $\varkappa_1=\frac{p\varkappa}{p+q},\varkappa_2=\frac{q\varkappa}{p+q}$ and $r=p+q$. Then, from (3) we can obtain the following:

$$\begin{aligned}&\Gamma(\mathscr{F}\varpi_n,\mathscr{G}v,\varkappa)+h(1-\max\{M(\mathscr{T}\varpi_n,\mathscr{G}v,\varkappa),\Gamma(\mathscr{S}v,\mathscr{F}\varpi_n,\varkappa),\Gamma(\mathscr{T}\varpi_n,\mathscr{S}v,\varkappa)\})\\ &\geq\psi\left(\Gamma\left(\mathscr{T}\varpi_n,\mathscr{F}\varpi_n,\frac{\varkappa}{r}\right),\Gamma\left(\mathscr{S}v,\mathscr{G}v,\frac{\varkappa}{r}\right)\right).\end{aligned}$$

By taking the limit as $n\to\infty$, we yield

$$\begin{aligned}&\Gamma(z,\mathscr{G}v,\varkappa)+h(1-\max\{\Gamma(z,\mathscr{G}v,\varkappa),\Gamma(\mathscr{S}v,z,\varkappa),\Gamma(z,\mathscr{S}v,\varkappa)\})\\ &\geq\psi\left(\Gamma\left(z,z,\frac{\varkappa}{r}\right),\Gamma\left(\mathscr{S}v,\mathscr{G}v,\frac{\varkappa}{r}\right)\right)\\ &=\psi\left(1,\Gamma\left(\mathscr{S}v,\mathscr{G}v,\frac{\varkappa}{r}\right)\right).\end{aligned}$$

Since $z=\mathscr{S}v$, the inequality above can be rewritten as

$$\begin{aligned}\Gamma(\mathscr{S}v,\mathscr{G}v,\varkappa)+h(1-\max\{\Gamma(\mathscr{S}v,\mathscr{G}v,\varkappa),1,1\})&\geq\psi\left(1,\Gamma\left(\mathscr{S}v,\mathscr{G}v,\frac{\varkappa}{r}\right)\right)\\ \Gamma(\mathscr{S}v,\mathscr{G}v,\varkappa)+h(1-1)&\geq\psi\left(1,\Gamma\left(\mathscr{S}v,\mathscr{G}v,\frac{\varkappa}{r}\right)\right)\\ \Gamma(\mathscr{S}v,\mathscr{G}v,\varkappa)&\geq\psi\left(1,\Gamma\left(\mathscr{S}v,\mathscr{G}v,\frac{\varkappa}{r}\right)\right).\end{aligned}$$

Due to Ψ-function's properties and Lemma 1, we yield

$$\begin{aligned}\Gamma(\mathscr{S}v,\mathscr{G}v,\varkappa) &\geq \psi\left(1,\Gamma\left(\mathscr{S}v,\mathscr{G}v,\frac{\varkappa}{r}\right)\right)\\ &\geq \psi\left(\Gamma\left(\mathscr{S}v,\mathscr{G}v,\frac{\varkappa}{r}\right),\Gamma\left(\mathscr{S}v,\mathscr{G}v,\frac{\varkappa}{r}\right)\right)\\ &> \Gamma\left(\mathscr{S}v,\mathscr{G}v,\frac{\varkappa}{r}\right)\\ &> \Gamma(\mathscr{S}v,\mathscr{G}v,\varkappa)\end{aligned}$$

which leads to a contradiction. As a result, $\Gamma(\mathscr{G}v,\mathscr{S}v,\varkappa) = 1$ for each $\varkappa > 0$. By using the condition (2) from Definition 3, we yield $\mathscr{G}v = \mathscr{S}v = z$. So v is a coincidence point of the pair $(\mathscr{G},\mathscr{S})$.

*Remark.*It is possible to obtain Theorem 2.2 in Choudhury et al. [5] if we let $\mathscr{F} = \mathscr{G}$, $\mathscr{T} = \mathscr{S}$ and $\max\{\Gamma(\mathscr{T}\varpi,\mathscr{G}\omega,\varkappa),\Gamma(\mathscr{S}\omega,\mathscr{F}\varpi,\varkappa),\Gamma(\mathscr{T}\varpi,\mathscr{S}\omega,\varkappa)\} = \max\{\Gamma(\mathscr{T}\varpi,\mathscr{G}\omega,\varkappa),\Gamma(\mathscr{S}\omega,\mathscr{F}\varpi,\varkappa)\}$ in our Theorem 1 above. In addition to that, they require the fuzzy metric space to be equipped with Hadzic type t-norm, whereas in our result the t-norm for fuzzy metric space picked is arbitrary. Hence, our results improvises their results without t-norm restriction and completeness on fuzzy metric space.

We deduce the subsequent corollary from Theorem 1.

Corollary 1.*Suppose that $(E,\Gamma,*)$ is a fuzzy metric space and $\mathscr{F},\mathscr{G},\mathscr{S},\mathscr{T}$ are self-mappings of E satisfying the following condition:*

$$\begin{aligned}&\Gamma(\mathscr{F}\varpi,\mathscr{G}\omega,\varkappa)+h(1-\max\{\Gamma(\mathscr{T}\varpi,\mathscr{G}\omega,\varkappa),\Gamma(\mathscr{S}\omega,\mathscr{F}\varpi,\varkappa),\Gamma(\mathscr{T}\varpi,\mathscr{S}\omega,\varkappa)\})\\ &\geq \psi\left(\Gamma\left(\mathscr{T}\varpi,\mathscr{F}\varpi,\frac{\varkappa_1}{p}\right),\Gamma\left(\mathscr{S}\omega,\mathscr{G}\omega,\frac{\varkappa_2}{q}\right)\right)\end{aligned}\tag{4}$$

for any $\varpi,\omega \in E$ and $\varkappa > 0$ where $h \geq 0, \varkappa, \varkappa_1, \varkappa_2 > 0$ with $\varkappa = \varkappa_1 + \varkappa_2$, $p,q > 0$ with $p+q \in (0,1)$ and $\psi \in \Psi_f$. Assume that $\mathscr{T}E$, $\mathscr{S}E$ are closed subsets of E and the pairs $(\mathscr{F},\mathscr{T})$, $(\mathscr{G},\mathscr{S})$ satisfy common (E.A.) property, then both pairs $(\mathscr{F},\mathscr{T})$ and $(\mathscr{G},\mathscr{S})$ have a coincidence point.

*Proof.*As both pairs $(\mathscr{F},\mathscr{T}),(\mathscr{G},\mathscr{S})$ fulfill common (E.A.) property, we have some sequences $\{\varpi_n\},\{\omega_n\} \subset E$ such that for all $\varkappa > 0$,

$$\lim_{n\to\infty}\Gamma(\mathscr{F}\varpi_n,z,\varkappa) = \lim_{n\to\infty}\Gamma(\mathscr{T}\varpi_n,z,\varkappa) = \lim_{n\to\infty}\Gamma(\mathscr{G}\omega_n,z,\varkappa) = \lim_{n\to\infty}\Gamma(\mathscr{S}\omega_n,z,\varkappa) = 1$$

for some z in E. This means that

$$\lim_{n\to\infty}\mathscr{F}\varpi_n = \lim_{n\to\infty}\mathscr{S}\omega_n = \lim_{n\to\infty}\mathscr{G}\omega_n = \lim_{n\to\infty}\mathscr{T}\varpi_n = z.$$

Given that $\mathscr{T}E$ is closed set, there is an element $u \in E$ satisfy $z = \mathscr{T}u$. Moreover, since $\mathscr{S}E$ is closed, we can identify an element $v \in E$ satisfy $z = \mathscr{S}v$. Hence, $z \in \mathscr{T}E \cap \mathscr{S}E$. This concludes that both pairs $(\mathscr{F},\mathscr{T}),(\mathscr{G},\mathscr{S})$ satisfy the $(\mathrm{CLR}_{\mathscr{T}\mathscr{S}})$ property. The remaining of this proof follows from Theorem 1.

Theorem 2.*Suppose that $(E,\Gamma,*)$ is a fuzzy metric space and $\mathscr{F},\mathscr{G},\mathscr{S},\mathscr{T}$ are self-mappings of E satisfying the following condition:*

$$\Gamma(\mathscr{F}\varpi,\mathscr{G}\omega,\varkappa) \geq \psi\left(\Gamma\left(\mathscr{T}\varpi,\mathscr{F}\varpi,\frac{\varkappa_1}{p}\right),\Gamma\left(\mathscr{S}\omega,\mathscr{G}\omega,\frac{\varkappa_2}{q}\right)\right)\tag{5}$$

for all $\varpi,\omega \in E$ and $\varkappa > 0$ where $t_1,t_2 > 0$ with $\varkappa = \varkappa_1 + \varkappa_2$, $p,q > 0$ with $p+q \in (0,1)$ and $\psi \in \Psi_f$. Assume that both pairs $(\mathscr{F},\mathscr{T})$ and $(\mathscr{G},\mathscr{S})$ satisfy the $(\mathrm{CLR}_{\mathscr{T}\mathscr{S}})$ property, then both pairs $(\mathscr{F},\mathscr{T})$ and $(\mathscr{G},\mathscr{S})$ have a coincidence point. Furthermore, if both pairs $(\mathscr{F},\mathscr{T})$ and $(\mathscr{G},\mathscr{S})$ are weakly compatible, this implies that mappings $\mathscr{F},\mathscr{T},\mathscr{G},\mathscr{S}$ have a unique common fixed point in E.

*Proof.*To show both pairs $(\mathscr{F},\mathscr{T}),(\mathscr{G},\mathscr{S})$ possess a coincidence point, consider $h = 0$ in (1) and the proof follows as in Theorem 1.

For the rest of the Theorem, as $(\mathscr{F},\mathscr{T})$ is weakly compatible and $\mathscr{F}u = \mathscr{T}u = z$, it follows that $\mathscr{T}z = \mathscr{T}\mathscr{F}u = \mathscr{F}\mathscr{T}u = \mathscr{F}z$. We say that point z is the common fixed point of $(\mathscr{F},\mathscr{T})$. Using (5) and Ψ-function's property, for each

$\varkappa > 0$, we yield

$$\begin{aligned}\Gamma(\mathscr{F}z,z,\varkappa) = \Gamma(\mathscr{F}z,\mathscr{G}v,\varkappa) &\geq \psi\left(\Gamma\left(\mathscr{T}z,\mathscr{F}z,\frac{\varkappa_1}{p}\right),\Gamma\left(\mathscr{S}v,\mathscr{G}v,\frac{\varkappa_2}{q}\right)\right)\\ &= \psi\left(\Gamma\left(\mathscr{F}z,\mathscr{F}z,\frac{\varkappa_1}{p}\right),\Gamma\left(z,z,\frac{\varkappa_2}{q}\right)\right)\\ &= \psi(1,1)\\ &= 1.\end{aligned}$$

Thus, $\Gamma(\mathscr{F}z,z,\varkappa) = 1$ for each $\varkappa > 0$, which means, $\mathscr{F}z = \mathscr{T}z = z$. So, z is a common fixed point of $\mathscr{F}$ and $\mathscr{T}$.

Also, since $(\mathscr{G},\mathscr{S})$ is weakly compatible and $\mathscr{G}v = \mathscr{S}v = z$, this implies that $\mathscr{S}z = \mathscr{S}\mathscr{G}v = \mathscr{G}\mathscr{S}v = \mathscr{G}z$. We say that point z is a common fixed point of pair $(\mathscr{G},\mathscr{S})$. Using (5) and Ψ-function's property, for each $\varkappa > 0$, it follows that

$$\begin{aligned}\Gamma(z,\mathscr{G}z,\varkappa) = \Gamma(\mathscr{F}z,\mathscr{G}z,\varkappa) &\geq \psi\left(\Gamma\left(\mathscr{T}z,\mathscr{F}z,\frac{\varkappa_1}{p}\right),\Gamma\left(\mathscr{S}z,\mathscr{G}z,\frac{\varkappa_2}{q}\right)\right)\\ &= \psi\left(\Gamma\left(z,z,\frac{\varkappa_1}{p}\right),\Gamma\left(\mathscr{G}z,\mathscr{G}z,\frac{\varkappa_2}{q}\right)\right)\\ &= \psi(1,1)\\ &= 1.\end{aligned}$$

As a result, $\Gamma(z,\mathscr{G}z,\varkappa) = 1$ for every $\varkappa > 0$, which means, $\mathscr{G}z = z = \mathscr{S}z$. Thus, z is a common fixed point of pair $(\mathscr{G},\mathscr{S})$. This shows that z is a common fixed point of mappings $\mathscr{F},\mathscr{G},\mathscr{T},\mathscr{S}$.

For the uniqueness, assume two common fixed points z_1, $z_2 \in E$ are distinct, for instance, $0 < \Gamma(z_1,z_2,\varkappa) < 1$ for some $\varkappa > 0$. Using (5), for any $\varkappa > 0$, we get

$$\begin{aligned}\Gamma(z_1,z_2,\varkappa) &= \Gamma(\mathscr{F}z_1,\mathscr{G}z_2,\varkappa)\\ &\geq \psi\left(\Gamma\left(\mathscr{T}z_1,\mathscr{F}z_1,\frac{\varkappa_1}{p}\right),\Gamma\left(\mathscr{S}z_2,\mathscr{G}z_2,\frac{\varkappa_2}{q}\right)\right)\\ &= \psi\left(\Gamma\left(z_1,z_1,\frac{\varkappa_1}{p}\right),\Gamma\left(z_2,z_2,\frac{\varkappa_2}{q}\right)\right)\\ &= \psi(1,1)\\ &= 1\end{aligned}$$

which is contradict to our assumption. Thus, $z_1 = z_2$ which proves the common fixed point is unique.

By substituting $\mathscr{G}$ with $\mathscr{F}$ and $\mathscr{S}$ with $\mathscr{T}$ in the theorem above, we deduce the subsequent corollary.

Corollary 2. *Suppose that $(E,\Gamma,*)$ is a fuzzy metric space and $\mathscr{F},\mathscr{T}$ are self-mappings of E satisfying the following condition:*

$$\Gamma(\mathscr{F}\varpi,\mathscr{F}\omega,\varkappa) \geq \psi\left(\Gamma\left(\mathscr{T}\varpi,\mathscr{F}\varpi,\frac{\varkappa_1}{p}\right),\Gamma\left(\mathscr{T}\omega,\mathscr{F}\omega,\frac{\varkappa_2}{q}\right)\right)$$

for all $\varpi,\omega \in E$ and $\varkappa > 0$ where $\varkappa_1,\varkappa_2 > 0$ with $\varkappa = \varkappa_1 + \varkappa_2$, $p,q > 0$ with $p+q \in (0,1)$ and $\psi \in \Psi_f$. Consider the pair $(\mathscr{F},\mathscr{T})$ satisfies ($CLR_{\mathscr{T}}$) property, then the pair $(\mathscr{F},\mathscr{T})$ has a coincidence point. Furthermore, if the pair $(\mathscr{F},\mathscr{T})$ is weakly compatible, this implies that both mappings $\mathscr{F}$ and $\mathscr{T}$ have a unique common fixed point.

Theorem 3. *Suppose that $(E,\Gamma,*)$ is a fuzzy metric space and $\mathscr{F},\mathscr{G},\mathscr{S},\mathscr{T}$ are self-mappings of E satisfying the following condition:*

$$\Gamma(\mathscr{F}\varpi,\mathscr{G}\omega,\varkappa) \geq \psi\left(\Gamma\left(\mathscr{T}\varpi,\mathscr{F}\varpi,\frac{\varkappa_1}{p}\right),\Gamma\left(\mathscr{S}\omega,\mathscr{G}\omega,\frac{\varkappa_2}{q}\right)\right)$$

for all $\varpi,\omega \in E$ and $\varkappa > 0$ where $\varkappa_1,\varkappa_2 > 0$ with $\varkappa = \varkappa_1 + \varkappa_2$, $p,q > 0$ with $p+q \in (0,1)$ and $\psi \in \Psi_f$. Assume that $\mathscr{T}E$ and $\mathscr{S}E$ are closed subsets of E and the pairs $(\mathscr{F},\mathscr{T})$ and $(\mathscr{G},\mathscr{S})$ satisfy common (E.A.) property, then both pairs $(\mathscr{F},\mathscr{T})$ and $(\mathscr{G},\mathscr{S})$ have a coincidence point. Furthermore, if both pairs $(\mathscr{F},\mathscr{T})$ and $(\mathscr{G},\mathscr{S})$ are weakly compatible, this implies that mappings $\mathscr{F},\mathscr{T},\mathscr{G},\mathscr{S}$ have a unique common fixed point.

Proof. As both $(\mathscr{F},\mathscr{T}),(\mathscr{G},\mathscr{S})$ satisfy common (E.A.) property, there exist $\{\varpi_n\},\{\omega_n\}\subset E$ such that for all $\varkappa>0$,

$$\lim_{n\to\infty}\Gamma(\mathscr{F}\varpi_n,z,\varkappa)=\lim_{n\to\infty}\Gamma(\mathscr{T}\varpi_n,z,\varkappa)=\lim_{n\to\infty}\Gamma(\mathscr{G}\omega_n,z,\varkappa)=\lim_{n\to\infty}\Gamma(\mathscr{S}\omega_n,z,\varkappa)=1$$

for some $z\in X$. This means that

$$\lim_{n\to\infty}\mathscr{F}\varpi_n=\lim_{n\to\infty}\mathscr{S}\omega_n=\lim_{n\to\infty}\mathscr{G}\omega_n=\lim_{n\to\infty}\mathscr{T}\varpi_n=z.$$

As $\mathscr{T}E$ is closed, there is an element $u\in E$ satisfy $z=\mathscr{T}u$. Moreover, since $\mathscr{S}E$ is closed, there is an element $v\in E$ satisfy $z=\mathscr{S}v$. Hence, $z\in\mathscr{T}E\cap\mathscr{S}E$ which means that both $(\mathscr{F},\mathscr{T}),(\mathscr{G},\mathscr{S})$ satisfy ($\text{CLR}_{\mathscr{T}\mathscr{S}}$) property. The rest of the proof follows from Theorem 2.

By substituting $\mathscr{G}$ with $\mathscr{F}$ and $\mathscr{S}$ with $\mathscr{T}$ in Theorem above, we obtain corollary below.

Corollary 3. *Suppose that $(E,\Gamma,*)$ is a fuzzy metric space and $\mathscr{F},\mathscr{T}$ are self-mappings of E satisfying the following condition:*

$$\Gamma(\mathscr{F}\varpi,\mathscr{F}\omega,\varkappa)\geq\psi\left(\Gamma\left(\mathscr{T}\varpi,\mathscr{F}\varpi,\frac{\varkappa_1}{p}\right),\Gamma\left(\mathscr{T}\omega,\mathscr{F}\omega,\frac{\varkappa_2}{q}\right)\right)$$

for all $\varpi,\omega\in E$ and $\varkappa>0$ where $\varkappa_1,\varkappa_2>0$ with $\varkappa=\varkappa_1+\varkappa_2$, $p,q>0$ with $p+q\in(0,1)$ and $\psi\in\Psi_f$. Assume that the pair $(\mathscr{F},\mathscr{T})$ satisfies the (E.A.) property, then the pair $(\mathscr{F},\mathscr{T})$ has a coincidence point. Furthermore, if the pair $(\mathscr{F},\mathscr{T})$ is weakly compatible, then mappings $\mathscr{F}$ and $\mathscr{T}$ have a unique common fixed point.

We will present an example below to demonstrate our Theorem 2.

Example 3. Suppose that $(E,\Gamma,*)$ is a fuzzy metric space with $E=[2,8)$, $*$ is a product continuous t-norm, that is, $a*b=ab$ for any $a,b\in[0,1]$ and $\Gamma(\varpi,\omega,\varkappa)=\frac{\varkappa}{\varkappa+|\varpi-\omega|}$ for every $\varpi,\omega\in E$, $\varkappa>0$. Let $\mathscr{F},\mathscr{G},\mathscr{T},\mathscr{S}:E\to E$ define as follows:

$$\mathscr{F}(\varpi)=\begin{cases}2 & \text{if } \varpi\in\{2\}\cup(7,8),\\ 2.3 & \text{if } \varpi\in(2,7],\end{cases}$$

$$\mathscr{G}(\varpi)=\begin{cases}2 & \text{if } \varpi\in\{2\}\cup(7,8),\\ 2.5 & \text{if } \varpi\in(2,7],\end{cases}$$

$$\mathscr{T}(\varpi)=\begin{cases}2 & \text{if } \varpi\in\{2\},\\ 4 & \text{if } \varpi\in(2,7),\\ 5 & \text{if } \varpi\in\{7\},\\ \frac{\varpi+3}{5} & \text{if } \varpi\in(7,8),\end{cases}$$

$$\mathscr{S}(\varpi)=\begin{cases}2 & \text{if } \varpi\in\{2\},\\ 6 & \text{if } \varpi\in(2,7),\\ 7 & \text{if } \varpi\in\{7\},\\ \frac{\varpi+3}{5} & \text{if } \varpi\in(7,8),\end{cases}$$

and $\psi(u,v)=\sqrt{uv}$ where $u,v\in E$. One can easily validate that inequality (5) is satisfied for every ϖ,ω in E and for all $\varkappa>0$. Now, we pick sequences $\{\varpi_n\}=\left\{7+\frac{1}{n}\right\}$ and $\{\omega_n\}=\{2\}$. It is clear that we have

$$\lim_{n\to\infty}\mathscr{F}\varpi_n=\lim_{n\to\infty}\mathscr{T}\varpi_n=\lim_{n\to\infty}\mathscr{G}\omega_n=\lim_{n\to\infty}\mathscr{S}\omega_n=2.$$

Since $2\in\mathscr{T}E\cap\mathscr{S}E$, it implies that both pairs $(\mathscr{F},\mathscr{T})$ and $(\mathscr{G},\mathscr{S})$ satisfy ($\text{CLR}_{\mathscr{T}\mathscr{S}}$) property. Furthermore, it is straightforward to verify that both pairs $(\mathscr{F},\mathscr{T})$ and $(\mathscr{G},\mathscr{S})$ are weakly compatible. Hence, each conditions of Theorem 2 hold. Furthermore, 2 is the unique common fixed point of $\mathscr{F},\mathscr{G},\mathscr{T}$ and $\mathscr{S}$. Figures 1, 2 and 3 provide a visual representation of the inequality with specific assigned values.

Remark. It is obvious that Theorem 3 cannot be applied on example above because both $\mathscr{T}E$, $\mathscr{S}E\subset E$ are not closed.

Before we proceed further, we present two lemmas that are needed the next results related to four mappings but only two mappings satisfying (CLR) or (E.A.) property, respectively.

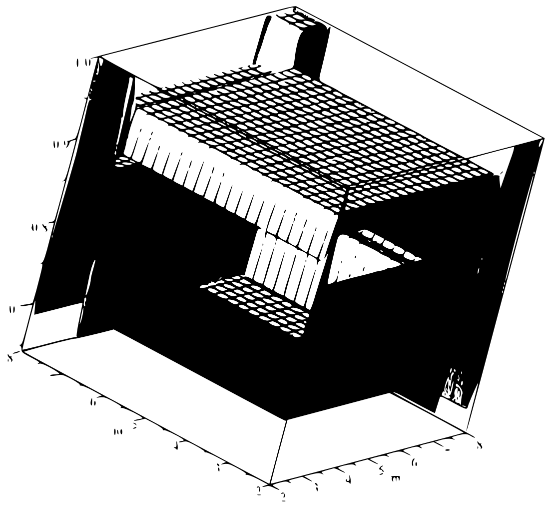

Fig. 1: Graphical view of inequality $\Gamma(\mathscr{F}\varpi,\mathscr{G}\omega,\varkappa) \geq \psi\left(\Gamma\left(\mathscr{T}\varpi,\mathscr{F}\varpi,\frac{\varkappa_1}{p}\right),\Gamma\left(\mathscr{S}\omega,\mathscr{G}\omega,\frac{\varkappa_2}{q}\right)\right)$, where the orange plane represents the left-hand side and the blue plane represents the right-hand side, with specific values assigned as follows: $\varkappa = 5, \varkappa_1 = 3, \varkappa_2 = 2, p = 0.5$, and $q = 0.3$.

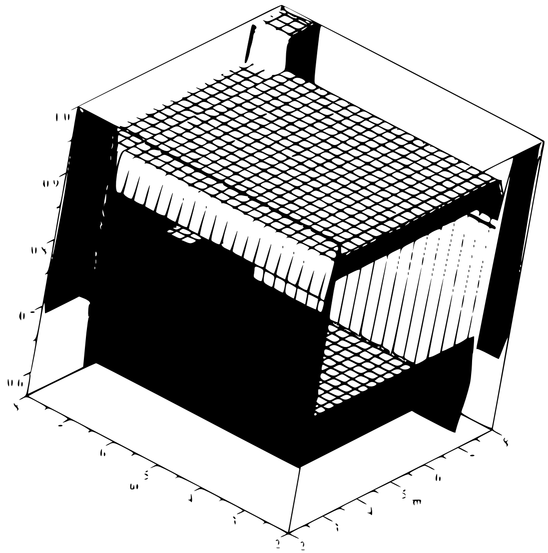

Fig. 2: Graphical view of inequality $\Gamma(\mathscr{F}\varpi,\mathscr{G}\omega,\varkappa) \geq \psi\left(\Gamma\left(\mathscr{T}\varpi,\mathscr{F}\varpi,\frac{\varkappa_1}{p}\right),\Gamma\left(\mathscr{S}\omega,\mathscr{G}\omega,\frac{\varkappa_2}{q}\right)\right)$, where the orange plane represents the left-hand side and the blue plane represents the right-hand side, with specific values assigned as follows: $\varkappa = 5, \varkappa_1 = 1, \varkappa_2 = 4, p = 0.5$, and $q = 0.3$.

Lemma 2. *Suppose that $(E,\Gamma,*)$ is a fuzzy metric space and $\mathscr{F},\mathscr{G},\mathscr{T},\mathscr{S}$ are four self-mappings of E such that the following conditions hold:*

1. the pair $(\mathscr{F},\mathscr{T})$ (or $(\mathscr{G},\mathscr{S})$) satisfies the $(CLR_{\mathscr{T}})$ (or $(CLR_{\mathscr{S}})$) property;

2. $\mathscr{F}E \subset \mathscr{S}E$ (or $\mathscr{G}E \subset \mathscr{T}E$);

3. $\mathscr{S}E \subset E$ closed;

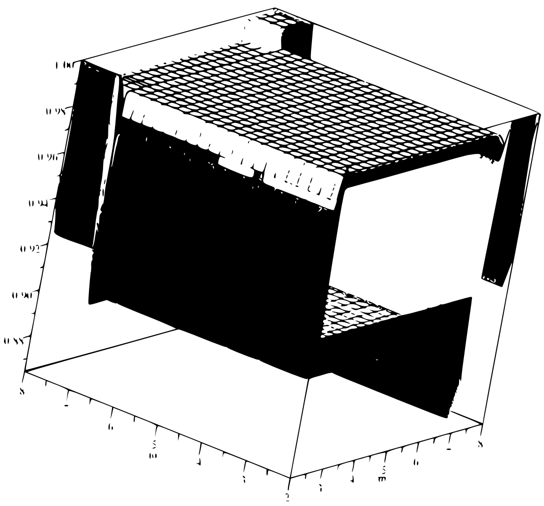

Fig. 3: Graphical view of inequality $\Gamma(\mathscr{F}\varpi,\mathscr{G}\omega,\varkappa) \geq \psi\left(\Gamma\left(\mathscr{T}\varpi,\mathscr{F}\varpi,\frac{\varkappa_1}{p}\right),\Gamma\left(\mathscr{S}\omega,\mathscr{G}\omega,\frac{\varkappa_2}{q}\right)\right)$, where the orange plane represents the left-hand side and the blue plane represents the right-hand side, with specific values assigned as follows: $\varkappa = 30, \varkappa_1 = 5, \varkappa_2 = 25, p = 0.5$, and $q = 0.3$.

4. *$\{\mathscr{G}\omega_n\}$ converges for all sequences $\{\omega_n\}$ in E provided $\{\mathscr{S}\omega_n\}$ converges (or $\{\mathscr{F}\omega_n\}$ converges for all sequences $\{\omega_n\}$ in E provided $\{\mathscr{T}\omega_n\}$ converges);*

5. *$\mathscr{F},\mathscr{G},\mathscr{T}$ and $\mathscr{S}$ satisfy inequality (5) for every $\varpi,\omega \in E$ and any $\varkappa > 0$.*

Then, both pairs $(\mathscr{F},\mathscr{T})$ and $(\mathscr{G},\mathscr{S})$ satisfy the $(CLR_{\mathscr{T}\mathscr{S}})$ property.

Proof. As $(\mathscr{F},\mathscr{T})$ satisfy ($CLR_{\mathscr{T}}$) property, there is $\{\varpi_n\} \subset E$ satisfy

$$\lim_{n\to\infty}\Gamma(\mathscr{F}\varpi_n,z,\varkappa) = \lim_{n\to\infty}\Gamma(\mathscr{T}\varpi_n,z,\varkappa) = 1$$

for some $z \in \mathscr{T}E$. This means that

$$\lim_{n\to\infty}\mathscr{F}\varpi_n = \lim_{n\to\infty}\mathscr{T}\varpi_n = z.$$

Since $\mathscr{F}E \subset \mathscr{S}E$, for each ϖ_n, there is an element $\omega_n \in E$ satisfy $\mathscr{F}\varpi_n = \mathscr{S}\omega_n$ for every $n \in \mathbb{N}$. Thus, we yield

$$\lim_{n\to\infty}\mathscr{F}\varpi_n = \lim_{n\to\infty}\mathscr{S}\omega_n = z.$$

Since $\mathscr{S}E$ is closed, the convergent point z is in $\mathscr{S}E$. Therefore we have $z \in \mathscr{T}E \cap \mathscr{S}E$ and

$$\mathscr{F}\varpi_n \to z, \mathscr{T}\varpi_n \to z, \text{ and } \mathscr{S}\omega_n \to z$$

as we let $n \to \infty$. Due to condition (4), sequence $\{\mathscr{G}\omega_n\}$ converges, which means, there is a point $\theta \in E$ satisfy

$$\lim_{n\to\infty}\mathscr{G}\omega_n = \theta.$$

We claim that $\theta = z$. Otherwise, let $\theta \neq z$. This implies that $0 < \Gamma(\theta,z,\varkappa) < 1$ for every $\varkappa > 0$. Using inequality (5), we have

$$\Gamma(\mathscr{F}\varpi_n,\mathscr{G}\omega_n,\varkappa) \geq \psi\left(\Gamma\left(\mathscr{T}\varpi_n,\mathscr{F}\varpi_n,\frac{\varkappa_1}{p}\right),\Gamma\left(\mathscr{S}\omega_n,\mathscr{G}\omega_n,\frac{\varkappa_2}{q}\right)\right)$$

where $\varkappa_1,\varkappa_2 > 0$ with $\varkappa = \varkappa_1 + \varkappa_2$, $p,q > 0$ with $p+q \in (0,1)$ and $\psi \in \Psi_f$. Let $\varkappa_1 = \frac{p\varkappa}{p+q}, \varkappa_2 = \frac{q\varkappa}{p+q}$ and $r = p+q$, the inequality above can be rewrite as

$$\Gamma(\mathscr{F}\varpi_n,\mathscr{G}\omega_n,\varkappa) \geq \psi\left(\Gamma\left(\mathscr{T}\varpi_n,\mathscr{F}\varpi_n,\frac{\varkappa}{r}\right),\Gamma\left(\mathscr{S}\omega_n,\mathscr{G}\omega_n,\frac{\varkappa}{r}\right)\right).$$

Let $n \to \infty$,

$$\begin{aligned}\Gamma(z,\theta,\varkappa) &\geq \psi\left(\Gamma\left(z,z,\frac{\varkappa}{r}\right),\Gamma\left(z,\theta,\frac{\varkappa}{r}\right)\right)\\ &= \psi\left(1,\Gamma\left(z,\theta,\frac{\varkappa}{r}\right)\right).\end{aligned}$$

Due to the properties of Ψ-function and Lemma 1, it follows that

$$\begin{aligned}\Gamma(z,\theta,\varkappa) &\geq \psi\left(1,\Gamma\left(z,\theta,\frac{\varkappa}{r}\right)\right)\\ &\geq \psi\left(\Gamma\left(z,\theta,\frac{\varkappa}{r}\right),\Gamma\left(z,\theta,\frac{\varkappa}{r}\right)\right)\\ &> \Gamma\left(z,\theta,\frac{\varkappa}{r}\right)\\ &> \Gamma(z,\theta,\varkappa)\end{aligned}$$

which leads to a contradiction. As a result, $\Gamma(w,z,\varkappa)=1$ for any $\varkappa>0$ which means $\theta=z$. Hence, we conclude

$$\lim_{n\to\infty}\Gamma(\mathscr{F}\varpi_n,z,\varkappa)=\lim_{n\to\infty}\Gamma(\mathscr{T}\varpi_n,z,\varkappa)=\lim_{n\to\infty}\Gamma(\mathscr{G}\omega_n,z,\varkappa)=\lim_{n\to\infty}\Gamma(\mathscr{S}\omega_n,z,\varkappa)=1$$

which means that both $(\mathscr{F},\mathscr{T}),(\mathscr{G},\mathscr{S})$ satisfy (CLR$_{\mathscr{T}\mathscr{S}}$) property.

Lemma 3. *Supose that $(E,\Gamma,*)$ is a fuzzy metric space and $\mathscr{F},\mathscr{G},\mathscr{T},\mathscr{S}$ are four self-mappings of E such that the following conditions hold:*

1. the pair $(\mathscr{F},\mathscr{T})$ (or $(\mathscr{G},\mathscr{S})$) satisfies (E.A.) property;

2. $\mathscr{F}E\subset\mathscr{S}E$ (or $\mathscr{G}E\subset\mathscr{T}E$);

3. $\{\mathscr{G}\omega_n\}$ converges for all sequences $\{\omega_n\}$ in E provided $\{\mathscr{S}\omega_n\}$ converges (or $\{\mathscr{F}\omega_n\}$ converges for all sequences $\{\omega_n\}$ in E provided $\{\mathscr{T}\omega_n\}$ converges);

4. $\mathscr{F},\mathscr{G},\mathscr{T}$ and $\mathscr{S}$ satisfy inequality (5) for every $\varpi,\omega\in E$ and any $\varkappa>0$.

Then, both pairs $(\mathscr{F},\mathscr{T})$, $(\mathscr{G},\mathscr{S})$ satisfy common (E.A.) property.

Proof. The proof is similar to Lemma 2 so we omit here to avoid repetition.

Theorem 4. *Suppose that $(E,\Gamma,*)$ is a fuzzy metric space and $\mathscr{F},\mathscr{G},\mathscr{S},\mathscr{T}$ are self-mappings of E satisfy the following conditions:*

1. the pair $(\mathscr{F},\mathscr{T})$ (or $(\mathscr{G},\mathscr{S})$) satisfies the (CLR$_{\mathscr{T}}$) (or (CLR$_{\mathscr{S}}$)) property;

2. $\mathscr{F}E\subset\mathscr{S}E$ (or $\mathscr{G}E\subset\mathscr{T}E$);

3. $\{\mathscr{G}\omega_n\}$ converges for all sequences $\{\omega_n\}$ in E provided $\{\mathscr{S}\omega_n\}$ converges (or $\{\mathscr{F}\omega_n\}$ converges for all sequences $\{\omega_n\}$ in E provided $\{\mathscr{T}\omega_n\}$ converges);

4. mappings $\mathscr{F},\mathscr{G},\mathscr{T}$ and $\mathscr{S}$ satisfy

$$\Gamma(\mathscr{F}\varpi,\mathscr{G}\omega,\varkappa)\geq\psi\left(\Gamma\left(\mathscr{T}\varpi,\mathscr{F}\varpi,\frac{\varkappa_1}{p}\right),\Gamma\left(\mathscr{S}\omega,\mathscr{G}\omega,\frac{\varkappa_2}{q}\right)\right)$$

for every $\varpi,\omega\in E$ and any $\varkappa>0$ where $\varkappa_1,\varkappa_2>0$ with $\varkappa=\varkappa_1+\varkappa_2$, $p,q>0$ with $p+q\in(0,1)$ and $\psi\in\Psi_f$.

Then, both pairs $(\mathscr{F},\mathscr{T})$ and $(\mathscr{G},\mathscr{S})$ have a coincidence point. Furthermore, if both pairs $(\mathscr{F},\mathscr{T})$ and $(\mathscr{G},\mathscr{S})$ are weakly compatible, this implies that mappings $\mathscr{F},\mathscr{T},\mathscr{G},\mathscr{S}$ have a unique common fixed point.

Proof. By Lemma 2, both pairs $(\mathscr{F},\mathscr{T}),(\mathscr{G},\mathscr{S})$ satisfy (CLR$_{\mathscr{T}\mathscr{S}}$) property. Hence, there are $\{\varpi_n\},\{\omega_n\}\subset E$ such that for all $\varkappa>0$,

$$\lim_{n\to\infty}\Gamma(\mathscr{F}\varpi_n,z,\varkappa)=\lim_{n\to\infty}\Gamma(\mathscr{T}\varpi_n,z,\varkappa)=\lim_{n\to\infty}\Gamma(\mathscr{G}\omega_n,z,\varkappa)=\lim_{n\to\infty}\Gamma(\mathscr{S}\omega_n,z,\varkappa)=1$$

for some $z\in\mathscr{T}E\cap\mathscr{S}E$. The remaining of this proof follows from Theorem 2.

Theorem 5. *Suppose that $(E,\Gamma,*)$ is a fuzzy metric space and $\mathscr{F},\mathscr{G},\mathscr{S},\mathscr{T}$ are self-mappings of E satisfy the following conditions:*

1. *the pair $(\mathscr{F},\mathscr{T})$ (or $(\mathscr{G},\mathscr{S})$) satisfies the (E.A.) property;*
2. *$\mathscr{F}E \subset \mathscr{S}E$ (or $\mathscr{G}E \subset \mathscr{T}E$);*
3. *$\{\mathscr{G}\omega_n\}$ converges for all sequences $\{\omega_n\}$ in E provided $\{\mathscr{S}\omega_n\}$ converges (or $\{\mathscr{F}\omega_n\}$ converges for all sequences $\{\omega_n\}$ in E provided $\{\mathscr{T}\omega_n\}$ converges);*
4. *mappings $\mathscr{F},\mathscr{G},\mathscr{T}$ and $\mathscr{S}$ satisfy*

$$\Gamma(\mathscr{F}\varpi,\mathscr{G}\omega,\varkappa) \geq \psi\left(\Gamma\left(\mathscr{T}\varpi,\mathscr{F}\varpi,\frac{\varkappa_1}{p}\right),\Gamma\left(\mathscr{S}\omega,\mathscr{G}\omega,\frac{\varkappa_2}{q}\right)\right)$$

for every $\varpi,\omega \in E$ and any $\varkappa > 0$ where $\varkappa_1,\varkappa_2 > 0$ with $\varkappa = \varkappa_1+\varkappa_2$, $p,q > 0$ with $p+q \in (0,1)$ and $\psi \in \Psi_f$.

Then, both pairs $(\mathscr{F},\mathscr{T})$ and $(\mathscr{G},\mathscr{S})$ have a coincidence point. Furthermore, if both pairs $(\mathscr{F},\mathscr{T})$ and $(\mathscr{G},\mathscr{S})$ are weakly compatible, this implies that mappings $\mathscr{F},\mathscr{T},\mathscr{G},\mathscr{S}$ have a unique common fixed point.

Proof. In view of Lemma 3, both $(\mathscr{F},\mathscr{T})$ and $(\mathscr{G},\mathscr{S})$ satisfy common (E.A.) property. Hence, there are $\{\varpi_n\}$, $\{\omega_n\} \subset E$ such that for all $\varkappa > 0$,

$$\lim_{n\to\infty}\Gamma(\mathscr{F}\varpi_n,z,\varkappa) = \lim_{n\to\infty}\Gamma(\mathscr{T}\varpi_n,z,\varkappa) = \lim_{n\to\infty}\Gamma(\mathscr{G}\omega_n,z,\varkappa) = \lim_{n\to\infty}\Gamma(\mathscr{S}\omega_n,z,\varkappa) = 1$$

for some $z \in E$. The remaining of this proof follows from Theorem 3.

Theorem 6. *Suppose that $(E,\Gamma,*)$ is a fuzzy metric space and $\mathscr{F},\mathscr{G},\mathscr{S},\mathscr{T}$ are self-mappings of E satisfy inequality (5) for every for every $\varpi,\omega \in E$ and any $\varkappa > 0$. Assume the pair $(\mathscr{F},\mathscr{T})$ satisfy (CLR$_{(\mathscr{F},\mathscr{T}),\mathscr{S}}$) property, then both pairs $(\mathscr{F},\mathscr{T})$ and $(\mathscr{G},\mathscr{S})$ have a coincidence point. Furthermore, if both pairs $(\mathscr{F},\mathscr{T})$ and $(\mathscr{G},\mathscr{S})$ are weakly compatible, this implies that mappings $\mathscr{F},\mathscr{T},\mathscr{G},\mathscr{S}$ have a unique common fixed point.*

Proof. Consider the pair $(\mathscr{F},\mathscr{T})$ satisfy (CLR$_{(\mathscr{F},\mathscr{T}),\mathscr{S}}$) property, we have a sequence $\{\varpi_n\} \in E$ such that for each $\varkappa > 0$,

$$\lim_{n\to\infty}\Gamma(\mathscr{F}\varpi_n,z,\varkappa) = \lim_{n\to\infty}\Gamma(\mathscr{T}\varpi_n,z,\varkappa) = 1$$

for some $z \in \mathscr{T}E \cap \mathscr{S}E$. This means that

$$\lim_{n\to\infty}\mathscr{F}\varpi_n = \lim_{n\to\infty}\mathscr{T}\varpi_n = z.$$

As $z \in \mathscr{S}E$, there is an element u in E satisfy $z = \mathscr{S}u$. We will show that $\mathscr{G}u = \mathscr{S}u$. Assume $\mathscr{G}u \neq \mathscr{S}u$, which means, $0 < \Gamma(\mathscr{G}u,\mathscr{S}u,\varkappa) < 1$ for some $\varkappa > 0$. Using inequality (5), for any $\varkappa > 0$, it leads to

$$\Gamma(\mathscr{F}\varpi_n,\mathscr{G}u,\varkappa) \geq \psi\left(\Gamma\left(\mathscr{T}\varpi_n,\mathscr{F}\varpi_n,\frac{\varkappa_1}{p}\right),\Gamma\left(\mathscr{S}u,\mathscr{G}u,\frac{\varkappa_2}{q}\right)\right). \tag{6}$$

Let $\varkappa_1 = \frac{p\varkappa}{p+q}, \varkappa_2 = \frac{q\varkappa}{p+q}$ and $r = p+q$. Then, we obtain

$$\Gamma(\mathscr{F}\varpi_n,\mathscr{G}u,\varkappa) \geq \psi\left(\Gamma\left(\mathscr{T}\varpi_n,\mathscr{F}\varpi_n,\frac{\varkappa}{r}\right),\Gamma\left(\mathscr{S}u,\mathscr{G}u,\frac{\varkappa}{r}\right)\right).$$

As we let $n \to \infty$, it follows that

$$\begin{aligned}\Gamma(\mathscr{S}u,\mathscr{G}u,\varkappa) &\geq \psi\left(\Gamma\left(z,z,\frac{\varkappa}{r}\right),\Gamma\left(\mathscr{S}u,\mathscr{G}u,\frac{\varkappa}{r}\right)\right)\\ &= \psi\left(1,\Gamma\left(\mathscr{S}u,\mathscr{G}u,\frac{\varkappa}{r}\right)\right).\end{aligned}$$

Due to properties of Ψ-function and Lemma 1, we yield

$$\begin{aligned}\Gamma(\mathscr{S}u,\mathscr{G}u,\varkappa) &\geq \psi\left(1,\Gamma\left(\mathscr{S}u,\mathscr{G}u,\frac{\varkappa}{r}\right)\right)\\ &\geq \psi\left(\Gamma\left(\mathscr{S}u,\mathscr{G}u,\frac{\varkappa}{r}\right),\Gamma\left(\mathscr{S}u,\mathscr{G}u,\frac{\varkappa}{r}\right)\right)\\ &> \Gamma\left(\mathscr{S}u,\mathscr{G}u,\frac{\varkappa}{r}\right)\\ &> \Gamma(\mathscr{S}u,\mathscr{G}u,\varkappa)\end{aligned}$$

which leads to a contradiction. Therefore, $\Gamma(\mathscr{S}u,\mathscr{G}u,\varkappa) = 1$ for any $\varkappa > 0$, which means, $\mathscr{G}u = \mathscr{S}u = z$. So u is a coincidence point of pair $(\mathscr{G},\mathscr{S})$.

Moreover, since $z \in \mathscr{T}E$, we can find an element v in E satisfy $z = \mathscr{T}v$. We will validate that $\mathscr{F}v = \mathscr{T}v$. Assume $\mathscr{F}v \neq \mathscr{T}v$, which means, $0 < \Gamma(\mathscr{F}v,\mathscr{T}v,\varkappa) < 1$ for some $\varkappa > 0$. Using inequality (5), for all $\varkappa > 0$, it leads to

$$\Gamma(\mathscr{F}v,\mathscr{G}u,\varkappa) \geq \psi\left(\Gamma\left(\mathscr{T}v,\mathscr{F}v,\frac{\varkappa_1}{p}\right),\Gamma\left(\mathscr{S}u,\mathscr{G}u,\frac{\varkappa_2}{q}\right)\right). \tag{7}$$

Again let $\varkappa_1 = \frac{p\varkappa}{p+q}, \varkappa_2 = \frac{q\varkappa}{p+q}$ and $r = p+q$. Then, we obtain

$$\Gamma(\mathscr{F}v,\mathscr{G}u,\varkappa) \geq \psi\left(\Gamma\left(\mathscr{T}v,\mathscr{F}v,\frac{\varkappa}{r}\right),\Gamma\left(\mathscr{S}u,\mathscr{G}u,\frac{\varkappa}{r}\right)\right) \quad = \psi\left(\Gamma\left(\mathscr{T}v,\mathscr{F}v,\frac{\varkappa}{r}\right),1\right).$$

Since $\mathscr{G}u = z = \mathscr{T}v$, by Ψ-function's properties and Lemma 1, it follows that

$$\begin{aligned}\Gamma(\mathscr{F}v,\mathscr{T}v,\varkappa) &\geq \psi\left(\Gamma\left(\mathscr{T}v,\mathscr{F}v,\frac{\varkappa}{r}\right),1\right)\\ &\geq \psi\left(\Gamma\left(\mathscr{T}v,\mathscr{F}v,\frac{\varkappa}{r}\right),\Gamma\left(\mathscr{T}v,\mathscr{F}v,\frac{\varkappa}{r}\right)\right)\\ &> \Gamma\left(\mathscr{T}v,\mathscr{F}v,\frac{\varkappa}{r}\right)\\ &> \Gamma(\mathscr{T}v,\mathscr{F}v,\varkappa)\\ &= \Gamma(\mathscr{F}v,\mathscr{T}v,\varkappa)\end{aligned}$$

which leads to a contradiction. As a result, $\Gamma(\mathscr{F}v,\mathscr{T}v,\varkappa) = 1$ for any $\varkappa > 0$, for instance, $\mathscr{F}v = \mathscr{T}v = z$. So v is a coincidence point of the pair $(\mathscr{F},\mathscr{T})$.

As $(\mathscr{F},\mathscr{T})$ is weakly compatible and $\mathscr{F}v = \mathscr{T}v$, these lead to $\mathscr{T}z = \mathscr{T}\mathscr{F}v = \mathscr{F}\mathscr{T}v = \mathscr{F}z$. We say that z is a common fixed point of $(\mathscr{F},\mathscr{T})$. Using inequality (5) and the property of Ψ-function, for any $\varkappa > 0$, we obtain

$$\begin{aligned}\Gamma(\mathscr{F}z,z,\varkappa) = \Gamma(\mathscr{F}z,\mathscr{G}u,\varkappa) &\geq \psi\left(\Gamma\left(\mathscr{T}z,\mathscr{F}z,\frac{\varkappa_1}{p}\right),\Gamma\left(\mathscr{S}u,\mathscr{G}u,\frac{\varkappa_2}{q}\right)\right)\\ &= \psi\left(\Gamma\left(\mathscr{F}z,\mathscr{F}z,\frac{\varkappa_1}{p}\right),\Gamma\left(z,z,\frac{\varkappa_2}{q}\right)\right)\\ &= \psi(1,1)\\ &= 1.\end{aligned}$$

Thus, $\Gamma(\mathscr{F}z,z,\varkappa) = 1$ for all $\varkappa > 0$,that is, $\mathscr{F}z = z = \mathscr{T}z$. Therefore, z is a common fixed point of the pair $(\mathscr{F},\mathscr{T})$.

Also, as the pair $(\mathscr{G},\mathscr{S})$ is weakly compatible and $\mathscr{G}u = \mathscr{S}u$, this implies that $\mathscr{S}z = \mathscr{S}\mathscr{G}u = \mathscr{G}\mathscr{S}u = \mathscr{G}z$. We say that z is a common fixed point of the pair $(\mathscr{G},\mathscr{S})$. Using inequality (5) and the property of Ψ-function, for any $\varkappa > 0$, we obtain

$$\begin{aligned}\Gamma(z,\mathscr{G}z,\varkappa) = \Gamma(\mathscr{F}z,\mathscr{G}z,\varkappa) &\geq \psi\left(\Gamma\left(\mathscr{T}z,\mathscr{F}z,\frac{\varkappa_1}{p}\right),\Gamma\left(\mathscr{S}z,\mathscr{G}z,\frac{\varkappa_2}{q}\right)\right)\\ &= \psi\left(\Gamma\left(z,z,\frac{\varkappa_1}{p}\right),\Gamma\left(\mathscr{G}z,\mathscr{G}z,\frac{\varkappa_2}{q}\right)\right)\\ &= \psi(1,1)\\ &= 1.\end{aligned}$$

Thus, $\Gamma(z,\mathscr{G}z,\varkappa) = 1$ for any $\varkappa > 0$, which means, $\mathscr{G}z = z = Sz$. So z is a common fixed point of the pair $(\mathscr{G},\mathscr{S})$. This shows that z is a common fixed point of mappings $\mathscr{F},\mathscr{G},\mathscr{T},\mathscr{S}$.

For the uniqueness, consider two common fixed points z_1, z_2 are distinct, which means that $0 < \Gamma(z_1,z_2,\varkappa) < 1$ for some $\varkappa > 0$. By inequality (5), for every $\varkappa > 0$, we have

$$\begin{aligned}\Gamma(z_1,z_2,\varkappa) &= \Gamma(\mathscr{F}z_1,\mathscr{G}z_2,\varkappa)\\ &\geq \psi\left(\Gamma\left(\mathscr{T}z_1,\mathscr{F}z_1,\frac{\varkappa_1}{p}\right),\Gamma\left(\mathscr{S}z_2,\mathscr{G}z_2,\frac{\varkappa_2}{q}\right)\right)\\ &= \psi\left(\Gamma\left(z_1,z_1,\frac{\varkappa_1}{p}\right),\Gamma\left(z_2,z_2,\frac{\varkappa_2}{q}\right)\right)\\ &= \psi(1,1)\\ &= 1\end{aligned}$$

which is contradict with our assumption. Thus, $z_1 = z_2$ which proves the common fixed point is unique.

4 Conclusion and Open Problem

Our paper generalized Kannan-type contractive mappings equipped with (CLR) or (E.A.) properties on fuzzy metric spaces and established several common fixed-point results of these mappings. Researchers can investigate the existence of fixed points for Kannan-type contractive mappings on more general setting, for example, fuzzy b-metric spaces, controlled fuzzy b-metric spaces, fuzzy bipolar metric spaces and etc. Additionally, Choudhury and Das [6] used h-coupled Kannan type mapping and obtained a common coupled fixed points for two mappings on partially ordered fuzzy metric space. This raise a question whether our results for four mappings able to expand to partially order fuzzy metric space. As mentioned in Section 1, Subrahmanyam [30] proved that the fixed point of Kannan-type contractive mappings implies the completeness for metric space. Therefore, we will end this paper with an open problem: Does the existence of fixed point for Kannan-type contractions imply the completeness on fuzzy metric space?

Declarations

Competing interests: The authors declare that there is no conflict of interest regarding the publication of this manuscript.
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