JJMS **18**, No. 1, 1-15 (2025)

Jordan Journal of Mathematics and Statistics.
Yarmouk University

DOI:https://doi.org/10.47013/18.1.1

New Two Parameter Inverse Gaussian Regression Estimators: Applications and Simulation

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Received: Sept. 9, 2023 Accepted: Oct. 10, 2024

Abstract: In situations where the response variable is positively skewed and has an inverse Gaussian distribution, the inverse Gaussian regression (IGR) model is employed. Due to the significant multicollinearity, the variance of the Maximum Likelihood (ML) Estimator is overestimated. In order to tackle multicollinearity in the IGR model, we present a new estimator in this study that combines two parameter estimators. In terms of mean squared error, the suggested estimator's performance is theoretically compared to that of the ML and a few other current estimators, as well as through Monte Carlo simulation and various real-data applications. The proposed estimators are found to perform better than the ML, ridge, Liu, Kibria-Lukman, and modified ridge type estimators under several circumstances.

Keywords: Multicollinearity; Inverse Gaussian ridge; Inverse Gaussian Regression model; hybrid estimator; Simulation.

2010 Mathematics Subject Classification.

1 Introduction

If the dependent variable has a normal distribution, the linear regression (LR) model is employed. Then, the generalized linear (GL) model is employed rather than the LR model since the dependent variable's assumption of normality may be broken, which will cause it to fit some of the exponential family distributions such as negative binomial, Poisson, gamma, and inverse gaussian. If the dependent variable behavior is positively skewed, the inverse gaussian regression (IGR) model is used in a variety of fields, including engineering, medical sciences, physical sciences, social sciences, environment, and business, [5], [6]. The maximum likelihood (ML) estimator, which is more practical than the ordinary least squares (OLS) estimator for describing and researching various phenomena, is used to estimate the IGR model parameters. The explanatory variables in the LR model may be correlated, which leads to an issue known as multicollinearity. The IGR model may also have this difficulty. The most widely used technique for estimating the unknown regression parameters in the IGR model is the ML estimator. The variances and standard errors of the regression parameters are likewise very significant when there is a multicollinearity issue [18]. The ordinary ridge regression (ORR) estimate approach, developed by Hoerl and Kennard [10], [11], is the most often used method for reducing the multicollinearity impact. The Liu estimator is yet another strategy suggested to address multicollinearity by Liu [14]. Next, the Kibria-Lukman estimator of Kibria and Lukman [13], followed by the modified ridge type estimator of Lukman et al. [15], and most recently a hybrid estimator of Shewa and Ugwuowo [20] are introduced and discussed in the LR model. On the other hand, Algamal [2] developed the ridge for the IGR model as an expansion of the biased estimators in the IGR model. Furthermore, Akram et al. [3] presented the Liu for the IGR model. Also, Lukman et al. [16] presented the Kibria-Lukman estimator for the IGR model. Akram et al. [4] recently presented the IGR model with the modified ridge type estimator. Another estimator, the hybrid estimator of Shewa and Ugwuowo [20], efficiently handles the problem of multicollinearity in the LR model by combining the Kibria-Lukman and modified ridge type estimators. Despite the fact that several biased estimators for the LR model have been proposed, there hasn't been a significant extension of these biased estimators for the IGR model. This is what motivates us to extend it for the IGR model, along with the analysis of these biased estimators for the IGR model and the hybrid estimator's superior performance in the LR model. This work employs the hybrid estimator for the

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IGR model and extracts its theoretical features in order to assess its effectiveness. We also compare the proposed hybrid estimator to various IGR estimating methods using the mean squared error criterion. The study is structured as follows: in Section 2, we define the model of interest and the IGR model estimation procedures. The existing IGR biased estimators and the novel IGR hybrid estimator are given in Section 3. The proposed IGR hybrid estimator is theoretically compared to other estimate approaches in Section 4, along with recommendations for the biasing parameters k and d. The design and outcomes of the Monte Carlo simulation are covered in Section 5. An empirical application is presented in Section 6. A few concluding conclusions are offered in Section 7 as the investigation comes to a close.

2 Methodology

Assume that y is a random variable with the inverse gaussian distribution and two positive parameters, the location parameter (γ) and the scale parameter (ϕ), which are represented by the notation IGR (γ, ϕ).

The probability density function for it is provided by

$$f(y,\gamma,\phi) = \frac{1}{\sqrt{2\pi y^3 \phi}} exp\left(-\frac{1}{2y}\left(\frac{y-\gamma}{\gamma\sqrt{\phi}}\right)^2\right), \quad y > 0. \tag{1}$$

The inverse gaussian distribution has a mean of $E(y) = \gamma$ and a variance of $var(y) = \phi\gamma^3$. The family of the GL models includes the IGR model.

Equation (1) is expressed in terms of the exponential family function as follows, using the GL model methodology:

$$f(y,\gamma,\phi) = \frac{1}{\phi}\left\{-\frac{y}{2\gamma^2}+\frac{1}{\gamma}\right\} + \left\{-\frac{1}{2}ln(2\pi y^3) - \frac{1}{2}ln(\phi)\right\}. \tag{2}$$

where $1/\gamma^2$ is the canonical link function and ϕ here is the dispersion parameter.
In GL models, the linear predictor is given by $\theta_i = x_i'\beta$, where x_i is considered the i-th row of x and β is determined as a $p\times 1$ vector of the unknown parameters. Since θ_i value depends on β and the mean of the response variable is $E(y_i) = \gamma_i = g^{-1}(\theta_i) = g^{-1}(x_i'\beta)$. In the IGR model, $\gamma = 1/\sqrt{x_i'\beta}$. In addition, the IGR models other link function is $\gamma = exp(x_i'\beta)$.
On the ML approach, the IGR model estimation is built. IGR log likelihood function is provided by under the canonical link function as follows:

$$\ell(\beta) = \sum_{i=1}^{n}\left\{\frac{1}{\phi}\left[\frac{y_i x_i'\beta}{2} - \sqrt{x_i'\beta}\right] - \frac{1}{2y_i\phi} - \frac{ln(\phi)}{2} - ln(2\pi y_i^3)\right\}. \tag{3}$$

Afterward, the ML estimator is obtained as follows by determining the first derivative of Equation (3) and setting it equal to zero:

$$\frac{\partial\ell(\beta)}{\partial\beta} = \sum_{i=1}^{n}\frac{1}{2\phi}\left[y_i - \frac{1}{\sqrt{x_i'\beta}}\right]x_i = 0. \tag{4}$$

The iteratively weighted least squares approach is utilized to obtain the ML estimators of the IGR parameters (IGML) estimator since Equation (4) is nonlinear in β and the first derivative cannot be analytically solved. Each time an iteration occurs, the following parameters are updated:

$$\beta^{(r+1)} = \beta^{(r)} + I^{-1}(\beta^{(r)})S(\beta^{(r)}), \tag{5}$$

where $S(\beta^{(r)})$ and $I^{-1}(\beta^{(r)})$ are $S(\beta^{(r)}) = \partial l(\beta)/\partial\beta$ and $I^{-1}(\beta^{(r)}) = (-E(\partial^2\ell(\beta)/\partial\beta\partial\beta^T))^{-1}$ evaluated at β^r, respectively. The final stage of the calculated parameters is defined as

$$\hat{\beta}_{IGML} = (X'\hat{U}X)^{-1}X'\hat{U}\hat{z}, \tag{6}$$

where $\hat{U} = diag(\gamma_i^3)$, $\hat{z}_i = \left(\frac{1}{\hat{\gamma}_i^2}\right) + \frac{y_i - \hat{\gamma}_i}{\hat{\gamma}_i^3}$ and $\hat{\gamma} = 1/\sqrt{x_i'\hat{\beta}}$.

Now, let $A'X'\hat{U}XA = Q = diag(q_1, q_2, \cdots, q_p)$ where $q_1 \geq q_2 \geq \cdots \geq q_p$ are the $X'\hat{U}X$ ordered eigenvalues and the orthogonal matrix A is given in which their columns are $X'\hat{U}X$ matrix's eigenvectors.

The IGML's matrix mean squared error (MMSE) and scalar mean squared error (SMSE) are expressed as follows:

$$MMSE(\hat{\beta}_{IGML}) = \phi AQ^{-1}A', \tag{7}$$

$$SMSE(\hat{\beta}_{IGML}) = \phi \sum_{j=1}^{p} \frac{1}{q_j}. \tag{8}$$

3 Available IGR Estimators and the Proposed IGR hybrid Estimator

3.1 IGR Ridge Estimator

The inverse Gaussian ridge regression (IGRR) estimator was introduced by Algamal [2] as an alternative to the IGML estimator and is denoted as follows:

$$\hat{\beta}_{IGRR} = (X'\hat{U}X + kI_p)^{-1} X'\hat{U}X\hat{\beta}_{IGML}, \tag{9}$$

additionally, the MMSE and SMSE of the IGRR estimator are provided as

$$MMSE(\hat{\beta}_{IGRR}) = \phi AFQF'A' + (AFQA' - I_p)\beta\beta'(AFQA' - I_p)', \tag{10}$$

where $F = (Q + kI_p)^{-1}, k > 0$, and

$$SMSE(\hat{\beta}_{IGRR}) = \phi \sum_{j=1}^{p} \frac{q_j}{(q_j + k)^2} + \sum_{j=1}^{p} \frac{k^2\beta_j^2}{(q_j + k)^2}. \tag{11}$$

3.2 IGR Liu Estimator

Next, Akram et al. [3] presented the inverse Gaussian Liu (IGL) estimator, which is defined as follows:

$$\hat{\beta}_{IGL} = (X'\hat{U}X + I_p)^{-1}(X'\hat{U}X + dI_p)\hat{\beta}_{IGML}, \tag{12}$$

$$MMSE(\hat{\beta}_{IGL}) = \phi ANQ^{-1}N'A' + (1-d)^2 A(Q + I_p)^{-1}\beta\beta'(Q + I_p)^{-1}A', \tag{13}$$

where $N = (Q + I_p)^{-1}(Q + dI_p), \quad 0 < d < 1$, and

$$SMSE(\hat{\beta}_{IGL}) = \phi \sum_{j=1}^{p} \frac{(q_j + d)^2}{q_j(q_j + 1)^2} + (1-d)^2 \sum_{j=1}^{p} \frac{\beta_j^2}{(q_j + 1)^2}. \tag{14}$$

3.3 IGR Kibria-Lukman Estimator

Lukman et al. [16] then provided the inverse Gaussian Kibria-Lukman (IGKL) estimator, which is as follows:

$$\hat{\beta}_{IGKL} = (X'\hat{U}X + kI_p)^{-1}(X'\hat{U}X - kI_p)\hat{\beta}_{IGML}, \tag{15}$$

$$MMSE(\hat{\beta}_{IGKL}) = \phi AMQ^{-1}M'A' + (AMA' - I_p)\beta\beta'(AMA' - I_p)', \tag{16}$$

where $M = (Q + kI_p)^{-1}(Q - kI_p)$, and

$$SMSE(\hat{\beta}_{IGKL}) = \phi \sum_{j=1}^{p} \frac{(q_j - k)^2}{q_j(q_j + k)^2} + 4k^2 \sum_{j=1}^{p} \frac{\beta_j^2}{(q_j + k)^2}. \tag{17}$$

3.4 IGR Modified Ridge-Type Estimator

Akram et al. [4] defined the estimator of the modified ridge-type for the IGR model (IGMRT):

$$\hat{\beta}_{IGMRT} = (X'\hat{U}X + k(1+d)I_p)^{-1}X'\hat{U}X\hat{\beta}_{IGML}, \quad (18)$$

$$MMSE(\hat{\beta}_{IGMRT}) = \phi AGQG'A' + (AGQA' - I_p)\beta\beta'(AGQA' - I_p)', \quad (19)$$

where $G = (Q + k(1+d)I_p)^{-1}$, and

$$SMSE(\hat{\beta}_{IGMRT}) = \phi \sum_{j=1}^{p} \frac{q_j}{(q_j + k(1+d))^2} + \sum_{j=1}^{p} \frac{(1+d)^2 k^2 \beta_j^2}{(q_j + k(1+d))^2}. \quad (20)$$

3.5 The Proposed IGR Hybrid Estimator

Shewa and Ugwuowo [20] demonstrated that the hybrid regression estimator they proposed performed better than a number of earlier LR estimators. To tackle this issue in the IGR model, we proposed expanding this estimator, also known as an IGH estimator and described by

$$\hat{\beta}_{IGH} = (X'\hat{U}X + kI_p)^{-1}(X'\hat{U}X - kI_p)(X'\hat{U}X + k(1+d)I_p)^{-1}X'\hat{U}X\hat{\beta}_{IGML}, \quad (21)$$

where $k > 0$ and $0 < d < 1$.

$$MMSE(\hat{\beta}_{IGH}) = \phi AMGQG'M'A' + (AMGQA' - I_p)\beta\beta'(AMGQA' - I_p)'. \quad (22)$$

$$SMSE(\hat{\beta}_{IGH}) = \phi \sum_{j=1}^{p} \frac{q_j(q_j - k)^2}{(q_j + k)^2(q_j + k(1+d))^2} + \sum_{j=1}^{p} \frac{(q_j(3+d) + k(1+d))^2 k^2 \beta_j^2}{(q_j + k)^2(q_j + k(1+d))^2}. \quad (23)$$

In the section that follows, some lemmas will be applied to the theoretical comparisons of the aforementioned accessible estimators.

Lemma 1: [9] Suppose the matrix W is an $n \times n$ positive definite (pd), which is $W > 0$ and w be some vector; so, $W - ww' > 0$ iff $w'W^{-1}w < 1$.

Lemma 2: [17] Let $w_i = W_i y$, $i = 1,2$ be two linear estimators of w. Let $D = Cov(\hat{w}_1) - Cov(\hat{w}_2) > 0$, where $Cov(\hat{w}_i) \quad i = 1,2$ be the covariance matrix of $\hat{w}_i$ and $b_i = Bias(\hat{w}_i) = (W_i X - I)w, \quad i = 1,2$. So,

$$\Delta(\hat{w}_1 - \hat{w}_2) = MMSE(\hat{w}_1) - MMSE(\hat{w}_2) = \sigma^2 D + b_1 b_1' - b_2 b_2' > 0 \quad (24)$$

iff $b_2'[\sigma^2 D + b_1 b_1']^{-1} b_2 < 1$ where $MMSE(\hat{w}_i) = Cov(\hat{w}_i) + b_i b_i'$.

4 Comparison among the Estimators

4.1 Comparison between $\hat{\beta}_{IGML}$ and $\hat{\beta}_{IGH}$.

Theorem 4.1: $MMSE(\hat{\beta}_{IGML}) - MMSE(\hat{\beta}_{IGH}) > 0$ iff

$$\beta'[AMGQA' - I_p]'[\phi(AQ^{-1}A' - AMGQG'M'A')]^{-1}[AMGQA' - I_p]\beta < 1. \quad (25)$$

proof.

$$\begin{aligned} Diff &= \phi(AQ^{-1}A' - AMGQG'M'A') \\ &= \phi A diag\left\{\frac{1}{q_j} - \frac{q_j(q_j-k)^2}{(q_j+k)^2(q_j+k(1+d))^2}\right\}_{j=1}^{p} A', \end{aligned} \quad (26)$$

where $AQ^{-1}A' - AMGQG'M'A'$ is pd iff $(q_j + k)^2(q_j + k(1+d))^2 - q_j^2(q_j - k)^2 > 0$ or $(q_j + k)(q_j + k(1+d)) - q_j(q_j - k) > 0$. It is clear that, for $k > 0$ and $0 < d < 1$, $(q_j + k)(q_j + k(1+d)) - q_j(q_j - k) = q_j k(3+d) + k^2(1+d) > 0$. By Lemma 2, the proof is completed.

***Theorem 4.2**: $MMSE(\hat{\beta}_{IGRR}) - MMSE(\hat{\beta}_{IGH}) > 0$ iff*

$$\beta'[AMGQA' - I_p]'[V_1 + (AFQA' - I_p)\beta\beta'(AFQA' - I_p)']^{-1}[AMGQA' - I_p]\beta < 1, \tag{27}$$

where $V_1 = \phi(AFQF'A' - AMGQG'M'A')$.
proof.

$$\begin{aligned} V_1 &= \phi(AFQF'A' - AMGQG'M'A') \\ &= \phi A diag\left\{\frac{q_j}{(q_j+k)^2} - \frac{q_j(q_j-k)^2}{(q_j+k)^2(q_j+k(1+d))^2}\right\}_{j=1}^{p} A', \end{aligned} \tag{28}$$

where $AFQF'A' - AMGQG'M'A'$ is pd iff $(q_j + k(1+d))^2 - (q_j - k)^2 > 0$ or $(q_j + k(1+d)) - (q_j - k) > 0$. It is clear that, for $k > 0$ and $0 < d < 1$, $(q_j + k(1+d)) - (q_j - k) = k(2+d) > 0$. By Lemma 2, the proof is completed.

***Theorem 4.3**: $MMSE(\hat{\beta}_{IGL}) - MMSE(\hat{\beta}_{IGH}) > 0$ iff*

$$\beta'[AMGQA' - I_p]'[V_2 + (ANA' - I_p)\beta\beta'(ANA' - I_p)']^{-1}[AMGQA' - I_p]\beta < 1, \tag{29}$$

where $V_2 = \phi(ANQ^{-1}N'A' - AMGQG'M'A')$.
proof.

$$\begin{aligned} V_2 &= \phi(ANQ^{-1}N'A' - AMGQG'M'A') \\ &= \phi A diag\left\{\frac{(q_j+d)^2}{q_j(q_j+1)^2} - \frac{q_j(q_j-k)^2}{(q_j+k)^2(q_j+k(1+d))^2}\right\}_{j=1}^{p} A', \end{aligned} \tag{30}$$

where $ANQ^{-1}N'A' - AMGQG'M'A'$ is pd iff $(q_j + d)^2(q_j + k)^2(q_j + k(1+d))^2 - q_j^2(q_j + 1)^2(q_j - k)^2 > 0$ or $(q_j + d)(q_j + k)(q_j + k(1+d)) - q_j(q_j + 1)(q_j - k) > 0$. It is clear that, for $k > 0$, $0 < d < 1$, and $k > \frac{1-d}{3+d}$, $(q_j + d)(q_j + k)(q_j + k(1+d)) - q_j(q_j + 1)(q_j - k) = q_j^2(k(3+d) + d - 1) + q_j(k[d(k+d+2) + k + 1]) + k^2 d(1+d) > 0$. By Lemma 2, the proof is completed.

***Theorem 4.4**: $MMSE(\hat{\beta}_{IGMRT}) - MMSE(\hat{\beta}_{IGH}) > 0$ iff*

$$\beta'[AMGQ\acute{A} - I_p]'[V_3 + (AGQA' - I_p)\beta\beta'(AGQ\acute{A} - I_p)']^{-1}[AMGQA' - I_p]\beta < 1, \tag{31}$$

where $V_3 = \phi(AGQG'A' - AMGQG'M'A')$.
proof.

$$\begin{aligned} V_3 &= \phi(AGQG'A' - AMGQG'M'A') \\ &= \phi A diag\left\{\frac{q_j}{(q_j+k(1+d))^2} - \frac{q_j(q_j-k)^2}{(q_j+k)^2(q_j+k(1+d))^2}\right\}_{j=1}^{p} A', \end{aligned} \tag{32}$$

where $AGQG'A' - AMGQG'M'A'$ is pd iff $(q_j + k)^2 - (q_j - k)^2 > 0$ or $(q_j + k) - (q_j - k) > 0$. It is clear that, for $k > 0$, and $0 < d < 1$, $(q_j + k) - (q_j - k) = 2k > 0$. By Lemma 2, the proof is completed.

***Theorem 4.5**: $MMSE(\hat{\beta}_{IGKL}) - MMSE(\hat{\beta}_{IGH}) > 0$ iff*

$$\beta'[AMGQ\acute{A} - I_p]'[V_4 + (AMA' - I_p)\beta\beta'(AMA' - I_p)']^{-1}[AMGQA' - I_p]\beta < 1, \tag{33}$$

where $V_4 = \phi(AMQ^{-1}M'A' - AMGQG'M'A')$.

proof.

$$V_2 = \phi(AMQ^{-1}M'A' - AMGQG'M'A')$$
$$= \phi A diag\left\{\frac{(q_j-k)^2}{q_j(q_j+k)^2} - \frac{q_j(q_j-k)^2}{(q_j+k)^2(q_j+k(1+d))^2}\right\}_{j=1}^{p} A', \tag{34}$$

where $AMQ^{-1}M'A' - AMGQG'M'A'$ *is pd iff* $(q_j+k(1+d))^2 - q_j^2 > 0$ *or* $(q_j+k(1+d)) - q_j > 0$. *It is clear that, for* $k > 0$, *and* $0 < d < 1$, $(q_j+k(1+d)) - q_j = k(1+d) > 0$. *By Lemma 2, the proof is completed.*

4.2 Selection of biasing parameters k and d

The earlier estimators have estimators for the biasing parameters in addition to the proposed IGH estimator.

As Hoerl and Kennard [10], $\hat{k}$ *for the IGRR estimator is chosen as*

$$\hat{k} = min\left\{\frac{\hat{\phi}_j}{\hat{\beta}^2_{IGML(j)}}\right\}_{j=1}^{p}, \tag{35}$$

where the dispersion parameter, ϕ, *is estimated by Uusipaikka [21].*

$$\hat{\phi} = \frac{1}{n-p}\sum_{i=1}^{n}\frac{(y_i-\hat{\gamma}_j)^2}{\hat{\gamma}_j^3}. \tag{36}$$

As Ozkale and Kaciranlar [19], $\hat{d}$ *for IGL estimator is chosen as*

$$\hat{d} = min\left\{\frac{\hat{\beta}^2_{IGML(j)}}{\hat{\beta}^2_{IGML(j)} + (\hat{\phi}_j/q_j)}\right\}_{j=1}^{p}. \tag{37}$$

As Kibria and Lukman [13], $\hat{k}_{IGKL}$ *for the IGKL estimator is chosen as*

$$\hat{k}_{IGKL} = min\left\{\frac{\hat{\phi}_j}{2\hat{\beta}^2_{IGML(j)} + (\hat{\phi}_j/q_j)}\right\}_{j=1}^{p}. \tag{38}$$

As Lukman et al. [15], $\hat{k}_{IGMRT}$ *for the IGMRT estimator is chosen as*

$$\hat{k}_{IGMRT} = min\left\{\frac{\hat{\phi}_j}{(1+\hat{d})\hat{\beta}^2_{IGML(j)}}\right\}_{j=1}^{p}. \tag{39}$$

where $\hat{d}$ *is stated in Equation (37).*

We suggest five different $\hat{k}_{IGH}$ *of the new IGH estimator as follows:*

$$\hat{k}_{IGH1} = \hat{k}. \tag{40}$$

$$\hat{k}_{IGH2} = \hat{k}_{IGKL}. \tag{41}$$

$$\hat{k}_{IGH3} = \hat{k}_{IGMRT}. \tag{42}$$

$$\hat{k}_{IGH4} = \frac{p\hat{\phi}_j}{\sum_{j=1}^{p}\hat{\beta}^2_{IGML(j)}}. \tag{43}$$

$$\hat{k}_{IGH5} = \sqrt{\frac{p\hat{\phi}_j}{\sum_{j=1}^{p}\hat{\beta}^2_{IGML(j)}}}. \tag{44}$$

5 Simulation Study

We should carry out the simulation study in order to compare the IGH estimators performance further. The sample size (n), multicollinearity levels (E), the number of explanatory variables (p), and dispersion parameter (ϕ) are all important variables to consider while creating the simulation. The performance of the estimators is evaluated using the MSE criterion using the following formula:

$$MSE(\hat{\beta}) = \frac{\sum_{r=1}^{MCN} (\hat{\beta}_r - \beta)'(\hat{\beta}_r - \beta)}{MCN}, \tag{45}$$

where $\hat{\beta}_r$ is considered as the estimated value of β in the rth replication and MCN is considered as the number of replications, which is set to 2000.

5.1 Simulation Design

The inverse Gaussian distribution is used to create the response variable, i.e. $y_i \sim IG(\gamma_i, \phi)$, where $\gamma_i = \frac{1}{\sqrt{\beta_0+\beta_1 x_{1i}+\beta_2 x_{2i}+\cdot++\beta_p x_{pi}}}$ with different sample sizes i.e. n= 25, 50, 100, 200 and 400.

A prevalent restriction in many investigations, such as those by Kibria [12], and Amin et al. [7], led to the selection of the parameters as $\beta'\beta = 1$.

The explanatory variables that are correlated are produced by

$$x_{ij} = (1-E^2)^{1/2} c_{ij} + E c_{i,j+1}; \quad i = 1,2,\cdots,n; \quad j = 1,2,\cdots,p, \tag{46}$$

where c_{ij} is a set of independent standard normal pseudo-random numbers and E is a specification such that the correlation between any two explanatory variables is provided by E^2. The simulation experiment takes into account two values of the dispersion parameter, ϕ = 0.1 and 0.25, as well as three distinct sets of explanatory variables, p = 5, 10, and 20, six different levels of correlation, E =0.50, 0.70, 0.80, 0.90, 0.95, and 0.99, and six different degrees of correlation.

Tables 1-6 display the calculated MSEs for the IGR model, which were derived using various estimating techniques. When evaluating the performance of the suggested IGR hybrid estimators, we take a variety of things into account. The key findings from the simulation study's findings are covered in the discussion that follows. The results shown in Tables 1-6 show that the suggested IGR hybrid estimators perform well in terms of reduced MSE when compared to other estimating techniques under examination. The outcomes also show that multicollinearity significantly affects the calculated MSEs of the studied estimators. Since they are slightly less affected by this influence when using our recommended IGR hybrid estimators, it is evident that IGR hybrid estimators function robustly in the presence of multicollinearity. It should be noted that the estimator most adversely affected by regressor correlation is the IGML. In the IGR model, our suggested IGR hybrid estimators outperform the other IGR biased estimators, minimizing the impact of regressor collinearity. With an increase in sample size (n), the MSE values of the IGR model for the evaluated estimators decrease. The sample size, meanwhile, indirectly influences the computed MSEs. Along with the correlation degree, scale parameter, and number of explanatory factors all increasing, the simulated MSE values of the IGR estimators also increased. This pattern is also clearly shown in Figure 1. Again, in this case the estimator that has been violated the most is the IGML. If we analyze how well the proposed IGR hybrid estimators perform in relation to the regressors, we can draw the conclusion from the results that they still provide a trustworthy estimation strategy when compared to other estimation approaches. The majority of the suggested IGR hybrid estimators beat the existing IGR biased estimators by providing MSE values that are less noticeable. The suggested IGR hybrid estimators (IGH4, IGH5) stand out among them as the top estimators. In conclusion, because IGR hybrid estimators greatly reduce MSEs, they should be used whenever multicollinearity is present along with adequate estimators of the biasing parameters.

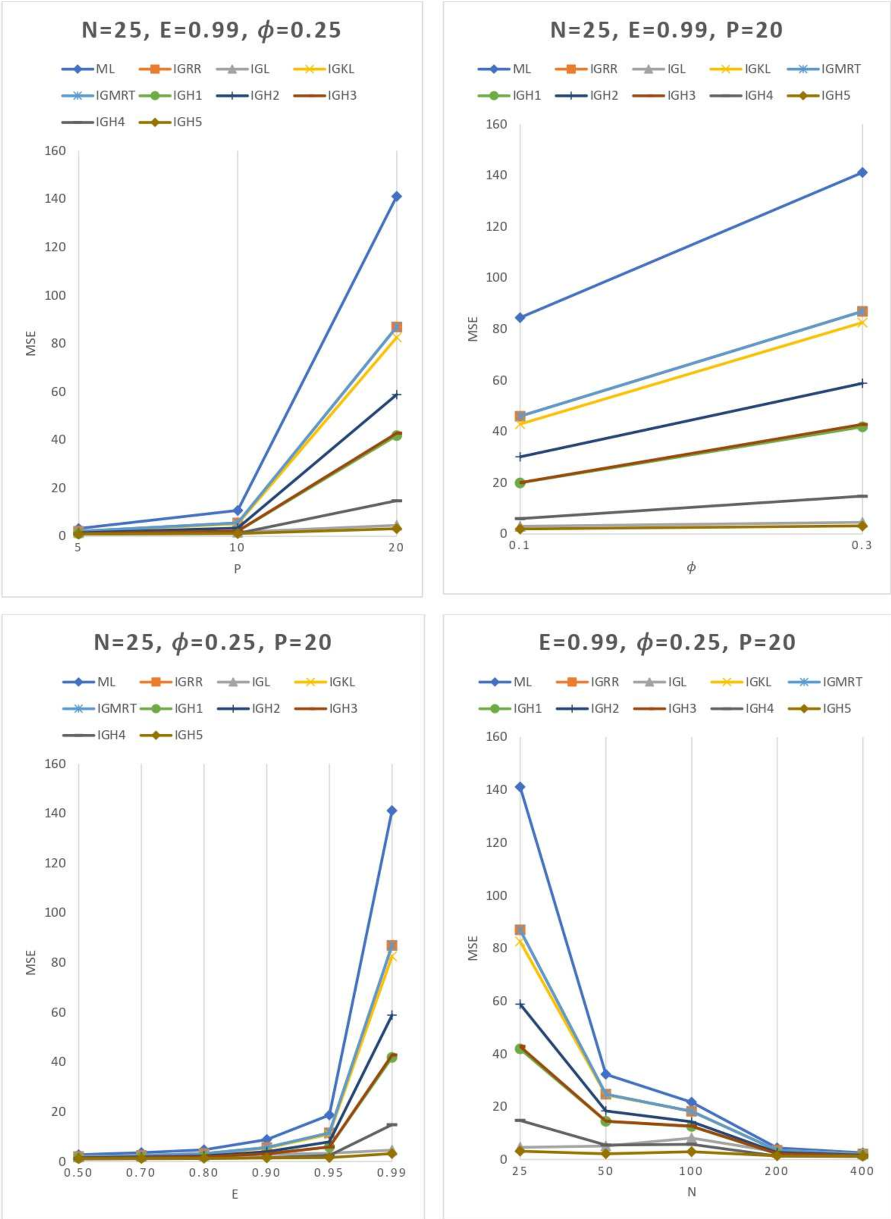

Fig. 1: Factors vs. MSE of the estimators

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		Available Estimators					*Proposed Estimators*				
n	*E*	*IGML*	*IGRR*	*IGL*	*IGKL*	*IGMRT*	*IGH1*	*IGH2*	*IGH3*	*IGH4*	*IGH5*
25	0.50	1.2856	1.1731	1.1765	1.1730	1.1731	0.9514	0.9572	0.9538	0.9340	0.9477
	0.70	1.3495	1.2258	1.2337	1.2254	1.2258	0.9875	0.9976	0.9916	0.9663	0.9856
	0.80	1.3765	1.2429	1.2555	1.2417	1.2429	0.9958	1.0092	0.9990	0.9748	0.9954
	0.90	1.4073	1.2494	1.2713	1.2452	1.2494	0.9899	1.0084	0.9914	0.9714	0.9879
	0.95	1.4643	1.2543	1.2853	1.2445	1.2543	0.9763	1.0021	0.9770	0.9623	0.9712
	0.99	2.7846	1.7864	1.4478	1.7281	1.7864	1.0273	1.2629	1.0457	0.9509	0.9499
50	0.50	1.1897	1.0917	1.0905	1.0915	1.0917	0.8951	0.8934	0.8950	0.8978	0.8926
	0.70	1.2304	1.1244	1.1278	1.1253	1.1244	0.9157	0.9198	0.9159	0.9091	0.9214
	0.80	1.2459	1.1381	1.1421	1.1391	1.1381	0.9262	0.9310	0.9265	0.9184	0.9328
	0.90	1.2697	1.1601	1.1638	1.1603	1.1601	0.9455	0.9484	0.9456	0.9390	0.9491
	0.95	1.2884	1.1734	1.1802	1.1733	1.1734	0.9540	0.9581	0.9541	0.9490	0.957
	0.99	1.3683	1.2056	1.2378	1.2049	1.2056	0.9640	0.9777	0.9642	0.9610	0.9651
100	0.50	1.0709	0.9874	0.9819	0.9872	0.9874	0.8175	0.8101	0.8162	0.8386	0.8087
	0.70	1.0758	0.9920	0.9863	0.9912	0.9920	0.8217	0.8133	0.8202	0.8382	0.8108
	0.80	1.0879	1.0014	0.9973	1.0003	1.0014	0.8274	0.8200	0.8265	0.8419	0.8182
	0.90	1.1040	1.0083	1.0115	1.0079	1.0083	0.8237	0.8239	0.8237	0.8298	0.8236
	0.95	1.1383	1.0259	1.0403	1.0256	1.0259	0.8261	0.8347	0.8267	0.8233	0.8314
	0.99	1.3298	1.1391	1.2399	1.1462	1.1391	0.8349	0.9012	0.8369	0.8258	0.8401
200	0.50	1.0466	0.9463	0.9531	0.9453	0.9463	0.7638	0.7688	0.7662	0.7664	0.7625
	0.70	1.0646	0.9506	0.9590	0.9488	0.9506	0.7541	0.7673	0.7596	0.7466	0.7474
	0.80	1.0684	0.9431	0.9511	0.9400	0.9431	0.7472	0.7603	0.7492	0.7441	0.7418
	0.90	1.0732	0.9318	0.9421	0.9281	0.9318	0.7409	0.7514	0.7411	0.7521	0.7382
	0.95	1.1326	0.9339	0.9522	0.9305	0.9339	0.7419	0.7505	0.7419	0.7564	0.7407
	0.99	1.2027	1.3789	1.0822	1.3955	1.3789	0.7935	1.0046	0.8012	0.7660	0.7614
400	0.50	0.9647	0.9043	0.8960	0.9039	0.9043	0.7556	0.7436	0.753	0.7782	0.7427
	0.70	0.9727	0.8974	0.8922	0.8968	0.8974	0.7447	0.7362	0.7429	0.7655	0.7371
	0.80	0.9771	0.8991	0.8959	0.8985	0.8991	0.7429	0.7367	0.7419	0.7608	0.7378
	0.90	0.9818	0.8993	0.8994	0.8986	0.8993	0.7393	0.7355	0.7388	0.7538	0.7363
	0.95	1.0142	0.9117	0.9226	0.9116	0.9117	0.7363	0.7419	0.7365	0.7416	0.7358
	0.99	1.1809	1.1438	1.1133	1.1160	1.1438	0.7695	0.8572	0.7728	0.7333	0.7341

Table 1: MSE of the estimators for ϕ=0.1 and p=5

		Available Estimators					*Proposed Estimators*				
n	*E*	*IGML*	*IGRR*	*IGL*	*IGKL*	*IGMRT*	*IGH1*	*IGH2*	*IGH3*	*IGH4*	*IGH5*
25	0.50	1.2945	1.1773	1.1837	1.1771	1.1773	0.9507	0.9594	0.9527	0.9318	0.9501
	0.70	1.3640	1.2323	1.2450	1.2317	1.2323	0.9875	1.0016	0.9900	0.9642	0.9896
	0.80	1.3983	1.2539	1.2725	1.2527	1.2539	0.9983	1.0165	1.0006	0.9730	1.0004
	0.90	1.4526	1.2752	1.3042	1.2719	1.2752	0.9993	1.0267	1.0016	0.9711	0.9946
	0.95	1.5596	1.3130	1.3438	1.3051	1.3130	0.9982	1.0431	1.0012	0.9637	0.9789
	0.99	3.2500	2.0816	1.5684	2.0166	2.0816	1.1566	1.4662	1.1871	0.9714	0.9540
50	0.50	1.1920	1.0926	1.0926	1.0926	1.0926	0.8945	0.8940	0.8945	0.8961	0.8939
	0.70	1.2334	1.1277	1.1305	1.1280	1.1277	0.9193	0.9220	0.9196	0.9145	0.9235
	0.80	1.2494	1.1417	1.1452	1.1420	1.1417	0.9300	0.9334	0.9304	0.9243	0.9348
	0.90	1.2751	1.1631	1.1686	1.1633	1.1631	0.9463	0.9503	0.9467	0.9403	0.9514
	0.95	1.2990	1.1784	1.1894	1.1784	1.1784	0.9547	0.9612	0.9553	0.9482	0.9602
	0.99	2.6291	1.2344	1.2824	1.2338	1.2344	0.9703	0.9954	0.9718	0.9599	0.9704
100	0.50	1.0749	0.9911	0.9855	0.9907	0.9911	0.8206	0.8130	0.8190	0.8394	0.8110
	0.70	1.1037	0.9945	0.9913	0.9940	0.9945	0.8202	0.8147	0.8193	0.8346	0.8133
	0.80	1.1272	1.0046	1.0039	1.0040	1.0046	0.8254	0.8219	0.8250	0.8362	0.8212
	0.90	1.1916	1.0167	1.0227	1.0160	1.0167	0.8275	0.8296	0.8278	0.8304	0.8287

	0.95	1.3746	1.0410	1.0601	1.0403	1.0410	0.8329	0.8449	0.8343	0.8264	0.8382
	0.99	2.4266	1.2144	1.3181	1.2159	1.2144	0.8595	0.9467	0.8663	0.8303	0.8482
200	0.50	1.0723	0.9613	0.9722	0.9597	0.9613	0.7729	0.7791	0.7755	0.7785	0.7690
	0.70	1.0814	0.9718	0.9864	0.9694	0.9718	0.7640	0.7814	0.7698	0.7562	0.7539
	0.80	1.0953	0.9765	0.9914	0.9733	0.9765	0.7617	0.7826	0.7661	0.7541	0.7491
	0.90	1.1167	0.9902	1.0027	0.9861	0.9902	0.7599	0.7880	0.7630	0.7553	0.7443
	0.95	1.1618	1.0411	1.0389	1.0366	1.0411	0.7644	0.8142	0.7692	0.7557	0.7445
	0.99	2.3031	2.1827	1.2961	2.1673	2.1827	0.9811	1.1732	1.0389	0.8084	0.7758
400	0.50	0.9838	0.9100	0.9024	0.9095	0.9100	0.7593	0.7478	0.7562	0.7857	0.7473
	0.70	0.9938	0.9045	0.9002	0.9038	0.9045	0.7497	0.7416	0.7480	0.7716	0.7417
	0.80	1.0047	0.9075	0.9059	0.9067	0.9075	0.7483	0.7428	0.7472	0.7662	0.7427
	0.90	1.0126	0.9107	0.9153	0.9098	0.9107	0.7445	0.7431	0.7442	0.7572	0.7414
	0.95	1.0493	0.9310	0.9500	0.9303	0.9310	0.7438	0.7541	0.7448	0.7462	0.7419
	0.99	1.7085	1.2274	1.2018	1.2062	1.2274	0.7966	0.9151	0.8054	0.7405	0.7395

Table 2: MSE of the estimators for ϕ=0.25 and p=5

		Available Estimators					*Proposed Estimators*				
n	*E*	*IGML*	*IGRR*	*IGL*	*IGKL*	*IGMRT*	*IGH1*	*IGH2*	*IGH3*	*IGH4*	*IGH5*
25	0.50	1.4687	1.3116	1.3042	1.3097	1.3116	1.0358	1.0603	1.0366	0.9970	1.0036
	0.70	1.5981	1.4043	1.3867	1.4007	1.4043	1.0852	1.1268	1.0866	1.0154	1.0278
	0.80	1.7056	1.4680	1.4379	1.4613	1.4680	1.1093	1.1675	1.1105	1.0268	1.0393
	0.90	1.9428	1.5748	1.5015	1.5554	1.5748	1.1303	1.2228	1.1313	1.0406	1.0491
	0.95	3.6007	1.7215	1.5339	1.6663	1.7215	1.1370	1.2775	1.1379	1.0497	1.0531
	0.99	7.0924	2.9446	1.4571	2.5349	2.9446	1.2427	1.7262	1.2446	1.0960	1.0875
50	0.50	1.4216	1.2888	1.2879	1.2885	1.2888	1.0322	1.0483	1.0339	0.9275	0.9870
	0.70	1.4946	1.3507	1.3463	1.3503	1.3507	1.0758	1.0968	1.0780	0.9552	1.0150
	0.80	1.5230	1.3687	1.3583	1.3680	1.3687	1.0805	1.1086	1.0821	0.9435	1.0016
	0.90	1.6096	1.4179	1.3849	1.4153	1.4179	1.0852	1.1370	1.0870	0.9150	0.9634
	0.95	3.1338	1.6164	1.5011	1.6053	1.6164	1.1467	1.2614	1.1519	0.9195	0.9515
	0.99	6.1053	3.8916	1.9570	3.7638	3.8916	1.7698	2.6338	1.8063	1.0493	0.9558
100	0.50	1.3356	1.2203	1.2241	1.2203	1.2203	0.9923	0.9968	0.9928	0.9779	0.9973
	0.70	1.3408	1.2245	1.2287	1.2245	1.2245	0.9952	1.0000	0.9958	0.9816	0.9998
	0.80	1.3537	1.2343	1.2403	1.2343	1.2343	1.0008	1.0074	1.0019	0.9861	1.0066
	0.90	1.4038	1.2741	1.2846	1.2740	1.2741	1.0257	1.0378	1.0265	1.0029	1.0326
	0.95	2.7065	1.3542	1.3720	1.3541	1.3542	1.0729	1.0982	1.0751	1.0272	1.0752
	0.99	4.9500	2.1325	2.0795	2.1315	2.1325	1.4807	1.6699	1.4976	1.2089	1.2869
200	0.50	1.2501	1.1396	1.1452	1.1396	1.1396	0.9234	0.9299	0.9242	0.9095	0.9288
	0.70	1.2767	1.1629	1.1688	1.1629	1.1629	0.9405	0.9486	0.9413	0.9151	0.9424
	0.80	1.3093	1.1913	1.1972	1.1914	1.1913	0.9613	0.9712	0.9624	0.9216	0.9574
	0.90	1.4024	1.2726	1.2773	1.2727	1.2726	1.0214	1.0360	1.0230	0.9465	0.9987
	0.95	2.5180	1.4060	1.4045	1.4062	1.4060	1.1171	1.1418	1.1190	0.9850	1.0547
	0.99	3.8339	2.2622	2.0154	2.2650	2.2622	1.6420	1.7980	1.6488	1.0911	1.1602
400	0.50	1.1057	1.0090	1.0120	1.0089	1.0090	0.8191	0.8237	0.8196	0.8072	0.8177
	0.70	1.1727	1.0625	1.0704	1.0624	1.0625	0.8519	0.8644	0.8532	0.8208	0.8495
	0.80	1.2298	1.1063	1.1181	1.1060	1.1063	0.8764	0.8968	0.8789	0.8307	0.8702
	0.90	1.3664	1.2042	1.2229	1.2036	1.2042	0.9220	0.9664	0.9291	0.8453	0.8995
	0.95	1.5817	1.3599	1.3589	1.3598	1.3599	0.9980	1.0795	1.0103	0.8501	0.9089
	0.99	3.1461	2.4644	1.7795	2.4668	2.4644	1.5242	1.8808	1.5370	0.8533	0.8566

Table 3: MSE of the estimators for ϕ=0.1 and p=10

		Available Estimators					*Proposed Estimators*				
n	*E*	*IGML*	*IGRR*	*IGL*	*IGKL*	*IGMRT*	*IGH1*	*IGH2*	*IGH3*	*IGH4*	*IGH5*
25	0.50	1.5454	1.3607	1.3576	1.3571	1.3607	1.0589	1.0936	1.0605	1.0028	1.0176

	0.70	*1.7113*	*1.4740*	*1.4571*	*1.4673*	*1.4740*	*1.1163*	*1.1731*	*1.1183*	*1.0223*	*1.0430*
	0.80	*1.8696*	*1.5643*	*1.5285*	*1.5526*	*1.5643*	*1.1503*	*1.2303*	*1.1528*	*1.0339*	*1.0546*
	0.90	*2.2757*	*1.7607*	*1.6378*	*1.7336*	*1.7607*	*1.2044*	*1.3444*	*1.2086*	*1.0500*	*1.0639*
	0.95	*5.4994*	*2.1230*	*1.7212*	*2.0645*	*2.1230*	*1.2880*	*1.5467*	*1.2957*	*1.0672*	*1.0669*
	0.99	*10.785*	*5.5554*	*1.6796*	*5.1810*	*5.5554*	*2.1260*	*3.4693*	*2.1611*	*1.2127*	*1.1242*
50	*0.50*	*1.4542*	*1.3130*	*1.3157*	*1.3125*	*1.3130*	*1.0441*	*1.0658*	*1.0461*	*0.9310*	*0.9998*
	0.70	*1.5379*	*1.3815*	*1.3817*	*1.3807*	*1.3815*	*1.0897*	*1.1186*	*1.0920*	*0.9594*	*1.0289*
	0.80	*1.5840*	*1.4113*	*1.4069*	*1.4098*	*1.4113*	*1.0989*	*1.1380*	*1.1016*	*0.9511*	*1.0186*
	0.90	*1.7261*	*1.4949*	*1.4688*	*1.4901*	*1.4949*	*1.1175*	*1.1887*	*1.1219*	*0.9309*	*0.9877*
	0.95	*4.5594*	*1.7623*	*1.6331*	*1.7478*	*1.7623*	*1.2128*	*1.3615*	*1.2215*	*0.9461*	*0.9807*
	0.99	*7.2014*	*4.8231*	*2.2068*	*4.7251*	*4.8231*	*2.3350*	*3.3696*	*2.3940*	*1.1696*	*0.9813*
100	*0.50*	*1.3395*	*1.2237*	*1.2276*	*1.2237*	*1.2237*	*0.9946*	*0.9994*	*0.9955*	*0.9791*	*0.9996*
	0.70	*1.3449*	*1.2278*	*1.2324*	*1.2278*	*1.2278*	*0.9972*	*1.0025*	*0.9981*	*0.9816*	*1.0020*
	0.80	*1.3580*	*1.2379*	*1.2441*	*1.2379*	*1.2379*	*1.0031*	*1.0101*	*1.0042*	*0.9849*	*1.0084*
	0.90	*1.4126*	*1.2815*	*1.2923*	*1.2815*	*1.2815*	*1.0303*	*1.0434*	*1.0321*	*1.0002*	*1.0352*
	0.95	*3.9321*	*1.3759*	*1.3942*	*1.3758*	*1.3759*	*1.0871*	*1.1148*	*1.0909*	*1.0269*	*1.0835*
	0.99	*5.1343*	*2.2929*	*2.2261*	*2.2919*	*2.2929*	*1.5921*	*1.7939*	*1.6241*	*1.2235*	*1.3163*
200	*0.50*	*1.2652*	*1.1532*	*1.1589*	*1.1532*	*1.1532*	*0.9340*	*0.9409*	*0.9350*	*0.9157*	*0.9385*
	0.70	*1.2974*	*1.1802*	*1.1874*	*1.1802*	*1.1802*	*0.9523*	*0.9620*	*0.9535*	*0.9237*	*0.9551*
	0.80	*1.3363*	*1.2131*	*1.2215*	*1.2131*	*1.2131*	*0.9751*	*0.9878*	*0.9768*	*0.9327*	*0.9734*
	0.90	*1.4045*	*1.3054*	*1.3146*	*1.3056*	*1.3054*	*1.0404*	*1.0607*	*1.0429*	*0.9626*	*1.0209*
	0.95	*3.0586*	*1.4548*	*1.4597*	*1.4552*	*1.4548*	*1.1402*	*1.1772*	*1.1436*	*0.9985*	*1.0775*
	0.99	*4.2136*	*2.5263*	*2.2258*	*2.5306*	*2.5263*	*1.7658*	*1.9886*	*1.7788*	*1.1360*	*1.1974*
400	*0.50*	*1.1264*	*1.0260*	*1.0304*	*1.0259*	*1.0260*	*0.8302*	*0.8368*	*0.8311*	*0.8135*	*0.8290*
	0.70	*1.2029*	*1.0876*	*1.0971*	*1.0874*	*1.0876*	*0.8687*	*0.8838*	*0.8707*	*0.8293*	*0.8646*
	0.80	*1.2721*	*1.1412*	*1.1548*	*1.1409*	*1.1412*	*0.8996*	*0.9238*	*0.9030*	*0.8418*	*0.8896*
	0.90	*1.4164*	*1.2703*	*1.2862*	*1.2697*	*1.2703*	*0.9689*	*1.0188*	*0.9756*	*0.8638*	*0.9289*
	0.95	*1.7597*	*1.4828*	*1.4703*	*1.4820*	*1.4828*	*1.0766*	*1.1737*	*1.0869*	*0.8830*	*0.9528*
	0.99	*3.3068*	*3.0011*	*2.0915*	*3.0024*	*3.0011*	*1.7812*	*2.2638*	*1.8073*	*0.9431*	*0.9053*

Table 4: MSE of the estimators for ϕ=0.25 and p=10

		Available Estimators					*Proposed Estimators*				
n	*E*	*IGML*	*IGRR*	*IGL*	*IGKL*	*IGMRT*	*IGH1*	*IGH2*	*IGH3*	*IGH4*	*IGH5*
25	*0.50*	*2.3292*	*1.7488*	*1.6887*	*1.7374*	*1.7488*	*1.2593*	*1.3765*	*1.2601*	*0.9951*	*1.0848*
	0.70	*2.8764*	*2.0909*	*1.9060*	*2.0677*	*2.0909*	*1.4199*	*1.6137*	*1.4210*	*1.0418*	*1.1274*
	0.80	*3.6599*	*2.5982*	*2.1312*	*2.5507*	*2.5982*	*1.6582*	*1.9566*	*1.6598*	*1.1027*	*1.1683*
	0.90	*6.5506*	*4.4000*	*2.5910*	*4.2798*	*4.4000*	*2.4750*	*3.1767*	*2.4780*	*1.3064*	*1.2605*
	0.95	*13.610*	*8.6973*	*3.0139*	*8.3782*	*8.6973*	*4.4257*	*6.0721*	*4.4379*	*1.7828*	*1.3974*
	0.99	*84.482*	*45.997*	*3.0174*	*42.917*	*45.997*	*19.967*	*30.126*	*20.077*	*6.0143*	*1.9755*
50	*0.50*	*1.8878*	*1.6630*	*1.6947*	*1.6621*	*1.6630*	*1.2669*	*1.3346*	*1.2696*	*1.0612*	*1.2203*
	0.70	*1.9522*	*1.7363*	*1.7375*	*1.7357*	*1.7363*	*1.3468*	*1.4004*	*1.3476*	*1.1348*	*1.2415*
	0.80	*2.1755*	*1.8726*	*1.7899*	*1.8673*	*1.8726*	*1.3744*	*1.4838*	*1.3771*	*0.9586*	*1.0863*
	0.90	*3.4200*	*2.8164*	*2.4001*	*2.7999*	*2.8164*	*1.9115*	*2.1795*	*1.9154*	*1.1400*	*1.2429*
	0.95	*6.0357*	*4.7651*	*3.1798*	*4.7219*	*4.7651*	*2.9859*	*3.6025*	*2.9904*	*1.5225*	*1.4444*
	0.99	*28.238*	*21.222*	*4.5203*	*21.065*	*21.222*	*11.965*	*15.702*	*12.015*	*4.7651*	*2.0272*
100	*0.50*	*1.6633*	*1.5109*	*1.5183*	*1.5109*	*1.5109*	*1.2157*	*1.2308*	*1.2160*	*1.1179*	*1.1939*
	0.70	*1.6980*	*1.5092*	*1.5391*	*1.5082*	*1.5092*	*1.1687*	*1.2161*	*1.1705*	*1.0299*	*1.1580*
	0.80	*2.1425*	*1.8763*	*1.8962*	*1.8758*	*1.8763*	*1.4115*	*1.5015*	*1.4171*	*1.1111*	*1.2974*
	0.90	*2.9891*	*2.5986*	*2.5083*	*2.5985*	*2.5986*	*1.9216*	*2.0716*	*1.9308*	*1.2966*	*1.5183*
	0.95	*4.8005*	*4.1381*	*3.5963*	*4.1379*	*4.1381*	*2.9998*	*3.2830*	*3.0210*	*1.7053*	*1.8555*
	0.99	*19.586*	*16.556*	*7.3679*	*16.556*	*16.556*	*11.493*	*12.999*	*11.617*	*4.9831*	*2.6687*
200	*0.50*	*1.5182*	*1.3585*	*1.3485*	*1.3576*	*1.3585*	*1.0642*	*1.0980*	*1.0650*	*0.8672*	*0.9581*
	0.70	*1.5954*	*1.4508*	*1.4594*	*1.4507*	*1.4508*	*1.1691*	*1.1823*	*1.1698*	*1.0923*	*1.1644*
	0.80	*1.6278*	*1.4812*	*1.4882*	*1.4811*	*1.4812*	*1.1951*	*1.2075*	*1.1955*	*1.1111*	*1.1818*
	0.90	*1.7593*	*1.5880*	*1.5958*	*1.5878*	*1.5880*	*1.2635*	*1.2897*	*1.2638*	*1.1251*	*1.2154*

	0.95	1.7935	1.5789	1.5482	1.5766	1.5789	1.2034	1.2656	1.2051	0.9054	1.0207
	0.99	3.5988	2.7820	2.4848	2.7730	2.7820	1.7148	2.1066	1.7248	1.1905	1.2615
400	0.50	1.5148	1.3811	1.3875	1.3811	1.3811	1.1179	1.1267	1.1196	1.0794	1.1217
	0.70	1.5566	1.4157	1.4251	1.4157	1.4157	1.1409	1.1536	1.1427	1.0941	1.1460
	0.80	1.5894	1.4422	1.4542	1.4421	1.4422	1.1577	1.1740	1.1595	1.1032	1.1629
	0.90	1.6154	1.4683	1.4861	1.4682	1.4683	1.1686	1.1924	1.1700	1.1017	1.1715
	0.95	1.6778	1.4963	1.5249	1.4962	1.4963	1.1723	1.2099	1.1733	1.0954	1.1704
	0.99	2.1707	1.8246	1.8343	1.8232	1.8246	1.3084	1.4395	1.3097	1.0949	1.1732

Table 5: MSE of the estimators for ϕ=0.1 and p=20

		Available Estimators					*Proposed Estimators*				
n	*E*	*IGML*	*IGRR*	*IGL*	*IGKL*	*IGMRT*	*IGH1*	*IGH2*	*IGH3*	*IGH4*	*IGH5*
25	0.50	2.7424	1.9600	1.8131	1.9196	1.9600	1.3512	1.5020	1.3525	1.0417	1.1299
	0.70	3.5610	2.4472	2.0663	2.3775	2.4472	1.5880	1.8312	1.5905	1.1218	1.1932
	0.80	4.7136	3.1382	2.3221	3.0312	3.1382	1.9125	2.2960	1.9164	1.2262	1.2552
	0.90	8.8093	5.6019	2.8404	5.3674	5.6019	3.0587	3.9535	3.0699	1.5738	1.3945
	0.95	18.636	11.505	3.3316	10.954	11.505	5.8322	7.9261	5.8685	2.4012	1.6025
	0.99	141.13	86.927	4.5791	82.562	86.927	41.841	58.836	42.820	14.796	3.1684
50	0.50	1.8261	1.5076	1.4994	1.5058	1.5076	1.1562	1.2108	1.1588	0.9131	1.0272
	0.70	2.0513	1.7826	1.7442	1.7790	1.7826	1.3289	1.4194	1.3332	0.9685	1.1055
	0.80	2.4992	2.1296	2.0217	2.1231	2.1296	1.5343	1.6785	1.5410	1.0331	1.1785
	0.90	3.9343	3.2334	2.7107	3.2169	3.2334	2.1798	2.5002	2.1911	1.2484	1.3470
	0.95	6.9428	5.5256	3.5984	5.4874	5.5256	3.4975	4.1991	3.5176	1.6997	1.5606
	0.99	32.284	24.727	5.0949	24.533	24.727	14.427	18.422	14.535	5.4777	2.1541
100	0.50	1.7632	1.5702	1.5972	1.5696	1.5702	1.2186	1.2663	1.2222	1.0603	1.1969
	0.70	1.9976	1.7649	1.7907	1.7641	1.7649	1.3492	1.4178	1.3537	1.1125	1.2834
	0.80	2.2982	2.0151	2.0284	2.0143	2.0151	1.5178	1.6128	1.5241	1.1809	1.3811
	0.90	3.2542	2.8172	2.7197	2.8166	2.8172	2.0660	2.2403	2.0792	1.4061	1.6353
	0.95	5.2713	4.5177	3.9269	4.5169	4.5177	3.2385	3.5729	3.2665	1.8923	2.0185
	0.99	21.694	18.233	8.1415	18.231	18.233	12.517	14.271	12.675	5.7634	2.9679
200	0.50	1.6358	1.4861	1.4959	1.4861	1.4861	1.1954	1.2105	1.1964	1.1139	1.1910
	0.70	1.6823	1.5262	1.5368	1.5261	1.5262	1.2248	1.2424	1.2258	1.1312	1.2137
	0.80	1.7269	1.5630	1.5746	1.5629	1.5630	1.2494	1.2710	1.2504	1.1385	1.2290
	0.90	1.8558	1.6651	1.6790	1.6648	1.6651	1.3113	1.3485	1.3127	1.1507	1.2612
	0.95	2.1205	1.8672	1.8747	1.8664	1.8672	1.4245	1.4991	1.4270	1.1718	1.3052
	0.99	4.3679	3.4698	2.9054	3.4633	3.4698	2.2029	2.6552	2.2168	1.3106	1.3709
400	0.50	1.5297	1.3944	1.4010	1.3944	1.3944	1.1284	1.1376	1.1295	1.0871	1.1319
	0.70	1.5785	1.4355	1.4448	1.4355	1.4355	1.1569	1.1698	1.1583	1.1040	1.1603
	0.80	1.6205	1.4704	1.4821	1.4704	1.4704	1.1803	1.1970	1.1818	1.1160	1.1824
	0.90	1.6781	1.5136	1.5312	1.5135	1.5136	1.2032	1.2289	1.2048	1.1178	1.2001
	0.95	1.7615	1.5707	1.5975	1.5706	1.5707	1.2271	1.2692	1.2289	1.1133	1.2100
	0.99	2.5252	2.1085	2.0852	2.1074	2.1085	1.4830	1.6566	1.4863	1.1257	1.2462

Table 6: MSE of the estimators for ϕ=0.25 and p=20

6 Application

The modeling performance of the suggested IGH estimators and other IGR biased estimators is evaluated using two applications.

6.1 Real Data I

The data of Algamal et al. [1] is used here to investigate the performance of the proposed IGH estimators. The explanatory variables are the fifteen molecular descriptors, and the response variable (y) is the biological activity

(IC50). Also, Algamal [2] showed that the response variable (y) fits very well to the inverse Gaussian distribution. In addition, the estimator of the dispersion parameter equals 2.737426. Moreover, the eigenvalues of $X'\hat{U}X$ are obtained as (1.629014e+08, 2.883754e+05, 1.828653e+04, 1.437752e+03, 8.611209e+01, 6.042544e+01, 3.405411e+01, 1.066466e+01, 6.686535e+00, 4.210205e+00, 2.176471e+00, 1.966039e+00, 1.333998e+00, 6.169995e-01, 3.767226e-01, and 8.399781e-02), where the condition number is calculated as 44038.09, indicating that the severe multicollinearity issue exists.

	Available Estimators					*Proposed Estimators*				
Coef.	*IGML*	*IGRR*	*IGL*	*IGKL*	*IGMRT*	*IGH1*	*IGH2*	*IGH3*	*IGH4*	*IGH5*
Inter.	0.0193	0.5562	0.4643	0.5327	0.5562	0.5180	0.5279	0.5180	0.3659	0.3624
X1	0.0163	0.0130	0.0115	0.0107	0.0130	0.0120	0.0106	0.0120	0.0085	0.0089
X2	1.2699	1.4157	1.0732	1.0607	1.4157	1.3112	1.0575	1.3112	0.9561	0.9844
X3	-0.6303	-0.1105	-0.1093	-0.1697	-0.1105	-0.1237	-0.1697	-0.1237	-0.1402	-0.1484
X4	-2.3938	-2.7886	-2.288	-1.2180	-2.7886	-2.2839	-1.1971	-2.2839	-0.8029	-0.9372
X5	-1.4131	-0.5261	-0.4565	-0.9124	-0.5261	-0.5476	-0.9027	-0.5476	-0.4798	-0.5269
X6	-2.2301	-0.0117	-0.9401	-0.1733	-0.0117	-0.0378	-0.1717	-0.0378	-0.2514	-0.3316
X7	-2.5112	-1.3886	-0.7902	0.1481	-1.3886	-1.005	0.1526	-1.0050	0.0238	-0.0976
X8	3.1058	2.3478	2.1615	1.0050	2.3478	1.9419	0.9906	1.9419	0.8508	0.9696
X9	0.4535	0.1972	0.2954	0.1978	0.1972	0.2098	0.1996	0.2098	0.2743	0.2754
X10	0.0015	0.0027	0.0041	0.0035	0.0027	0.0034	0.0035	0.0034	0.0059	0.0056
X11	-1.2407	0.4156	0.5561	0.2381	0.4156	0.3507	0.2346	0.3507	0.1980	0.2376
X12	-0.0016	0.0022	0.0006	0.0077	0.0022	0.0021	0.0075	0.0021	0.0002	0.0002
X13	-0.7685	-0.5745	-0.4214	1.3969	-0.5745	-0.2973	1.3704	-0.2973	0.2329	0.1937
X14	-0.1514	-0.1643	-0.2092	-0.7656	-0.1643	-0.2231	-0.7538	-0.2231	-0.2873	-0.2928
X15	-0.5245	-0.1116	-0.0212	-0.0325	-0.1116	-0.0856	-0.0313	-0.0856	-0.0003	-0.0076
MSE	50.4703	11.2091	5.2296	11.1511	11.2091	4.7892	9.2655	4.7893	4.7890	9.2655

Table 7: The coefficients and the MSE of the estimators

Table 7 lists the estimated coefficients and the MSE values of the estimators. The estimators of the biasing parameters are calculated as, $\hat{k}$=0.2837919, $\hat{d}$=1.311941e-06, $\hat{k}_{IGKL}$=0.08260334, $\hat{k}_{IGMRT}$=0.2837916, $\hat{k}_{IGH1}$=0.2837919, $\hat{k}_{IGH2}$=0.08260334, $\hat{k}_{IGH3}$=0.2837916, $\hat{k}_{IGH4}$=1.315407, $\hat{k}_{IGH5}$=1.146912. The findings in Table 7 demonstrate that most of the suggested IGR hybrid estimators have MSEs that are significantly lower than those of the IGML and other biased estimators. The IGML, due to the exaggerated behavior of the MSE, is the estimator that is most adversely affected when the regressors are highly correlated with each other. The MSE is the highest for the IGML estimator when compared to IGRR, IGL, IGKL, IGMRT, and the suggested IGH estimators. All five of the suggested IGR hybrid estimators (IGH1, IGH2, IGH3, IGH4, and IGH5) perform better than the previously defined estimators (IGML, IGRR, IGKL, and IGMRT). The performance of the suggested IGR hybrid estimators in general is better than that of the others, especially the IGH1, IGH3, and IGH4. The IGH4 estimator, which has the lowest MSE value, has been suggested as the most effective IGR hybrid estimator.

6.2 Real Data II

Here, we use the Liver Cirrhosis Death Rate data set from Brownlee [8] to assess the performance of the estimators. The sample size consists of 46 observations, and the response variable y shows the standard mortality rate from liver cirrhosis with four explanatory variables, where x_1 denotes the population's proportion, x_2 is the number of children born to women aged 45-49 years, x_3 is the wine consumed by each person, and x_4 is the alcoholic beverage consumed. According to Akram et al. [3], the response variable (y) fits the inverse Gaussian distribution quite well. The estimator of the dispersion parameter equals 46.79699. Also, the eigenvalues of $X'\hat{U}X$ are obtained as (3.923936e+07, 1.021376e+06, 1.409038e+05, 3.813459e+04, and 7.688713e+01), where the condition is calculated as 714.388, indicating that the severe multicollinearity issue exists.

	Available Estimators					*Proposed Estimators*				
Coef.	*IGML*	*IGRR*	*IGL*	*IGKL*	*IGMRT*	*IGH1*	*IGH2*	*IGH3*	*IGH4*	*IGH5*
Inter.	-1.7525	-1.5415	-1.7295	-1.5284	-1.5415	-1.1683	-1.4298	-1.1709	-0.6471	-1.4366
X1	0.1292	0.1311	0.1294	0.1312	0.1311	0.1345	0.1321	0.1345	0.1395	0.1320

X2	*1.3131*	*1.3045*	*1.3122*	*1.3039*	*1.3045*	*1.2891*	*1.2999*	*1.2892*	*1.2671*	*1.3002*
X3	*2.1395*	*2.1394*	*2.1395*	*2.1394*	*2.1394*	*2.139*	*2.1393*	*2.139*	*2.1378*	*2.1393*
X4	*0.0589*	*0.0598*	*0.0590*	*0.0598*	*0.0598*	*0.0613*	*0.0602*	*0.0613*	*0.0637*	*0.0602*
MSE	*0.6103*	*0.4758*	*0.5948*	*0.4680*	*0.4758*	*0.2793*	*0.4115*	*0.2805*	*0.2790*	*0.4115*

Table 8: The coefficients and the MSE of the estimators

Table 8 states the estimated coefficients and MSE values of the estimators. The estimators of the biasing parameters are obtained as, $\hat{k}$= *10.22299,* $\hat{d}$= *0.00566305,* $\hat{k}_{IGKL}$=*5.110812,* $\hat{k}_{IGMRT}$= *10.16543,* $\hat{k}_{IGH1}$= *10.22299,* $\hat{k}_{IGH2}$=*5.110812,* $\hat{k}_{IGH3}$=*10.16543,* $\hat{k}_{IGH4}$=*24.90995,* $\hat{k}_{IGH5}$=*4.990987. The findings in Table 8 demonstrate that all of the suggested IGR hybrid estimators have MSEs that are significantly lower than those of the IGML estimator and other biased estimators. The IGML, due to the exaggerated behavior of the MSE, is the estimator that is most adversely affected when the regressors are highly correlated with each other. The MSE is the highest for the IGML estimator when compared to IGRR, IGL, IGKL, IGMRT, and the suggested IGH estimators. All five of the suggested IGR hybrid estimators (IGH1, IGH2, IGH3, IGH4, and IGH5) perform better than the previously defined estimators (IGML, IGRR, IGKL, and IGMRT). Moreover, the performance of the suggested IGR hybrid estimators is better than that of the others, which have the lowest MSE values.*

7 Conclusion

We present a novel IGR hybrid estimator in this study. We list the features of our suggested IGR hybrid estimator. The suggested IGR hybrid estimator was also conceptually compared to the traditional IGML and other well-known biased estimators such as IGRR, IGL, IGKL, and IGMRT. Through a simulation, the performance of our suggested IGR hybrid estimators with respect to other estimators as well as among themselves was evaluated. A variety of elements are used in the simulation experiment to monitor the performance of the suggested IGR hybrid estimators from various angles. Based on the results of the simulation, we discovered that at least one of the suggested IGR hybrid estimators performed better than traditional estimators in terms of decreased MSE and might be advised. Not to mention, an empirical application demonstrates the value of the suggested IGR hybrid estimators. In terms of decreased MSE, the suggested IGH for the IGR model significantly outperformed the IGML and the other IGR-biased estimators in this situation. As a result, we advise researchers to employ IGH with appropriate biasing parameter estimators if they attempt to fit the IGR model with the multicollinearity problem.

Declarations

Competing interests*: The author declares that he has no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.*
Authors' contributions*: Issam Dawoud: Conceptualization, Methodology, Software, Validation, Formal Analysis, Resources, Original Draft Writing Preparation, Review and Editing, Visualization, and Supervision.*
Funding*: This research received no external funding.*
Availability of data and materials*: Data and materials will be made available on request.*
Acknowledgments*: Author is grateful to anonymous referees and editor for their valuable comments and suggestions, which certainly improved the presentation and quality of the paper.*

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