Jordan Journal of Mathematics and Statistics.
Yarmouk University

DOI:https://doi.org/10.47013/17.4.11

Variable Fluid Properties and Partial Slip Effect of MHD Flow of Nanofluids over a permeable Stretching Sheet with Heat Radiation and Viscous Dissipation

Khaled K. Jaber[1,*]

Department of Mathematics, Faculty of Science, Isra University, Amman, Jordan

Received: July 4, 2023 Accepted : Oct. 17, 2023

Abstract: The simultaneous effects of variable fluid properties and velocity slip on a nanofluid flow over a continuously stretching semi-infinite horizontal sheet are investigated. The contribution of thermal radiation term is taken into account in the temperature equation. Viscous dissipation, Brownian motion and thermophoresis effects are taking into account. The flow is subjected to a transverse uniform magnetic field. Thermal conductivity and fluid viscosity are assumed to related linearly and inversely linear with temperature, respectively. The system of equations governed the problem is transformed into a dimensionless system of ordinary differential equations. This system is solved using shooting technique in conjunction with Rung-Kuta method. Convergence of the obtained solutions is examined. It is found that the velocity improved due to increasing θ_r, whereas increasing t_v yields to decrease the velocity. Increasing volume fraction parameter φ and nanolayer thickness t_v tend to reduce the temperature. We discuss in detail the effect of involved parameters on the velocity, temperature, and concentration profiles. In addition, we provide discussions and tabulated data on skin friction, Nusselt number, and Sherwood numbers.

Keywords: MHD; Nanofluids; thermal radiation; variable fluid properties; viscous dissipation.

2010 Mathematics Subject Classification. ;

1 Introduction

Convection heat transfer in a fluid flow past a semi-infinite permeable continuously stretching surface nestled in a porous medium is of great importance due to its huge range of applications in industrial, environmental and engineering. P.S.Gupta and A.S.Gupta [1] investigated the suction and blowing effect on heat and mass transfer near a stretching sheet. Shanshan Yao et al. [2] derived the convective boundary condition effect on the boundary layer of a generalized stretching/shrinking wall. Liancun Zheng et al. [3] analyzed the time dependent flow and heat transfer with heat source or sink over a permeable stretching wall. The forced convection flow of a Jeffrey fluid past a continuously stretching plate was studied by Nemat Dalir [4]. Nanotechnology deals with the manipulation and control of materials with volume at 1 to 100 nanometers. The behavior of a material nano-particles differs from their behavior at a larger scale. Nanofluid is one of the most interesting applications of nanotechnology that has revolutionized various industries, including electronics, medicine, energy, and materials science. Choi et al. [5] is the first who introduced the word nanofluid. Magneto-hydrodynamics flow of a Nanofluid attract the attention of many researchers such as Hamad [6] who presented a solution of the MHD free convective flow of a nanofluid. The effect of stagnation-point on MHD heat transfer past a stretching surface was studied by many researchers. M. Turky ilmazoglu and I. Pop [7] studied the stagnation point on the heat transfer of a Jeffrey fluid flow past a continuously stretching plate. F. M. Ali et al. [8], G.K. Ramesh et al. [9] studied the problem with non-uniform source/sink. N. S. Akbar et al. [10] studied the heat transfer of Prandtl fluid past a shrinking plate. Behrouz Raftari and K.Vajravelu [11] presented numerical solution for the problem of MHD fluid flow in a stretchable wall channel using homotopy method. Umar Khan et al. [12] investigated stagnation point flow of a MHD nanofluid with thermal diffusing near a stretching sheet. Rana and Bhargava [13] used finite difference method to

* Corresponding author e-mail: khaledjaber4@yahoo.com

derive heat transfer of a nanofluid flow near a nonlinearly stretching surface. For more researches deal with this problem, see the following references [14,15,16]. Jaber [17] investigated the simultaneous effects of Hall currents and heat radiation on MHD flow of a fluid with variable properties near a vertical plate stretched uniformly. Due to the important role of viscous dissipation in processes subjected to big contrast of gravitational force, viscous dissipation has attracted the attention of many researchers. Fluid of variable properties considering viscous dissipation also studied by Tasawar Hayat et al. [18] and Jaber [19] who take into account the joule heating on MHD flow past a stretching vertical plate. Mohsen Sheikholeslami and Davood Domiri Ganji [20] considered the heat transfer of a nanofluid flow between parallel plates. S. A. Shehzad et al. [21] examined Newtonian heating and viscous dissipation effect on the third grade nanofluid flow. Swati Mukhopadhyay [22] studied the heat transfer past a wall stretched exponentially and embedded in a thermally stratifies medium. For more reading in thermal radiation and variable fluid properties, see the following reference [23,24,25,26,27]. Shravani et al. [28] used keller box numerical method to derive nanofluid flow. Swati Mukhopadhyay [29] studied slip effects, suction/blowing and thermal radiation on flow past a wall stretched exponentially. Wubshet Ibrahim and Bandari Shankar [30] consider the thermal, velocity and melt slip conditions on MHD nanofluid flow over a permeable stretching sheet. T. Hayat et al. [31] investigated hall current, ionslip abd Joule heating effects on peristaltic nanofluid flow. Jawad Raza et al. [32] derived heat and mass transfer of MHD nanofluid flow with slip effect in a rotating channel. Radiation and Joule heating effects on micropolar fluid was considered by M. Ramzan et al. [33]. Partial slip effect was studied by Tasawar Hayat et al. [34] who considered Ferro-fluid flow past rotating stretchable disks, Aminreza Noghrehabadi et al. [35] who considered nanofluid flow past continuously stretching surface and Das [36] who derived the flow past non-linearly moving sheet.

When large temperature differences and radiative heat transfer are encountered, the fluid properties deviate from stability, which makes Boussinesq's approximation inapplicable. Thus, researchers have gone deeper into exploring the effect of these factors on fluid flow behavior and heat transfer.

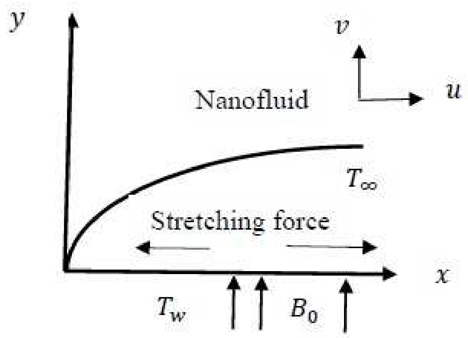

Fig. 1: Physical model and coordinate.

2 Mathematical Formulation

We consider a laminar steady state slip flow of a nanofluid over a permeable continuously stretching sheet. The fluid properties are taken to vary as follows. The thermal conductivity and temperature related linearly, while the fluid viscosity varies linearly with the reciprocal of the temperature. We considered the size and volume of nanoparticles, volume fraction and nanolayer, the viscous dissipation and heat radiation. We assume that the sheet slip velocity to be a constant multiple of x, the wall velocity is $u_w = ax$. A magnetic field with a uniform strength B_0 is applied transversely to the plate. Due to the small magnetic Reynolds number, we did not take the induced magnetic field into account. The x -axis is taken along the plate in the direction of flow, the y-axis perpendicular to it. The tangential and normal fluid velocities are u and v, respectively.

The continuity equation, the momentum equation, the energy equation and the concentration equation governing the problem under consideration are written as:

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0, \tag{1}$$

$$\rho_{nf}\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \frac{\partial}{\partial y}\left(\mu(T)\frac{\partial u}{\partial y}\right) - \sigma B_0^2 u - \frac{\mu_{nf}}{K'}u, \tag{2}$$

Deanship of Research and Graduate Studies, Yarmouk University, Irbid, Jordan.

$$(\rho C)_{nf}\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\frac{\partial}{\partial y}\left(K(T)\frac{\partial T}{\partial y}\right)+(\rho C)_p\left(D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right)+\mu_{nf}\left(\frac{\partial u}{\partial y}\right)^2-\frac{\partial q_r}{\partial y}, \tag{3}$$

$$\left(u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}\right)=D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2}. \tag{4}$$

The associated boundary conditions:

$$u=\delta u_w(x)+u_s, v=v_w(x), -k_f\frac{\partial T}{\partial y}=h(T_w-T_\infty), \quad C=C_w\text{asy}=0 \tag{5}$$

$$y=0u\to 0, v\to 0, T\to T_\infty, C\to C_\infty\text{asy}\to\infty$$

Where $D_B=\frac{KTC}{3\pi\mu d_p}$ and $u_w(x)$, $v_w(x)$ are the horizontal and vertical components of the variable velocity. The suction represented when $v_w(x)<0$, while $v_w(x)>0$ represents injection.

The fluid viscosity assumed as Ling and Dybbs [37] suggested of the form

$$\frac{1}{\mu}=\frac{1+\gamma(T-T_\infty)}{\mu_\infty}=a(T-T_r). \tag{6}$$

Where the value of the constants $a=\frac{\gamma}{\mu_\infty}$ and $T_r=T_\infty-\frac{1}{\gamma}$ depend on a reference state. In general, $a>0$ for liquid and $a<0$ for gasses. The constants μ_∞, γ and T_∞ are the dynamic viscosity, the fluid thermal property and fluid temperature away from the plate, respectively. The linear relation between thermal conductivity and temperature according to [37] is written as

$$K(T)=k_{nf}[1+\varepsilon\theta)]. \tag{7}$$

Here ε is the thermal conductivity factor. According to Rosseland approximation the radiation can be written as

$$q_r=-\frac{4\sigma^*}{3K^*}\frac{\partial T^4}{\partial y}. \tag{8}$$

Where K^* is the absorption coefficient. The temperature difference is expanded as a Taylor series of T^4 about T_∞ neglecting higher orders as

$$T^4\equiv 4T_\infty^3 T-3T_\infty^4. \tag{9}$$

The density of the nanofluid taking into consideration the base fluid density, the size and volume of nanoparticles, volume fraction and nanolayer according to Mohsen Sharifpur [38] is given as:

$$\rho_{nf}=\frac{(1-\varphi)\rho_f+\varphi\rho_p}{(1-\varphi)+\left(1+\frac{t_v}{r_p}\right)^3}, \tag{10}$$

the approximation of the nanolayer thickness t_v according to Mohsen Sharifpur [38] and μ_{nf} are given by:

$$t_v=-0.0002833r_p^2+0.0475r_p-0.1417, \tag{11}$$

where the average effective size of Fe_3O_4 particle in range of 21-123 nm, and the average effective size of Al_2O_3 particle size 39.1 nm. Also,

At constant pressure the nanofluid thermal conductivity and the specific heat k_{nf}, $(C_p)_{nf}$ respectively with reference to [39] are given by

$$(\rho C_p)_f=(1-\varphi)(\rho C_p)_{nf}+\varphi(\rho C_p)_s, \qquad \frac{k_{nf}}{k_f}=\frac{k_s+2k_f-2\varphi(k_f-k_s)}{k_s+2k_f+\varphi(k_f-k_s)}. \tag{12}$$

To non-dimensionalize the governing equations we introduce the following non dimensional variables and similarity transform:

$$\eta=y\sqrt{\frac{C(n+1)}{2\nu_f}}x^{\frac{n-1}{2}}, \quad \psi=\rho\sqrt{\frac{2C\nu_f}{n+1}x^{n+1}}f(\eta), \quad \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}, \quad \phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty}.$$

Where the stream function ψ satisfies the continuity equation so, $\frac{\partial\psi}{\partial y} = \rho u$ and $\frac{\partial\psi}{\partial x} = -\rho v$. Then the dimensionless temperature $\theta(\eta)$ can be written as

$$\theta(\eta) = \frac{T - T_r}{T_w - T_\infty} + \theta_r, \tag{13}$$

where $\theta_r = \frac{T_r - T_\infty}{T_w - T_\infty}$. Using this formula equation (6) can be written as

$$\mu_{nf} = \frac{\mu_f \theta_r}{\theta_r - \theta}. \tag{14}$$

Executing the previous similarity variables, we get:

$$u = cx^n f', v = -\sqrt{\frac{c(n+1)\nu_f}{2}} x^{\frac{n-1}{2}} \left(f + \frac{n-1}{n+1}\eta f' \right). \tag{15}$$

In view of the previous dimensionless quantities Equations (1) (4) reduced to the following system of non-dimensional ordinary differential equations

$$\left(1-\phi+\phi\frac{\rho_p}{\rho_f}\right)\left(\frac{2n}{n+1}f'^2 - ff'' - \frac{\theta_r}{\theta_r-\theta}\left(f''' + \frac{1}{\theta_r}f''\right)\right) + \left(1-\phi+\phi\left(1+\frac{t_v}{r_p}\right)^3\right)\left(M + \frac{\theta_r}{\theta_r-\theta}k\right)f' = 0, \tag{16}$$

$$\frac{k_{nf}}{k_f}\frac{1}{Pr}\left(\varepsilon\theta'^2 + (1+\varepsilon\theta)\theta''\right) + [(1-\phi)+\tau\phi] f\theta' + \left(N_b\theta'\phi' + N_t\theta'^2\right) + \frac{\theta_r}{\theta_r-\theta}Ec\, f''^2 - 4R\theta'' = 0, \tag{17}$$

$$f'\phi' + Sc\phi'' + Sr\theta'' = 0. \tag{18}$$

Where prime denotes differentiation with respect to η, $M = \frac{\sigma B_0^2 x}{\rho u_w}$ is the magnetic number, $k = \frac{2\nu_f\ x}{K\prime u_w(n+1)}$ is Porosity parameter, $Ec = \frac{u_w^2}{(T_w - T_\infty)(C_P)_f}$ is the Eckert number, $N_b = \frac{(\rho C_P)_f}{(\rho C_P)_{nf}} \frac{D_B(C_w - C_\infty)}{\nu_f}$, $N_t = \frac{(\rho C_P)_f}{(\rho C_P)_{nf}} \frac{D_T}{T_\infty \nu_f}(T_w - T_\infty)$ are the Brownian motion and thermophoresis parameters, $Sr = \frac{D_B K_T (T_w - T_\infty)}{T_m \upsilon (C_w - C_\infty)}$ is the Soret coefficient and $Sc = \frac{D_B}{\upsilon}$ is the Schmidt number. The non-dimensional fluid viscosity parameter θ_r and the thermal radiation parameter R are given by

$$\theta_r = \frac{T_r - T_\infty}{T_w - T_\infty} = \frac{-1}{\gamma(T_w - T_\infty)}, \qquad R = \frac{4\sigma^* T_\infty^3}{3\mu C_P k^*}. \tag{19}$$

θ_r is determined by thermal property of the fluid and temperature differences. Variable viscosity has a significant effect in case of large temperature differences, so small values of θ_r will be considered. The boundary conditions are transformed to the following non-dimensional form

$$f = S, f'(0) = \chi + \lambda f''(0),\ \theta'(0) = -\beta(1-\theta(0)), f'(\eta) = \theta(\eta) = \phi(\eta) = 0 \text{as} \eta \to \infty. \tag{20}$$

Where $S = \frac{-v_w}{\nu_f}\left[\frac{c(n+1)x^{n-1}}{2}\right]^{-\frac{1}{2}}$, $\beta = \frac{h}{k_f}\left[\frac{c(n+1)x^{n-1}}{2\nu_f}\right]^{-\frac{1}{2}}$.

The physically important quantities namely, the skin friction coefficient which is the resistant force acting on a particle moving in a fluid, Nusselt number and Sherwood number are expressed as:

$$C_f = \frac{2\tau_w}{\rho_f\, u_w^2}, \quad Nu = \frac{xq_w}{k_f(T_w - T_\infty)}, \quad Sh = \frac{xq_m}{D_B(C_w - C_\infty)} \tag{21}$$

Where τ_w is the wall shear stress, q_w is the wall heat flux, q_m is the mass flux, are given by:

$$\tau_w = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0}, \quad q_w = -k_{nf}\left.\frac{\partial T}{\partial y}\right|_{y=0}, \quad q_m = -D_B\left.\frac{\partial \phi}{\partial y}\right|_{y=0}. \tag{22}$$

Based on equations (21) and (22) the non-dimensional skin friction coefficient is expressed as

$$C_f = \frac{2\mu}{\rho_f\, u(x)^2}\left.\frac{\partial u}{\partial y}\right|_{y=0} = \sqrt{\frac{2x^{n-1}(n+1)\nu_f}{c}}\frac{\mu_{nf}}{\mu_f}f''(0). \tag{23}$$

The Nusselt number is given by

$$Nu = \frac{-k_{nf}x}{k_f(T_w - T_\infty)} \left.\frac{\partial T}{\partial y}\right|_{y=0} = -\sqrt{\frac{c\, x^{n-1}\,(n+1)}{2\nu_f}} \frac{K_{nf}}{K_f} \theta^{\prime}\,(0)\,. \tag{24}$$

Sherwood number is expressed as:

$$Sh = -\sqrt{\frac{cx^{n+1}\,(n+1)}{2\nu_f}} \phi'\,(0)\,. \tag{25}$$

Model	$C_P(Kg^{-1}K^{-1})$	$\rho(Kgm^{-3})$	$k(Wm^{-1}K^{-1})$
H_2O	4179	997.1	0.613
Al_2O_3	765	3970	40
Fe_3O_4	670	5180	9.7

Table 1: Thermo-physical properties of the water (base fluid) and nanoparticles (Al_2O_3 and Fe_3O_4) (see [40]).

3 Solution Scheme

The system of ordinary differential equations and boundary conditions (16)–(18) is solved to investigate the effect of all involved parameters numerically using Mathematica software to run the 4th order Runge-Kutta algorithm by shooting technique. To investigate the effect of the parameters under study, we fixed the parameter values $\theta_r = 1.2$, $R = 0.1$, $\varphi = 0.01, t_v = 0.01$, $Sr = 0.2$, $Pr = 0.72$, $Ec = 0.1$, $n = 0.1$ during the calculations and took different values for the parameter under consideration. To ensure convergence in the calculations, a criterion that the difference between any two consecutive iterations is less than 10^{-9} is employed.

4 Results And Discussions

The effect on velocity, heat transfer and mass transfer is presented graphically, the effect on skin friction, Nusselt number and Sherwood number is tabulated.

Figure 2 shows the effect of the Magnetic Number M on the flow velocity within the boundary layer. It is seen that increasing the magnetic number M decreases the dimensionless velocity $f\prime$. Figure 3 established the influence of the fluid viscosity parameter θ_r. It is seen that the fluid velocity increases with increasing values of θ_r. This behavior due to fact that the increasing in θ_r actually yields to decrease the dynamic viscosity as obvious from expression (14) so the fluid velocity will increase. Figure 4 shows the impact of the power index parameter n, it is clear that increasing n decreases the velocity. While it is evident from Figure 5 that the temperature decreases with an increase in values of R the thermal radiation parameter. Figure 6 plotted to study the effect of Eckert number on temperature transfer profile. It is perceived that increasing Eckert number significantly increases temperature transfer. Figure 7 reveals that the temperature transfer decreases due to increasing Prandtl number Pr. Figure 8 reveals that mass transfer enhanced due to the increase in the Sherwod number.

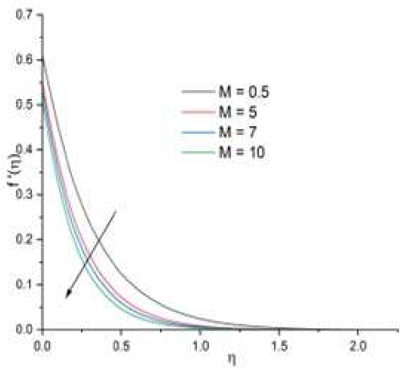

Fig. 2: Effect of M on the velocity profiles with $\theta_r = 1.2, R = 0.1, \varphi = 0.01, t_v = 0.01, Sr = 0.2, Pr = 0.72, Ec = 0.1, n = 0.1$.

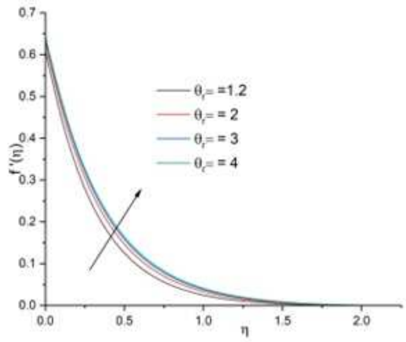

Fig. 3: Effect of θ_r on velocity profiles with $M = 0.5, R = 0.1, \varphi = 0.01, t_v = 0.01, Sr = 0.2, Pr = 0.72, Ec = 0.1, n = 0.1$.

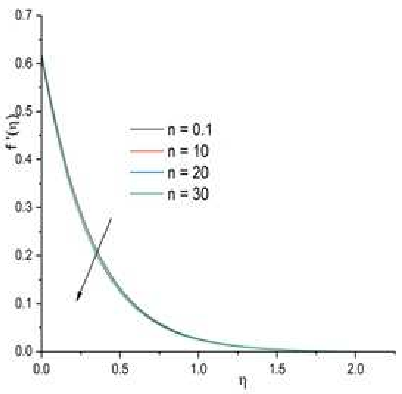

Fig. 4: Effect of n on velocity profiles with $\theta_r = 1.2, R = 0.1, \varphi = 0.01, t_v = 0.01, Sr = 0.2, Pr = 0.72, Ec = 0.1, n = 0.1, M = 0.5$.

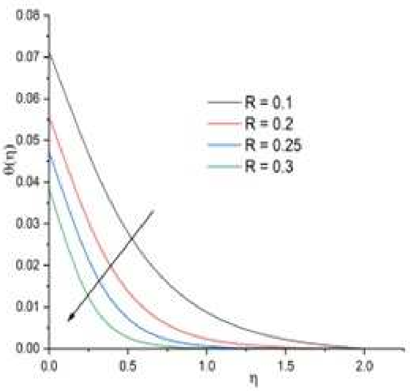

Fig. 5: Effect of R on temperature profiles with $\theta_r = 1.2, R = 0.1, \varphi = 0.01, t_v = 0.01, Sr = 0.2, Pr = 0.72, Ec = 0.1, n = 0.1, M = 0.5$.

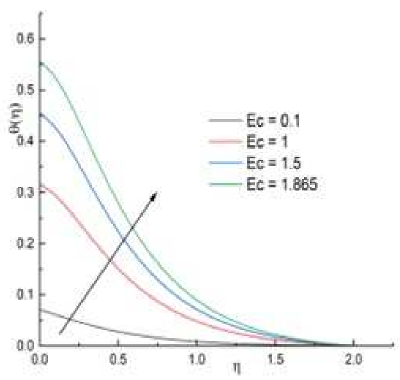

Fig. 6: Effect of Ec on temperature profiles with $\theta_r = 1.2, R = 0.1, \varphi = 0.01, t_v = 0.01, Sr = 0.2, Pr = 0.72, n = 0.1, M = 0.5$.

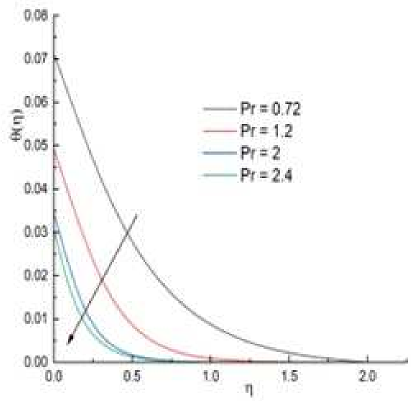

Fig. 7: Effect of Pr on temperature profiles with $\theta_r = 1.2, R = 0.1, \varphi = 0.01, t_v = 0.01, Sr = 0.2, Ec = 0.1, n = 0.1, M = 0.5$.

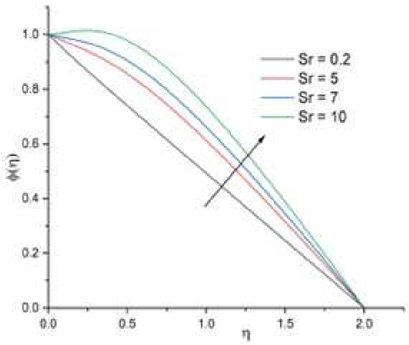

Fig. 8: Effect of Sr on concentration profiles with $\theta_r = 1.2$, $R = 0.1$, $\varphi = 0.01, t_v = 0.01$, $Pr = 0.72$, $Ec = 0.1$, $n = 0.1$, $M = 0.5$.

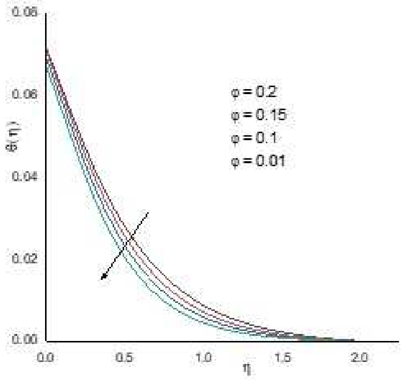

Fig. 9: Effect of φ on temperature profiles with $\theta_r = 1.2$, $R = 0.1$, $t_v = 0.01$, $Sr = 0.2$, $Pr = 0.72$, $Ec = 0.1$, $n = 0.1$, $M = 0.5$.

Figure 9 shows that the temperature transfer decreases by increasing φ. Figures 10–12 demonstrate the influence of the nanolayer thickness t_v. The impression of rising t_v has the same effect to decrease the velocity, temperature and concentration profiles. By comparing Figure 11 with Figures 10 and 12 it is clearly that nanolayer thickness t_v has greater effect in reducing fluid temperature.

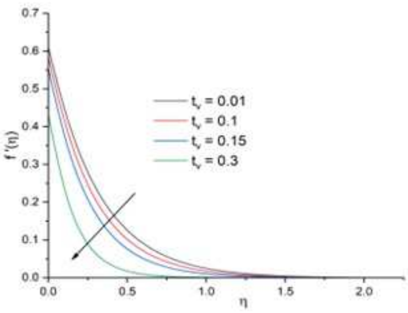

Fig. 10: Effect of t_v on velocity profiles with $\theta_r = 1.2$, $R = 0.1$, $\varphi = 0.01$, $Sr = 0.2$, $Pr = 0.72$, $Ec = 0.1$, $n = 0.1$, $M = 0.5$.

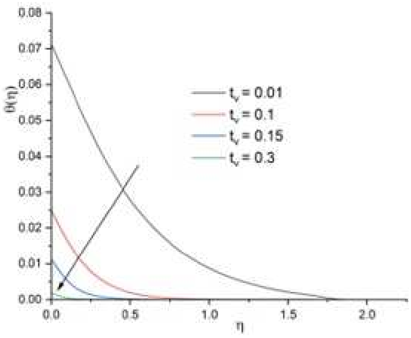

Fig. 11: Effect of t_v on temperature profiles with $\theta_r = 1.2$, $R = 0.1$, $\varphi = 0.01$, $Sr = 0.2$, $Pr = 0.72$, $Ec = 0.1$, $n = 0.1$, $M = 0.5$.

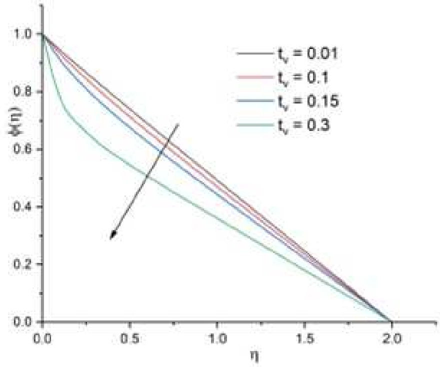

Fig. 12: Effect of t_v on concentration profiles with $\theta_r = 1.2$, $R = 0.1$, $\varphi = 0.01$, $Sr = 0.2$ $Pr = 0.72$, $Ec = 0.1$, $n = 0.1$, $M = 0.5$.

The effect of viscosity/temperature parameter θ_r, thermal radiation parameter R, Volume fraction parameter φ, nanolayer thickness t_v and soret coefficient Sr on skin-friction coefficient C_f, Nusselt number Nu and Sherwood number Sh are tabulated and presented in table 2. The numerical computations reveals that the improvement in φ increases the skin friction, Nusselt and Sherwood numbers, while these numbers decrease with increasing in nanolayer thickness t_v. The increase in viscosity/temperature parameter θ_r increases the skin friction and decreases Nusselt and Sherwood numbers. High thermal radiation parameter R and soret number Sr have the same effect in reducing Nusselt and Sherwood numbers and to increase skin friction coefficient.

θ_r	R	φ	t_v	Sr	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
1.2	0.1	0.01	0.01	0.2	−1.9608	0.092759	0.564379
2	0.1	0.01	0.01	0.2	−1.86046	0.09291	0.57281
3	0.1	0.01	0.01	0.2	−1.8105	0.09298	0.5771
1.2	0.2	0.01	0.01	0.2	−1.95997	0.09443	0.5623
1.2	0.3	0.01	0.01	0.2	−1.96989	0.09616	0.5597
1.2	0.1	0.15	0.01	0.2	−1.9496	0.09333	0.54969
1.2	0.1	0.2	0.01	0.2	−1.94801	0.09312	0.55271
1.2	0.1	0.2	0.15	0.2	−2.23533	0.09887	1.01484
1.2	0.1	0.01	0.3	0.2	−2.84538	0.099805	3.91634
1.2	0.1	0.01	0.01	7	−1.95065	0.092908	0.163023
1.2	0.1	0.01	0.01	10	−1.95072	0.092922	−0.014969

Table 2: Variation of skin friction, Nusselt number and Sherwood number with θ_r, R, φ and t_v

5 Conclusion and Remarks

The study derived the simultaneous effects of variable fluid properties on laminar steady state slip of a nanofluid flow past a permeable continuously stretching sheet. The flow passes through a uniform transverse magnetic field of strength B_0. The relation between thermal conductivity and temperature is considered linear, whereas the relation between fluid viscosity and temperature is assumed to be inversely linear. We assume the sheet slip velocity as a constant multiple of x, $u_w = ax$. To obtain accurate results, the density of the base fluid, the size and volume of the nanoparticles, the volume fraction and the nanoscale are taken into account to estimate the density of variable fluids. Using appropriate non-dimensional quantities and boundary layer assumptions the equations of the mathematical modeling non-dimensionalized then solved numerical using 4^{th} order Runge Kutta by shooting.

The computations shown that the flow velocity improved due to the increase in viscosity/temperature parameter θ_r, whereas growing Power index parameter n and nanolayer thickness t_v tend to depress the velocity. the temperature transfer increased due to the increase in Eckert number Ec. While the improvement in thermal radiation parameter R, Prandtl number Pr, Volume fraction parameter φ and nanolayer thickness t_v tend to reduce the temperature. the concentration improved due to the increase in soret number Sr. Growing t_v tend to reduce the concentration. the skin friction increased due to growing θ_r and φ, whereas it is decreased due to the increase in R, soret number Sr, and t_v. the Nusselt number decreased owing to the increase in R, soret number Sr, and t_v, whereas it is increased due to the increase in φ. the Sherwood number Sh increased by the growing of the values R, soret number Sr, whereas it is decreased owing to the increased in θ_r and θ_r.

Nomenclature

B_0	Constant magnetic field	X	Streamwise coordinate
c	Constant		**Greek Symbols**
C	Concentration	δ	Stretching/shrinking parameter
C_f	Coefficient of skin friction	ε	Thermal conductivity factor
C_p	Specific heat at constant pressure	ϕ	Dimensionless concentration
D_T	Thermodiffusion coefficient	η	Pseudo similar variable
Ec	Eckert number	φ	Volume fraction parameter
f	Dimensionless stream function	μ	Dynamical viscosity
g	Gravity acceleration	ν	Kinematical viscosity
Gr	Grashof number	θ	Dimensionless temperature
K	thermal conductivity	θ_r	Viscosity/temperature parameter
N	Power index parameter	ρ	Density
Nu	Nusselt number	σ	Stefan-Boltzmann constant
Pr	Prandtl number	τ	Shearing stress
r	Radius	ψ	Stream function
S	Heat source/sink parameter		**Subscripts**
Sc	Schmidt number	f	Base fluid
Sr	soret coefficient	nf	nanofluid
t_v	Nanolayer thickness	p	Solid particles
u	Flow velocity in x-direction	W	property at the wall
u_s	Slip velocity	∞	freestream condition
v	Flow velocity in y-direction		**Superscripts**
T	Temperature	$'$	differentiation with respect to η only

Declarations

Competing interests: The author declares no competing interests.
Authors' contributions: The author, Dr, Khaled Jaber, drafted the manuscript and do the final revision for publication.
Funding: No funding or grants was received.
Availability of data and materials: All data and conclusions used or analyzed in this study are available from the corresponding author, Dr, Khaled Jaber, upon reasonable request.
Acknowledgments:The author would like express his gratitude to the editor and referees for their valuable comments.

References

[1] P. S. Gupta, A. S. Gupta, Heat and mass transfer on a stretching sheet with suction or blowing, *the Canadian Journal of Chemical Engineering*, **55** (1977), 744.
[2] S. Yao, T. Fang, Y. Zhong, Heat transfer of a generalized stretching/shrinking wall problem with convective boundary conditions, *Communications in Nonlinear Science and Numerical Simulation*, **16(2)** (2011), 752–760.
[3] L. Zheng, L. Wang, X. Zhang, Analytic solutions of unsteady boundary flow and heat transfer on a permeable stretching sheet with non-uniform heat source/sink, *Communications in Nonlinear Science and Numerical Simulation*, **16(2)** (2011), 731–740.
[4] Nemat Dalir, Numerical study of entropy generation for forced convection flow and heat transfer of a Jeffrey fluid over a stretching sheet, *Alexandria Engineering Journal*, **53(4)** (2014), 769–778.
[5] M. Turkyilmazoglu, I.Pop, Exact analytical solutions for the flow and heat transfer near the stagnation point on a stretching/shrinking sheet in a Jeffrey fluid, *International Journal of Heat and Mass Transfer*, **57(1)** (2013), 82–88.
[6] M.A.A. Hamad, Analytical solution of natural convection flow of a nanofluid over a linearly stretching sheet in the presence of magnetic field, *International Communications in Heat and Mass Transfer*, **38(4)** (2011), 487–492.
[7] F. M. Ali, R. Nazar, N. M. Arifin, I. Pop, MHD stagnation-point flow and heat transfer towards stretching sheet with induced magnetic field, *Applied Mathematics and Mechanics*, **32(4)** (2011), 409–418.
[8] G.K. Ramesh, B.J. Gireesha, C.S. Bagewadi, MHD flow of a dusty fluid near the stagnation point over a permeable stretching sheet with non-uniform source/sink, *International Journal of Heat and Mass Transfer*, **55(17–18)** (2012), 4900–4907.

[9] N. S. Akbar, Z. H. Khan, R. U. Haq, S. Nadeem, Dual solutions in MHD stagnation-point flow of Prandtl fluid impinging on shrinking sheet, *Applied Mathematics and Mechanics*, **35(7)** (2014), 813–820.
[10] B. Raftari, K. Vajravelu, Homotopy analysis method for MHD viscoelastic fluid flow and heat transfer in a channel with a stretching wall, *Communications in Nonlinear Science and Numerical Simulation*, **17(11)** (2012), 4149–4162.
[11] S. U. S. Choi, Enhancing Thermal Conductivity of Fluids with Nanoparticles, *Proceedings, ASME International Mechanical Engineering Congress and Exposition*, San Francisco, Cal., USA, ASME, FED, MD **231** (1995), 99–105.
[12] U. Khan, N. Ahmed, S. I. U. Khan, S. T. Mohyud-din, Thermo-diffusion effects on MHD stagnation point flow towards a stretching sheet in a nanofluid, *Propulsion and Power Research*, **3(3)** (2014), 151–158.
[13] P. Rana, R. Bhargava, Flow and heat transfer of a nanofluid over a nonlinearly stretching sheet: A numerical study, *Communications in Nonlinear Science and Numerical Simulation*, **17(1)** (2012), 212–226.
[14] M. Govindaraju, N. Vishnu Ganesh, B. Ganga, A.K. Abdul Hakeem, Entropy generation analysis of magneto hydrodynamic flow of a nanofluid over a stretching sheet, *Journal of the Egyptian Mathematical Society*, **23(2)** (2015), 429–434.
[15] N. S. Akbar, D. Tripathi, Z. H. Khan, O. A. Bg, A numerical study of magnetohydrodynamic transport of nanofluids over a vertical stretching sheet with exponential temperature-dependent viscosity and buoyancy effects, *Chemical Physics Letters*, **661** (2016), 20–30.
[16] M. Sheikholeslami, M. Gorji-Bandpy, D.D. Ganji, Lattice Boltzmann method for MHD natural convection heat transfer using nanofluid, *Powder Technology*, **254** (2014), 82–93.
[17] K. K. Jaber, Combined effects of Hall current and variable viscosity on Non-Newtonian MHD flow past a stretching vertical plate, *Journal of Advances in Mathematics*, **3(7)** (2014).
[18] K. K. Jaber, Effect of viscous dissipation and Joule heating on MHD flow of a fluid with variable properties past a stretching vertical plate, *European Scientific Journal*, **33(10)** (2014).
[19] M. Sheikholeslami, D. D. Ganji, Nanofluid flow and heat transfer between parallel plates considering Brownian motion using DTM, *Computer Methods in Applied Mechanics and Engineering*, **283** (2015), 651–663.
[20] T. Hayat, M. I. Khan, M. Waqas, T. Yasmeen, A. Alsaedi, Viscous dissipation effect in flow of magnetonanofluid with variable properties, *Journal of Molecular Liquids*, **222** (2016), 47–54.
[21] S. A. Shehzad, Tariq Hussain, T. Hayat, M. Ramzan, A. Alsaedi, Boundary layer flow of third grade nanofluid with Newtonian heating and viscous dissipation, *Journal of Central South University*, **22(1)** (2015), 360–367.
[22] S. Mukhopadhyay, MHD boundary layer flow and heat transfer over an exponentially stretching sheet embedded in a thermally stratified medium, *Alexandria Engineering Journal*, **52(3)** (2013), 259–265.
[23] M. Sheikholeslami, D. D. Ganji, M.Y. Javed, R. Ellahi, Effect of thermal radiation on magnetohydrodynamics nanofluid flow and heat transfer by means of two phase model, *Journal of Magnetism and Magnetic Materials*, **374** (2015), 36–43.
[24] M.M. Rashidi, N.Vishnu Ganesh, A.K. Abdul Hakeem, B. Ganga, Buoyancy effect on MHD flow of nanofluid over a stretching sheet in the presence of thermal radiation, *Journal of Molecular Liquids*, **198** (2014), 234–238.
[25] O.D.Makinde, W.A. Khan, J.R.Culham, MHD variable viscosity reacting flow over a convectively heated plate in a porous medium with thermophoresis and radiative heat transfer, *International Journal of Heat and Mass Transfer*, **93** (2016), 595–604.
[26] M. M. Rashidi, B. Rostami, N. Freidoonimehr, S. Abbasbandy, Free convective heat and mass transfer for MHD fluid flow over a permeable vertical stretching sheet in the presence of the radiation and buoyancy effects, *Ain Shams Engineering Journal*, **5(3)**, 901–912, September 2014.
[27] R. Cortell, MHD (magneto-hydrodynamic) flow and radiative nonlinear heat transfer of a viscoelastic fluid over a stretching sheet with heat generation/absorption, *Energy*, **74** (2014), 896905.
[28] I. Shravani, R. Dodda, J. Sucharitha, Boundary Layer Flow of Heat Transfer and Nanofluids Over a Nonlinear Stretching Sheet with Presence of Magnetic Field and Viscous Dissipation, *Journal of Nanofluids*, **6(3)** (2017), 567–573.
[29] S. Mukhopadhyay, Slip effects on MHD boundary layer flow over an exponentially stretching sheet with suction/blowing and thermal radiation, *Ain Shams Engineering Journal*, **4(3)** (2013), 485–491.
[30] W.t Ibrahim, B. Shankar, MHD boundary layer flow and heat transfer of a nanofluid past a permeable stretching sheet with velocity, thermal and solutal slip boundary conditions, *Computers & Fluids*, **75** (2013), 1–10.
[31] T. Hayat, M. Shafique, A. Tanveer, A. Alsaedi, Hall and ion slip effects on peristaltic flow of Jeffrey nanofluid with Joule heating, *Journal of Magnetism and Magnetic Materials*, **407** (2016), 51–59.
[32] J. Raza, A. M. Rohni, Z. Omar, M. Awais, Heat and mass transfer analysis of MHD nanofluid flow in a rotating channel with slip effects, *Journal of Molecular Liquids*, **219** (2016), 703–708.
[33] M. Ramzan, M. Farooq, T. Hayat, Jae Dong Chung, Radiative and Joule heating effects in the MHD flow of a micropolar fluid with partial slip and convective boundary condition, *Journal of Molecular Liquids*, **221** (2016), 394–400.
[34] T.r Hayat, S. Qayyum, M. Imtiaz, F. Alzahrani, A. Alsaedi, Partial slip effect in flow of magnetite-Fe3O4 nanoparticles between rotating stretchable disks, *Journal of Magnetism and Magnetic Materials*, **413** (2016), 39–48.
[35] A. Noghrehabadi, R. Pourrajab, M. Ghalambaz, Effect of partial slip boundary condition on the flow and heat transfer of nanofluids past stretching sheet prescribed constant wall temperature, *International Journal of Thermal Sciences*, **54** (2012), 253–261.
[36] K. Das, Nanofluid flow over a non-linear permeable stretching sheet with partial slip, *J Egypt Math Soc*, **23** (2015), 451–456.
[37] J.X. Ling, A. Dybbs, Forced convection over a flat plate submersed in a porous medium: Variable viscosity case, *ASME, Paper 87-WA/HT-23*, ASME, winter annual meeting, Boston, Massachusetts, (1987).
[38] M. Sharifpur, S. Yousefi, J. P. Meyer, A new model for density of nanofluids including nanolayer, *International Communications in Heat and Mass Transfer*, **78** (2016), 168–174.

[39] M.I. Anwar, I. Khan, M.Z. Salleh, A. Hasnain, S. Shafie, Magnetohydrodynamic effects on stagnation-point flow of Nanofluids towards a non-linear stretching sheet, *Wulfenia J.*, **19** (2012), 367.
[40] M. Turkyilmazoglu, Unsteady convection flow of some nanofluids past a moving vertical flat plate with heat transfer, *Journal of Heat Transfer*, **136** (2014), 031704–031711.