Jordan Journal of Mathematics and Statistics.
Yarmouk University

DOI:https://doi.org/10.47013/18.1.7

Stochastic Analysis of Repairable System Comprised Two Subsystems With k-out-of-n: G Operational Policy and Copula Repair Approach

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Received: Nov. 6, 2023 Accepted:July 2,2024

Abstract: The finest performable systems are needed everywhere. The present paper deals with system performance assessment consisting of two subsystems in a series arrangement. The first subsystem follows the k-out-of-n: G work policy, and the second subsystem consists of three units and operates under the 2-out-of-4: G scheme. The failure rates are considered constants and obey negative exponential distribution. Two types of repair, general repair, and copula, are employed for repairing the models partial failure and complete breakdown states. Supplementary variable and Laplace transformation methods were used to study the traditional reliability measures for different values of system parameters. The probabilistic estimates, like system availability and reliability analysis, MTTF, and profit analysis, have been computed with Maple software

Keywords: System Availability; MTTF; System Reliability; Cost Analysis; Gumbel-Hougaard family copula;

1 INTRODUCTION

Modeling is intricate, but predicting is even more difficult due to the unknown factors of the system and sworking environment. Because of the vast number of complex systems and applications in everyday safety and economic welfare, reliability computing, which entails extensive modeling and prediction, has recently sparked a lot of attention. Any system where the majority of the units are designed to be repairable has to have its transient availability modeled in order to assess and enhance its efficacy. One popular technique that helps increase the systems availability and dependability is redundancy. A wide range of designs, including those for motor vehicles, streetlight systems, parking systems, aircraft, nuclear power plants, and many more, have been found to use redundancy. Moreover, the most prevalent kinds of redundancy are cold standby, worm, and active. Functional redundancy is prepared to act at all times. A k-out-of-n: G redundancy requires that k of n units remain active at all times. In terms of system reliability, failure is a natural phenomenon. The nature of the failure rate is constant, even it may follow various distribution patterns; in contrast, the repair rate is both constant and changeable. Whenever the system failure rates are consistent and repair rates are variable, the system performance is studied using the Markova method with the supplementary variable approach. A wide range of literature has been studied using Markova methods and Laplace transforms implications.
In the literature concerning the system performance assessment in the nineteenth century, the researchers

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assessed the repairable system with the idea of a particular restoration, which was unsuitable when the system was utterly broken downstage. In numerous real-world scenarios, it is imperative to promptly restore the entire damaged state; in the unfortunate circumstance that such a scenario is identified; the system state must be restored by utilizing a copula, Nelson R. B. [8]. To cite a few works of literature with k-out-of-n: G/F operational scheme Chao[2], Singh and Rawal [21], Singh[17] [18] Monika[3] amp;[4], Poonia [11], [12] and others studied the system performance under different types of failure and multi-repair tactic. Alka Munjal and S. B. Singh [7] analyzed a complex system composed of two 2-out-of-3: G subsystems in the parallel configuration using a supplementary variable approach with the general repair. A deliberation on reliability evaluation and optimization of a series-parallel system with k-out-of-n: G subsystems and mixed redundancy have been presented by Boddu and Xing [1].
Reliability computations for systems with outsourced components have been studied by Yangquan Sun[22]. Singh and Ram [17] investigated a three-state system with two subsystems in series under distinctive types of failure and two types of restoration. A new method for analytically estimating the reliability of consecutive k-out-of-n: F systems were devised by G. Gokhan[5]. Considering the assumption of cost-free repair across warranty policy, authors Ram Niwas and Harish Garg [10] have examined the industrial systems efficiency and dependability using the Markov method and supplemental variable methodology. Singh analysis [15] used supplementary variables and the Laplace transform to examine a complex system in a degraded state. The traditional reliability measures were computed for different failures and copula repair approaches. Singh[19] employed a copula linguistics repair approach to study a computer-based test (CBT) model system by forecasting performance for various system parameter values. Niaki and Yaghoubi [9] estimated the mean time to failures (MTTF) and reliability of a 1-out-of-n: G cold standby system with imperfect switching using a precise approach and a closed form. Copula repair was applied by Praveen Poonia [11] to analyze the performance of multistate computer networks in series configuration. The operational reliability metrics of linear sequential 2-out-of-4 systems coupled to a 2-out-of-4 supporting device was studied by Ibrahim Yusuf[23]. Reliability metrics of the complex system in combinations of two subsystems in a series configuration and copula repair scheme were investigated by Dhruv Raghav [13], [14]. Recently a specific type of k –out –of –n: G; type of configuration in the system with three subsystems with k degraded working states have been addressed by authors H. I. Ayagi [6]

1.1 Description of the model

There is widespread literature on system presentation evaluations through the traditional measures for repairable systems presented in which most five units are taken to the studies. Treating the above discussion in view, in this paper, the authors have examined the performance of a complex system having two subsystems in a series arrangement. The first subsystem consists of n units and employs the k-out-of-n G policy; however, the second subsystem comprises four vague units and operates on a 2-out-of-4 G scheme.
The switching device regulates the operation of the subsystem units, and a switching failure is regarded as a total failure. A human operator is supposed to operate the system, and any human error or fault is treated as a complete shutdown state.Here are four different states for the system: completely failed, minor degraded, major degraded, and perfect. In order to restore failed states, two distinct two distinct repair types are utilized: copula repair and general repair.

By the probability influences, the following states are possible as; $S_0, S_1, S_2, S_3, ..., S_10$, presented in the state transition diagram can be categorized as the set of states A, B, C, D denoted as;

A= Set of perfect state= S_0

B= Set of minor degraded states= $S_1, S_2, S_5,$

C= Set of major degraded states=$S_3, S_6,$

D= complete failed states= $S_4, S_7, S_8, S_9, S_{10}$.

Fig. 1: State Transition Diagram of Model

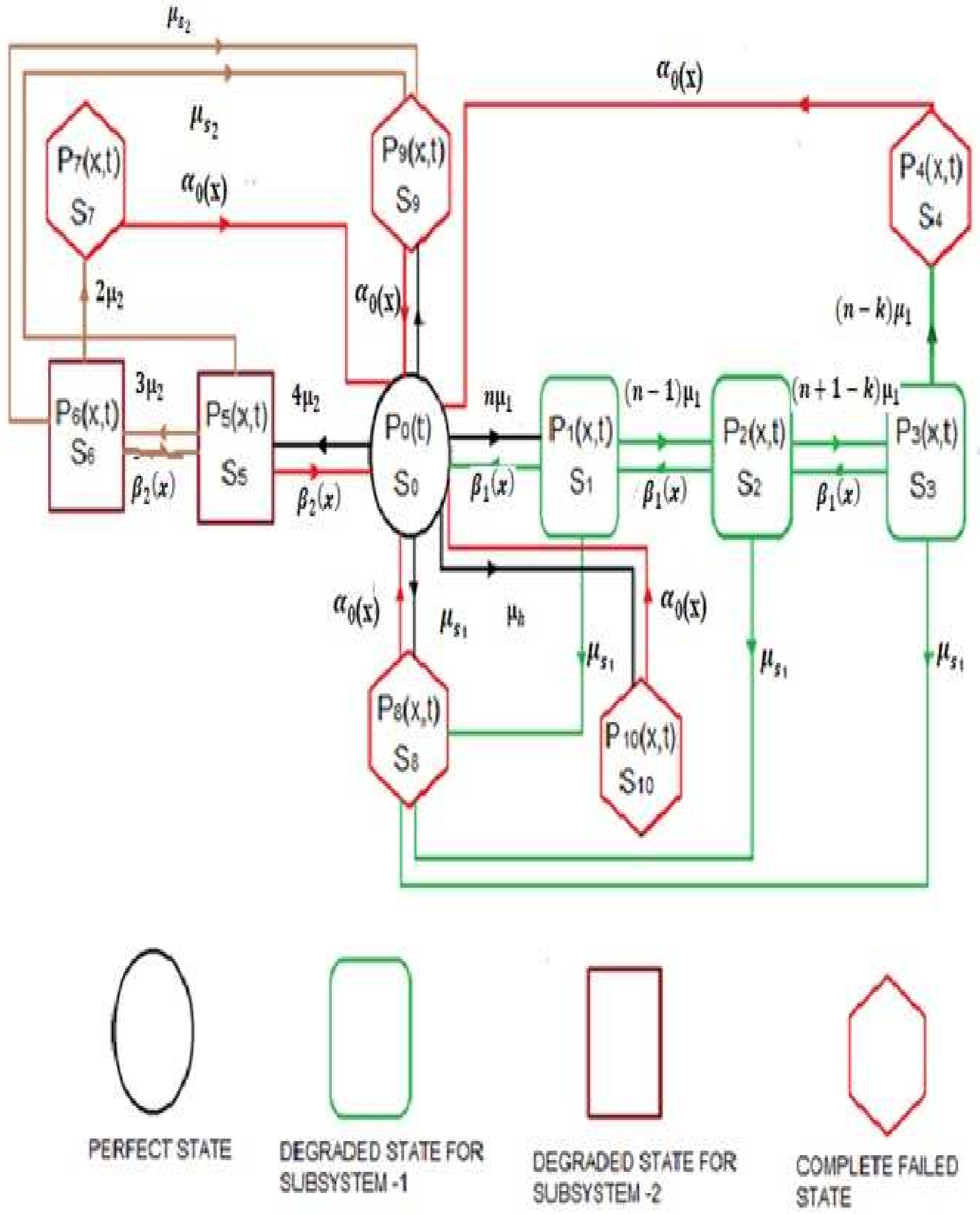

Table 1: State Description in detail

State No	Description	State No.	Description
S_0	Perfect state	S_5	Minor degraded state due to 1 unit fail in subsystem 2
S_1	Minor degraded 1 unit fail in subsystem 1	S_6	Major degraded state due to 2 unit fail in subsystem 2
S_2	Minor degraded two unit fail in subsystem 1	S_7	Complete failed state due to 3 unit fail in subsystem 2
S_3	Major degraded k unit fail in subsystem 1	S_8	Complete failed state due to switch failure in subsystem1
S_4	Complete fail state due to (k+1) failing in subsystem 1	S_9	Complete failed state due to switch failure in subsystem 2
S_{10}	Complete failed state due to human failure		

1.2 Expectations secondhand for the study of the model:

While studying the model, the formerly mentioned assumption is prepared.

1.The system is first in the state S_0, in which all subsystems and switching devices are pristine,with probability P_0 (0) =1.

2.When the partial failure surpasses operating policy k-out-of-n: G, the system ceases to perform functions, and its ability to perform is reduced.

3.A total failure circumstance renders the system inaccessible.

4.General distribution repairs partially failed states, but copula distribution improves complete failure.

5.All failure rates are constant and follow an exponential distribution.

6.At least k units are prerequisite to be operative to keep subsystem 1 properly functioning.

7.Repaired system components work resembling new ones, and fixing does not loss anything.

1.3 Notations used for the study of the model

Table 2: Terminology of system variables and representations

t	The variable t represents time variable for all expressions.
s	Laplace transform variable.
μ_1 / μ_2	Failure rates for both subsystem units respectively.
μ_{s_1}/ μ_{s_2}/μ_h	The failure rates of the switch for both subsystems 1&2 and human failure.
$\beta_1(x)$/β_2 (x)	Repair rates of units of subsystem 1/ subsystem 2.
α_0(x)	Repair rate for all complete failed states of the system, i.e., S_4, S_7, S_8, S_9, and S_{10}.
P_0(t)	This represent the state transition probability of S_0 state.
$\overline{P}(s)$	It is the notation of Laplace transformation of state transition P (t).
P_i(x, t)	The probability that a system is in state S_i, undergoing repair, with an elapsed repair time of x,t, for $i = 1$ to 10.
$E_p(t)$	Represents the predictable profit through the interval [0, t).
K_1 / K_2	K_1/K_2 stands revenue/service cost per unit time.in the time interval [0, t) respectively.
$\alpha_0(x) = C_\theta(u_1(x), u_2(x))$	The joint probability function (from failed state S_j to good state S_0) is defined as follows using the Gumbel-Hougaard family copula: for $1 \leq \theta \leq \infty$, $C_\theta(u_1, u_2(x)) = \exp\left[x^\theta + \{\log\beta(x)\}^\theta\right]^{\frac{1}{\theta}}$, $u_1 = \beta(x)$, and $u_2 = e^{\beta x}$

2 Mathematical Modelling of the System

The system cannot move to any other state if it is in the state S_0 at time t and will remain there until the time interval [t, t + Δt]. If it is in another failed state, it must return to S_0 after being restored. Assuming the condition of transition of the preexisting mathematical model a combination of probability constraints, the following mentioned state equations can be develop from figure 1.

$$S_0:\left(\frac{\partial}{\partial t}+n\mu_1+\mu_{s_1}+\mu_{s_2}+4\mu_2+\mu_h\right)Po\,(t)=\int_0^\infty\beta_1\,(x)\,P_1\,(x,t)\,dx+\int_0^\infty\beta_2\,(x)\,P_5\,(x,t)\,dx+\int_o^\infty\alpha_0\,(x)\,P_4\,(x,t)\,dx$$
$$+\int_o^\infty\alpha_0\,(x)\,P_7\,(x,t)\,dx+\int_o^\infty\alpha_0\,(x)\,P_8\,(x,t)\,dx+\int_o^\infty\alpha_0\,(x)\,P_9\,(x,t)\,dx+\int_o^\infty\alpha_0\,(x)\,P_{10}\,(x,t)\,dx \qquad (1)$$

$$S_1:\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+(n-1)\,\mu_1+\mu_{s_1}+\beta_1\,(x)\right)P_1\,(x,t)=0 \qquad (2)$$

$$S_2:\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+(n-k)\mu_1+\mu_{s_1}+\beta_1\,(x)\right)P_2\,(x,t)=0 \qquad (3)$$

$$S_3:\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+(n-(k+1))\mu_1+\mu_{s_1}+\beta_1\,(x)\right)P_3\,(x,t)=0 \qquad (4)$$

$$S_4:\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\alpha_0\,(x)\right)P_4\,(x,t)=0 \qquad (5)$$

$$S_5:\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\;3\mu_2+\mu_{s_2}+\beta_2\,(x)\right)P_5\,(x,t)=0 \qquad (6)$$

$$S_6:\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\;2\mu_2+\mu_{s_2}+\beta_2\,(x)\right)P_6\,(x,t)=0 \qquad (7)$$

$$S_7:\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\alpha_0\,(x)\right)P_7\,(x,t)=0 \qquad (8)$$

$$S_8:\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\alpha_0\,(x)\right)P_8\,(x,t)=0 \qquad (9)$$

$$S_9:\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\alpha_0\,(x)\right)P_9\,(x,t)=0 \qquad (10)$$

$$S_{10}:\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\alpha_0\,(x)\right)P_{10}\,(x,t)=0 \qquad (11)$$

Equations for the Boundary Conditions

$$P_1\,(0,t)=n\mu_1P_0\,(t)\,,\quad P_2\,(0,t)=n(n-1)\mu_1^2\,P_0(t)\;,\;P_3\,(0,t)=n\,(n-1)\,(n-k)\,\mu_1^3\,P_0\,(t)\,,$$
$$P_4\,(0,t)=n\,(n-1)\,(n-k)\,(n-1-k)\,\mu_1^4\,P_0\,(t)\,,\;P_5\,(0,t)=4\mu_2P_0\,(t)\,,\;P_6\,(0,t)=12\mu_2^2P_0\,(t)\,,\;P_7\,(0,t)=\;24\mu_2^3\,P_0(t),$$
$$P_8\,(0,t)=\mu_{s_1}[1+n\mu_1+n\,(n-1)\,\mu_1^2+(\mathrm{n-k})\,\mu_1^3]\,P_0(t),$$
$$P_9\,(0,t)=\mu_{s_2}\left(1+4\mu_1+12\mu_1^2\right)P_0\;(t)\,,P_{10}\,(0,t)=\mu_h\,P_0(t) \qquad (12)$$

Initial condition

$$P_0\;(0)=1,\;P_j\,(x,0)=0\;for\;j=1,2,\;3,\;4\ldots\ldots..10 \qquad (13)$$

3 Solution of the Model

Through using the Laplace transform of equations (1) - (11) and the initial condition, namely, $P_0(0) = 1$ and other state transition probability at t=0 are zero, one can derive the equations in the following manner:
$[L[P_0(t)] = \overline{P}_0(s)]$ $[L[\acute{P}_0(t)] = s\overline{P}_0(s) - P_0(0)]$.

$$(s+n\mu_1+\mu_{s_1}+\mu_{s_2}+4\,\mu_2+\mu_h)\,\overline{P}_0(s) = 1+\int_0^\infty \beta_1(x)\,\overline{P}_1(x,s)\,dx+\int_0^\infty \beta_2(x)\,\overline{P}_5(x,s)\,dx+\int_0^\infty \alpha_0(x)\,\overline{P}_4(x,s)\,dx$$

$$+\int_0^\infty \alpha_0(x)\,\overline{P}_8(x,s)\,dx+\int_0^\infty \alpha_0(x)\,\overline{P}_7(x,s)\,dx+\int_0^\infty \alpha_0(x)\,\overline{P}_9(x,s)dx+\int_0^\infty \alpha_0(x)\,\overline{P}_{10}(x,s)dx \tag{14}$$

$$\left(s+\frac{\partial}{\partial x}+(n-1)\mu_1+\mu_{s_1}+\beta_1(x)\right)\overline{P}_1(x,s)=0 \tag{15}$$

$$\left(s+\frac{\partial}{\partial x}+(n-1)\mu_1+\mu_{s_1}+\beta_1(x)\right)\overline{P}_2(x,s)=0 \tag{16}$$

$$\left(s+\frac{\partial}{\partial x}+(n-k)\mu_1+\mu_{s_1}+\beta_1(x)\right)\overline{P}_3(x,s)=0 \tag{17}$$

$$\left(s+\frac{\partial}{\partial x}+\alpha_0(x)\right)\overline{P}_4(x,s)=0 \tag{18}$$

$$\left(s+\frac{\partial}{\partial x}+4\mu_2+\mu_{s_2}+\beta_2(x)\right)\overline{P}_5(x,s)=0 \tag{19}$$

$$\left(s+\frac{\partial}{\partial x}+3\mu_2+\mu_{s_2}+\beta_2(x)\right)\overline{P}_6(x,s)=0 \tag{20}$$

$$\left(s+\frac{\partial}{\partial x}+\alpha_0(x)\right)\overline{P}_7(x,s)=0 \tag{21}$$

$$\left(s+\frac{\partial}{\partial x}+\alpha_0(x)\right)\overline{P}_8(x,s)=0 \tag{22}$$

$$\left(s+\frac{\partial}{\partial x}+\alpha_0(x)\right)\overline{P}_9(x,s)=0 \tag{23}$$

$$\left(s+\frac{\partial}{\partial x}+\alpha_0(x)\right)\overline{P}_{10}(x,s)=0 \tag{24}$$

Laplace Transform of the Boundary Conditions

$$\overline{P}_1(0,s)=n\mu_1\overline{P}_0(s),\ \overline{P}_2(0,s)=n(n-1)\mu_1^2\,\overline{P}_0(s)\ ,\overline{P}_3(0,s)=n(n-1)(n-k)\mu_1^3\overline{P}_0(s)$$

$$\overline{P}_4(0,s)=n(n-1)(n-k)(n-1-k)\mu_1^4\overline{P}_0(s),\ \overline{P}_5(0,s)=4\mu_2\overline{P}_0(s)$$

$$\overline{P}_6(0,s)=12\mu_2^2\overline{P}_0(s),\overline{P}_7(0,s)=\ 24\mu_2^3\overline{P}_0(s)\,.\overline{P}_8(0,s)=\mu_{s_1}[1+n\mu_1+n(n-1)\mu_1^2+(\mathrm{n-k})\mu_1^3]\,\overline{P}_0(s)$$

$$\overline{P}_9(0,s)=\mu_{s_2}\left(1+4\mu_1+12\mu_1^2\right)\overline{P}_0\ (s)\,,\overline{P}_{10}(0,s)=\mu_h\,P_0(s) \tag{25}$$

Solving equation (11)- (24) with consequence of equation (25) and representations

$$S_\beta(x)=\beta(x)e^{-\int_0^\infty \beta(x)}\quad ,\overline{S}_\beta(s)=\int_0^\infty e^{-sx}S_\beta(x)dx,\overline{S}_\beta(s)=\int_0^\infty (e^{-sx}\beta(x)e^{\int_0^\infty \beta(x)dx})dx$$

$$\overline{P}_0\,(s) = \frac{1}{D\,(s)} \tag{26}$$

$$\overline{P}_1\,(s) \;= \frac{n\mu_1}{D(s)} \left[\frac{1-\overline{S}_{\beta 1}(s+(n-1)\,\mu_1+\mu_{s_1})}{(s+(n-1)\,\mu_1+\mu_{s_1})}\right] \tag{27}$$

$$\overline{P}_2\,(s) = \; \frac{n(n-1)\mu_1^2}{D(s)} \; \left[\frac{1-\overline{S}_{\beta 1}\,(s+(n-k)\,\mu_1+\mu_{s_1})}{(s+(n-k)\,\mu_1+\mu_{s_1})}\right] \tag{28}$$

$$\overline{P}_3\,(s) = \frac{n(n-1)(n-k)\mu_1^3}{D(s)} \left[\frac{1-\overline{S}_{\beta 1}\,(s+(n-1-k)\,\mu_1+\mu_{s_1})}{(s+(n-1-k)\,\mu_1+\mu_{s_1})}\right] \tag{29}$$

$$\overline{P}_4\,(s) = \frac{n(n-1k)(n-k)(n-1-k)}{D(s)} \left(\frac{1-\overline{S}_{\alpha_0(s)}}{s}\right) \tag{30}$$

$$\overline{P}_5\,(s) = \; \frac{4\mu_2}{D(s)} \left(\frac{1-\overline{S}_{\beta_1}(s+3\mu_2+\mu_{s_2})}{(s+3\mu_2+\mu_{s_2})}\right) \tag{31}$$

$$\overline{P}_6\,(s) = \frac{12\mu_2^2}{D(s)} \; \left(\frac{1-\overline{S}_{\beta_1}(s+2\mu_2+\mu_{s_2})}{(s+2\lambda_2+\lambda_{s_2})}\right) \tag{32}$$

$$\overline{P}_7\,(s) = \; \frac{24\mu_2^3}{D(s)} \; \left(\frac{1-\overline{S}_{\alpha_0(s)}}{s}\right) \tag{33}$$

$$\overline{P}_8\,(s) = \; \frac{A}{D(s)} \quad \left(\frac{1-\overline{S}_{\alpha_0(s)}}{s}\right) \tag{34}$$

$$\overline{P}_9\,(s) = \frac{\mathrm{B}}{D(s)} \left(\frac{1-\overline{S}_{\alpha_0(s)}}{s}\right) \tag{35}$$

$$\overline{P}_{10}\,(s) = \frac{\mu_h}{D(s)} \left(\frac{1-\overline{S}_{\alpha_0(s)}}{s}\right) \tag{36}$$

Where; A=μ_{s1} [1+$n\mu_1+n\,(n-1)\,\mu_1^2+(\mathrm{n}-\mathrm{k})\,\mu_1^3$], B=$\mu_{s2}\left(1+4\mu_1+12\mu_1^2\,\right)$

$$\overline{P}_{up}\,(s) = \sum \overline{P}_i\,(s)\,,$$

Sum of Laplace transform of state transition probabilities of operational states, i.e.,

$$S_0,\; S_1,\; S_2, S_3, S_4,\; S_5,\; S_6$$

$$\overline{P}_{down}\,(s) = 1-\overline{P}_{up}\,(s)$$

$$\overline{P}_{up}\,(s) = \frac{1}{D(s)} \left[\begin{array}{c} 1+\frac{n\ \ \mu_1}{(s+(n-1)\mu_1+\mu_{s1}+\beta_1)}+\frac{n(n-1)\mu_1^2}{(s+(n-1)\mu_1+\mu_{s1}+\beta_1)}+ \\ \frac{n(n-1)(n-k)\mu_1^3}{(s+(n-k)\mu_1+\mu_{s_1}+\beta_1)}+\frac{4\mu_2}{\left(s+3\mu_2+\mu_{s_2}+\beta_2\right)}+\frac{12\mu_2}{\left(s+3\mu_2+\mu_{s_2+}\beta_2\right)} \end{array}\right] \tag{37}$$

4 Analytical Study of the Model for the Particular Cases;

4.1 Availability analysis:

When the repairs follow a copula distribution. Setting: $\overline{S}_{\alpha_0}(s)=\frac{exp[\ x^{\theta}+\{log\beta_1\ (x)\}^{\theta}]^{1/\theta}}{s+exp[\ x^{\theta}+\{log\beta_1\ (x)\}^{\theta}]^{1/\theta}},\ \ \overline{S}_{\beta_i}(s)=\frac{\beta_i}{s+\beta_i},$
i= 1, 2 and using the following values for failure and repair rates; $\mu_1=0.01,\ \ \mu_2=0.02, \mu_{s_1}=0.03,\ \ \mu_{s_2}=0.03,\ \ \mu_h=0.025,\ \beta_1=1,\ \beta_2=1,\ \alpha_0=2.7183$ in equation (37) one can obtain the different expressions of system performance of a repairable system by using inverse Laplace transform.
Availability analysis of the system via copula repair for different failures:
$A=Case1.$
$\mu_1=0.01,\ \mu_2=0.02,\ \ \mu_h=0.025,\ \mu_{s_1}=0.03,\ \ \mu_{s_2}=0.03,\ \beta_1=1,\ \ \beta_2=1,\ \alpha_0=2.7183,$ *n=50, k=30*

$$A:=0.032974e^{-2.8333t}-0.11167e^{-1.8469t}-0.14283e^{-1.0379t}+1.2212e^{-0.27523t}+0.0020373e^{-1.0700t}-0.0016654e^{-1.2300t}-0.00001679e^{-1.2200t}\qquad(38a)$$

$B=Case1.$
$\mu_1=0.01,\ \mu_2=0.02,\ \ \mu_h=0.025,\ \mu_{s_1}=0.03,\ \ \mu_{s_2}=0,\ \beta_1=1,\ \ \beta_2=1,\ \alpha_0=2.7183,$ *n=50, k=30)*

$$B:=0.0019771e^{-1.0400t}+0.032102e^{-2.88316t}-0.11205e^{-1.8425t}-0.15073e^{-1.0054t}+1.2304e^{-0.25382t}-0.0017057e^{-1.2300t}-0.000017287e^{-1.2200t}\qquad(38b)$$

$C=Case1.$
$\mu_1=0.01,\ \ \mu_2=0.02, \mu_3=0.03,\ \ \mu_h=0.025,\ \ \mu_{s_1}=0,\ \ \mu_{s_2}=0,\ \ \beta_1=1,\ \beta_2=1,\ \ \alpha_0=2.7183,$ *n=50, k=30*

$$C:=-0.000017171e^{-1.1900t}-0.0017032e^{-1.2000t}+0.0019892e^{-1.0400t}+0.011314e^{-2.7563t}-0.10724e^{-1.8426t}-0.16317e^{-1.0057t}+1.2488e^{-0.26874t}\qquad(38c)$$

$D=Case1.$
$\mu_1=0.01,\ \mu_2=0.02, \mu_{s_1}=0,\ \ \mu_{s_2}=0,\ \ \mu_h=0,\ \beta_1=1\ \ \beta_2=1,\ \alpha_0=2.7183,$ *n=50, k=30)*

$$D:=-0.000017287e^{-1.1900t}-0.0017147e^{-1.2000t}+0.00008895e^{-2.7186t}-0.10514e^{-1.8536t}-0.15609e^{-1.0051t}+1.2609e^{-0.27101t}+0.0019765e^{-1.0400t}\qquad(38d)$$

Applying the following equations 38a, 38b, 38c, and 38d, respectively, we obtain different values of availability for different values of time variable, as shown Figure 2a.

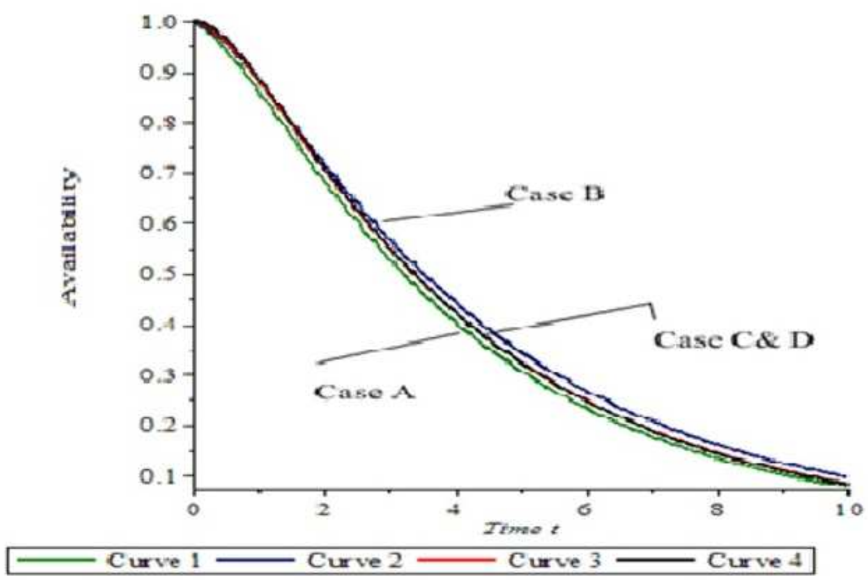

Fig. 2: (2a) Availability variation for copula repair

4.2 Availability for the general repair

When all repair trails general distribution keeping all failure rates the same for copula repair, one can obtain the subsequent expressions presented in (39a, 39b, 39c & 39d) for general repair distribution;

$A = Case1$.

$\mu_1 = 0.01,\ \mu_2 = 0.02,\ \mu_h = 0.025,\ \mu_{s_1} = 0.03,\ \mu_{s_2} = 0.03,\ \beta_1 = \beta_2 = 1,\ \alpha_0 = 2.7183, n = 50, k = 30$

$$A = 0.001336e^{-1.0700t} - 0.000020124e^{-1.2200t} - 0.089531e^{-1.9159t} - 0.016063e^{-0.053739t} - 0.016063e^{-1.0497t} + 1.1223e^{-0.25983t} - 0.00198e^{-1.2300t} \quad (39a)$$

B= Case 2.

$\mu_1 = 0.01,\ \mu_2 = 0.02,\ \mu_h = 0.025,\ \mu_{s_1} = 0.03,\ \mu_{s_2} = 0,\ \beta_1 = \beta_2 = 1,\ \alpha_0 = 2.7183,$ *n=50, k=30)*

$$B := 8.47380*10^{-9}\mathrm{e}^{-1.04900t} + 0.0014628\mathrm{e}^{-1.040000t} + 0.00000763\mathrm{e}^{-1.050000t} - 0.0511981\mathrm{e}^{-1.100494t} - 0.026778\mathrm{e}^{-1.0497t} - 0.0267779\mathrm{e}^{-1.0497t} - 1.10328377\mathrm{e}^{-0.152219t} \quad (39b)$$

$C = Case\ 3$.

$\mu_1 = 0.01,\ \mu_2 = 0.02,\ \mu_h = 0.025,\ \mu_{s_1} = 0,\ \mu_{s_2} = 0,\ \beta_1 = 1,\ \beta_2 = 1,\ \alpha_0 = 2.7183,$ *n=50, k=30) B:=*

$$-0.99615\mathrm{e}^{-1.8657t} - 0.056608\mathrm{e}^{-1.01290t} - 0.056608\mathrm{e}^{-1.01290t} + 1.2131\mathrm{e}^{-0.26356t} - 0.0015566\mathrm{e}^{-1.0400t} - 0.0018086\mathrm{e}^{-1.2000t} - 0.00001828\mathrm{e}^{-1.1900t} \quad (39c)$$

$D = Case4$.

$\mu_1 = 0.01,\ \mu_2 = 0.02,\ \mu_\mathrm{h} = 0,\ \mu_{\mathrm{s}_1} = 0,\ \mu_{\mathrm{s}_2} = 0,\ \beta_1 = 1,\ \beta_2 = 1,\ \alpha_0 = 2.7183, n = 50, k = 30)$

$$D = -0.000017287e^{-1.1900t} + 0.0017147e^{-1.2000t} + 0.000088949e^{-2.7186t} - 0.105147e^{-1.8536t} - 0.15609e^{-1.0051t} + 1.2609e^{-0.2710t} + 0.0019765e^{-1.0400t} \quad (39d)$$

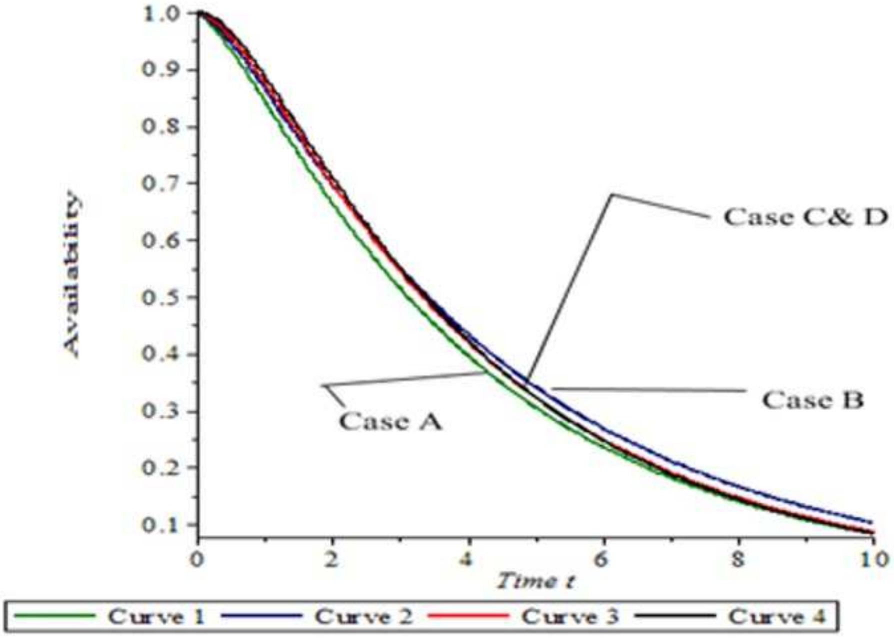

Fig. 3: (2b) Availability variation corresponds to General repair

4.3 Reliability

The system performance is affirmed to be reliability metric when all repairs have been deemed to be zero. Applying zero to each repair in equation (36) of $P_{up}(s)\,expression.$, i.e., $\beta_1 = \beta_2 = 0$ *and* $\alpha_0(x) = 0$ and for

some values of failure rates as $\mu_1 = 0.01, \ \mu_2 = 0.02, \ \mu_h = 0.025, \ \ \mu_{s_1} = 0.03, \ \ \mu_{s_2} = 0.03.$
The following four case have studied for the same values of failure and repair rates keeping n=50 & k=30.
$Case\,A : \mu_1 = 0.01, \ \mu_2 = 0.02, \ \mu_h = 0.025, \ \ \mu_{s_1} = 0.03, \ \ \mu_{s_2} = 0.03, n = 50, k = 30$
$Case\,B : \mu_1 = 0.01, \ \mu_2 = 0.02, \ \mu_h = 0.025, \ \ \mu_{s_1} = 0.03, \ \ \mu_{s_2} = 0, \ n = 50, k = 30$
$Case\,C : \mu_1 = 0.01, \ \mu_2 = 0.02, \ \mu_h = 0.025, \ \ \mu_{s_1} = 0, \ \ \mu_{s_2} = 0, \ n = 50, k = 30$
$CaseD : \mu_1 = 0.01, \ \mu_2 = 0.02, \ \mu_h = 0.025, \ \ \mu_{s_1} = 0, \ \ \mu_{s_2} = 0, \ n = 50, k = 30$

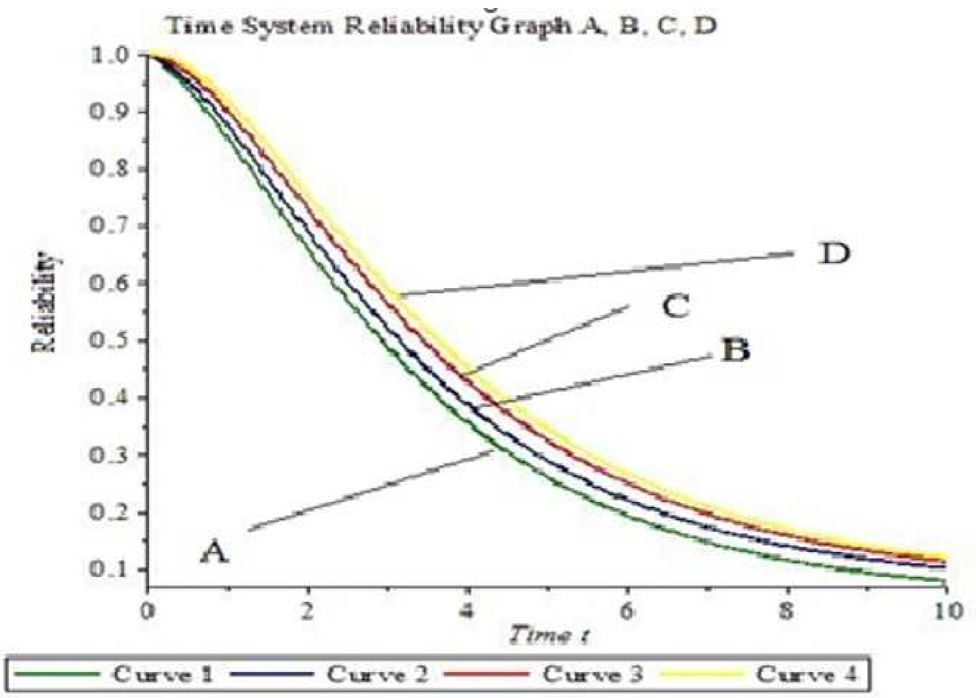

Fig. 4: (2c) Reliability variation of the non-repairable system

5 Profit Analysis/Cost Analysis:

Formulation (40) can be utilized in order to determine the expected profit $E_p(t)$, if revenue generation K_1 and service expenses K_2 are both per unit time in the interval [0, t]. Concerning the acquisition of parametric failures and repaired rates metrics, $\mu_1 = 0.01, \ \mu_2 = 0.02, \mu_h = 0.025, \ \ \mu_{s_1} = 0.03, \ \ \mu_{s_2} = 0.03, \ \ \beta_1, = 1, \ \beta_2 = 1, \ \ \alpha_0 = 2.7183$. Employing equation (36), one is able to obtain the expression for an expected profit $E_p(t)$, from system operation in the interval [0,T), as specified in equation (41) using Maple software output, assuming that the maintenance facility is always easily accessible.

$$E_p(t) = K_1 \int_0^t P_{up}(t) - K_2 t \tag{40}$$

5.1 Cost Analysis for copula repair

Presume for the sake of argument all the repair satisfies the general and Gumbel Hougaard family copula distributions. As a consequence, we get the following expression employing equation (36) in the equation used in the cost function.

$$E_p(t).E_p(t) = K_1[0.18216x10^{-3}e^{-1.1050t} + 0.19294x10^{-5}e^{1.1450t} - 0.013276e^{-2.8314t} + 0.032900e^{-1.6345t}\}$$

$$+K_1[-0.10870x10^{-3}e^{-1.1291t} - 17.444e^{-0.058264t} + 0.88994x10^{-4}e^{-1.1550\,t} + 17.423] - K_2t \qquad (41)$$

Table 3 is obtained by setting K_1= 1 and K_2 = 0.6, 0.4, 0.2, and 0 and varying time t. The profit variation concerning time t is depicted in Figure 3 and table 2.

Table 3: : Expected profit for copula repair

Time t	Expected Profit for K_1=1& K_2			
	K_2=0.6	K_2=0.4	K_2=0.2	K_2=0
0	0	0	0	0
1	0.357	0.572	0.756	0.957
2	0.609	1.009	1.409	1.809
3	0.756	1.356	1.956	2.556
4	0.807	1.607	2.407	3.207
5	0.775	1.775	2.775	3.775
6	0.670	1.869	3.070	4.270
7	0.500	1.900	3.301	4.701
8	0.276	1.876	3.476	5.076
9	0.004	1.804	3.603	5.403
10	-0.31	1.689	3.689	5.700

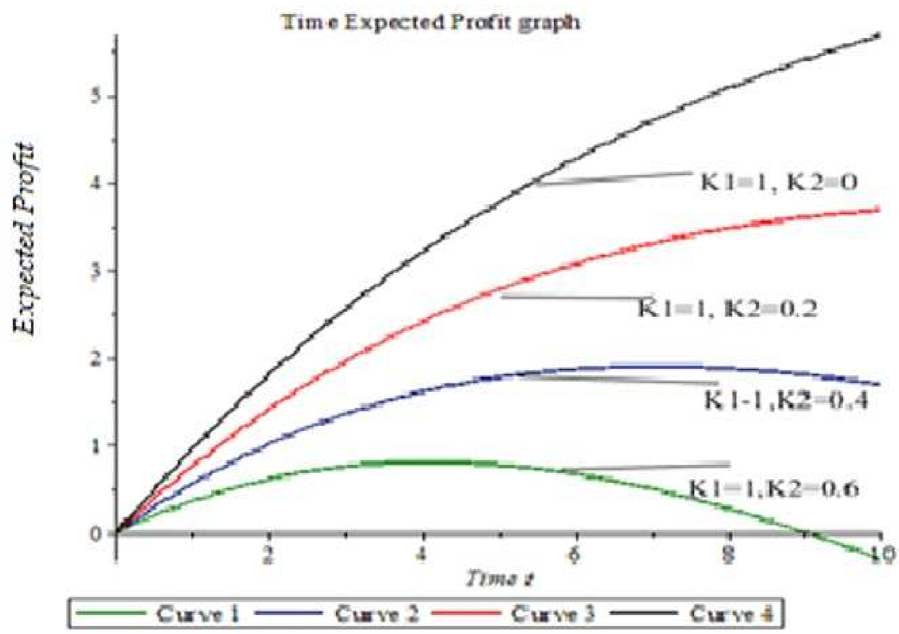

Fig. 5: (3) Expected profit graph for copula repair

5.2 Cost Analysis for General Repair

While the repair exclusively follows a general distribution, one may utilize expression (36) in our computation for the cost function $E_p(t)$and obtain the following expression for fixed values of failure rates when they are

Table 4: Expected profit in [0,t), $t = 0, 1, 2, 3, \cdots, 10$

Time t	Expected Profit K_1=1, and K_2			
	K_2=0.6	K_2=0.4	K_2=0.2	K_2=0
0	0	0	0	0
1	0.348	0.548	0.748	0.948
2	0.580	0.980	1.380	1.780
3	0.705	1.305	1.905	2.505
4	0.739	1.539	2.339	3.138
5	0.693	1.693	2.693	3.693
6	0.579	1.779	2.979	4.179
7	0.405	1.805	3.204	4.605
8	0.178	1.778	3.378	4.978
9	-0.095	1.705	3.504	5.305
10	-0.490	1.591	3.591	5.591

in the availability analysis section.

$$E_P(t) = K_1[0.010637e^{-1.7884t} - 0.0030391e^{-1.1402t} - 0.026476e^{-1.0251t} - 12.167e - {}^{0.081279t}$$
$$+0.79927x10^{-4}e^{-1.2550t} + 0.16203x10^{-5}e^{-1.2450t} + 0.37118x10^{-3}e^{-\ 1.1050t} + 12.186] - K_1t \quad (42)$$

Using different values of time t in equation (42) as; t = 0, 1, 2, 3. . . .10, one can obtain the values of expected profit presented in table 3b and the corresponding table 3 and figure 6..

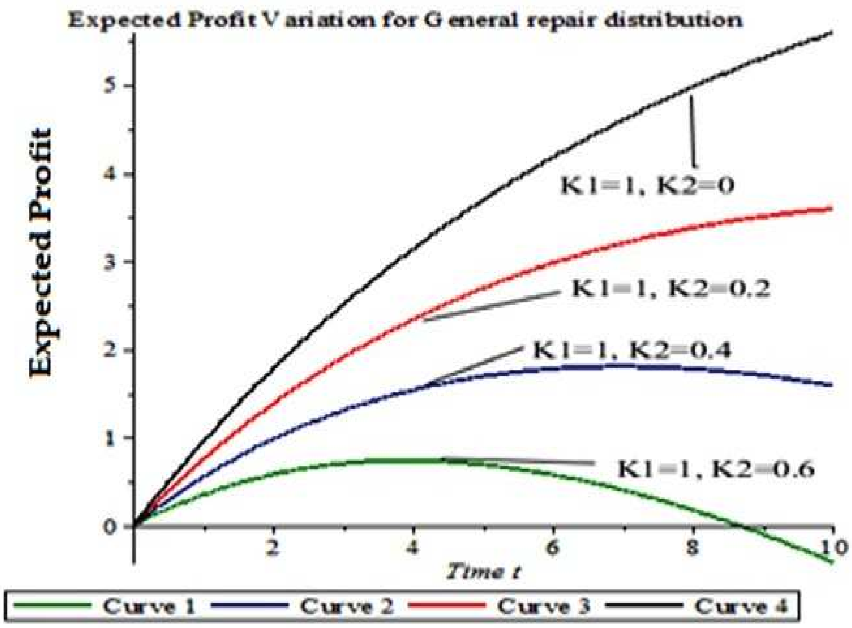

Fig. 6: Expected profit graph for General repair

5.3 Mean Time to failure (MTTF)Analysis

The (MTTF) is an extremely significant indicator in system operation that reveals which subsystem or unit requires considerably greater attention. The system's average failure time is determined by the rates of failure of its subsystems. An expression of MTTF corresponding to malfunction rates can be obtained mathematically by setting the limit of s tend to zero and taking all repair rates, i.e., $\beta_1,\ \beta_2$ & α_0 to zero in equation (42). Essentially., $MTTF = \overline{P}_{up}(s)$, with all repair zero.

$$MTTF = \frac{1}{(n\mu_1 + 4\mu_2 + \mu_h + \mu_{s_1} + \mu_{s_2})}\left[1 + \frac{n\mu_1}{(n-1)\mu_1+\mu_{s_1}} + \frac{n(n-1)\mu_1^2}{(n+1-k)\mu_1+\mu_{s_1}} + \frac{n(n-1)(n+1-k)\mu_1^3}{(n+1-k)\mu_1+\mu_{s_1}} + \frac{4\mu_2}{3\mu_2+\mu_{s_2}} + \frac{12\mu_2^2}{2\mu_2+\mu_{s_2}}\right] \quad (43)$$

We can find the values of the MTTF corresponding to the failure rate μ_1 by setting $\mu_1 = 0.01,\ \mu_2 = 0.02, \mu_h = 0.025, \mu_{s_1} = 0.03,\ \ \mu_{s_2} = 0.03$ and varying the failure rates μ_j from 0.01 to 0.1 with an incremental variation of 0.01 in each next value, in equation (43) the interpretation of MTTF with respective failure rate have computed in the table.

Table 5: Variation of MTTF with failure rates.

Failure Rates	MTTF μ_1	MTTF μ_2	MTTF μ_h	MTTF μ_{s_1}	MTTF μ_{s_2}
0.01	4.421	4.277	4.523	4.621	4.995
0.02	2.569	4.421	4.454	4.520	4.676
0.03	1.818	4.401	4.388	4.421	4.421
0.04	1.413	4.327	4.323	4.327	4.211
0.05	1.158	4.235	4.261	4.238	4.034
0.06	0.985	4.137	4.200	4.142	3.881
0.07	0.858	4.041	4.141	4.069	3.747
0.08	0.762	3.950	4.083	4.00	3.628
0.09	0.687	3.858	4.027	3.912	3.521
0.1	0.626	3.773	3.973	3.838	3.424

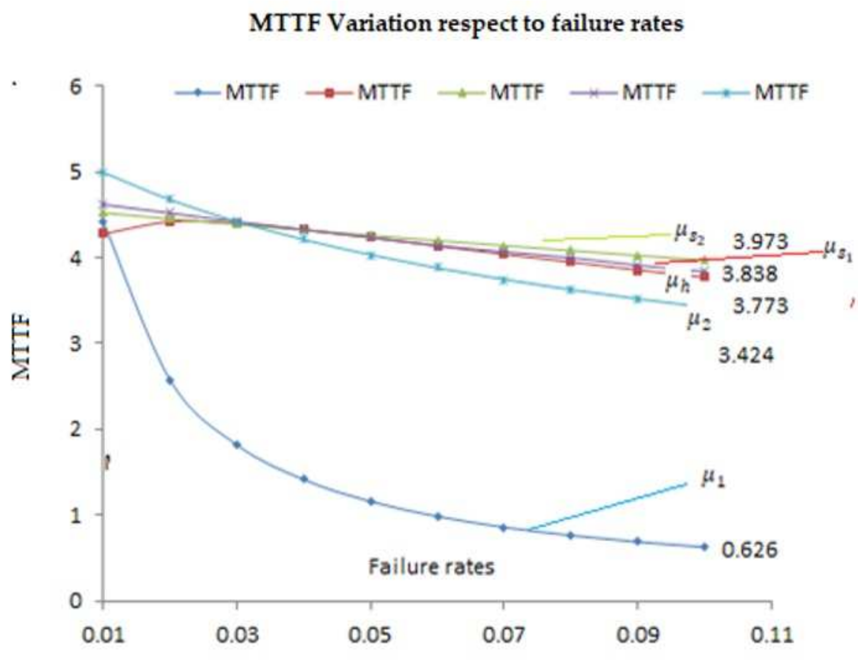

Fig. 7: MTTF variation conceiving to failure rates

5.4 Result Discussion and Conclusions

When failure rates are fixed at different levels, as $\mu_1 = 0.01,\ \mu_2 = 0.02, \mu_h = 0.03, \mu_{s_1} = 0.03,\ \ \mu_{s_2} = 0.025,\ \beta_1, = 1,\ \beta_2 = 1,\ \alpha_0 = 2.7183,\ n = 50,\ k = 30$ the Figure.2 demonstrates the availability variation of the system for copula repair four different cases, however, the figure 3 give information of system performance for availability analysis concern to the single repair facility. From the adjacent graph in Figures 2& 3, the availability of case A is lower, and case B's is higher, but for a general repair, it is lower than copula repair. The variation for patients C& D is almost similar for both restorations. The system's availability drops for the precise predicted failure rate values and, provided sufficient time, subsequently settles at zero. As a result, one can accurately predict the system's future behavior at any time for any given set of parametric values, exhibited by the graphical reflection of the model. It observed from keeping the fixed value of n=50 and varying k =30 that the availability decreases in both the cases of copula repair and general repair. It also noticed that system performance is better when the repair follows two types of distribution. The vacillation in the reliability of a non-repairable system is shown in Figure 4, and the system performance is relatively low compared to a repairable system. In the reliability study, we observed that the system reliability for case A is the lowest, and for case D it is the highest. It is a prediction that when failure effects are ignored, the system reliability improves. When failure rates are fixed at different levels, as $\mu_1 = 0.01,\ \mu_2 = 0.02, \mu_h = 0.03, \mu_{s_1} = 0.03, \mu_{s_2} = 0.025,\ \beta_1, = 1,\ \beta_2 = 1,\ \alpha_0 = 2.7183,\ n = 50,\ k = 30$ the Figure.2 demonstrates the availability variation of the system for copula repair four different cases, however, the figure 3 give information of system performance for availability analysis concern to the single repair facility. Tables 3a and 3b show the predicted profit from the system's operation using the same set of system variables. It demonstrates how the anticipated profit increases over time. Furthermore, it's important to remember that profit reduces as service costs rise. Generally, when low service costs are compared to high service costs, the predicted profit is significant. The system's mean-time-to-failure (MTTF) for variations in $\mu_1,\ \mu_2, \mu_h, \mu_{s_1},\ and\ \mu_{s_2}$ is shown in figure 4. The rest of the parameters are kept constant. The failure rate linked to μ_1 is the one in which the variation in MTTF is lowest and decreases at a rapid rate. For $\mu_h,\ \mu_2, \mu_{s1},\ and\ \mu_{s2}$ the MTTF is higher but shows a decline in trend.

Declarations

We should like to thank the Editor in Chief of the journal, Prof. Mohammed Al- Refai, an Associate Editor, and the anonymous referees for their constructive comments and very useful suggestions and remarks which were most valuable for improvement in the final version of the paper. Authors also ensure that no funding is associate with this research paper.

References

[1] Boddu, P. and Xing, L., (2012). Reliability evaluation and optimization of series-parallel systems with k-out-of-n: G subsystems and mixed redundancy types. Proceedings of the Institution of Mechanical Engineers Part O Journal of Risk and Reliability 227(2):187-198

[2] Chao, M. T., Fu, J. C. and Koutras, M. V, (1995). Survey of Reliability studies of consecutive- k-out-of-n: F & related systems. IEEE Transactions on Reliability, 44(1): 120-127.

[3] Gahlot, M, Singh, V. V., Ayagi H. Ismail and Abdullahi, I. (2020). Stochastic Analysis of a two unit's complex repairable system with Switch and Human failure using Copula Approach. Life Cycle Reliability, and Safety Engineering, 9(1): 1-11.

[4] Gahlot, M, Singh, V. V., Ayagi H. Ismail and Goel, C. K. (2018). Performance assessment of repairable system in the series configuration under different types of failure and repair policies using Copula Linguistics. International Journal of Reliability and Safety, 12(4): 348-374.

[5] Gokdere, G., Gurcan, M, Kilic, M. B, (2016). A new method for computing the reliability of consecutive k-out-of-n: F systems. Open Physics, 14(1): 166-170.

[6] Hamisu Ismial Ayagi, V.V. Singh, Wan Zhong, (2023), Performance Assessment of k operational state system with k-out-of-n: G working scheme with implementing copula, International Journal of Reliability Quality and Safety Engineering, Vol. 30(2),: https://doi.org/10.1142/S0218539323500043.

[7] Munjal, A. and Singh, S. B. (2014). Reliability analysis of a complex repairable system composed of two 2-out-of-3: G subsystems connected in parallel. Journal of Reliability and Statistical Studies, 6: 89-111.

[8] Nelsen R. B. An Introduction to Copulas. 2nd edition. New York, Springer, 2006.

[9] Niaki, S. T. and Yaghoubi, A. (2021). Exact equations for the reliability and mean time to failure of 1-out-of-n cold standby system with imperfect switching. Journal of Optimization of Industrial Engineering, 14(2): 197-203.
[10] Niwas, R. and Garg, H. (2018). An approach for analyzing the reliability and profit of an industrial system based on the cost-free warranty policy. Journal of Brazilian Mechanical Sciences and Engineering, 40(5): 1-9.
[11] Poonia, P K. and Sirohi, A. (2020). Cost-benefit analysis of a k-out-of-n: G type of warm standby series system under catastrophic failure using copula linguistic. Int. J Reliab Risk Safety Theory and Applications, 3(1): 35-44.
[12] Poonia, P K., Sirohi, A. and Kumar, A. (2021). Cost analysis of a reparable warm standby k-out-of-n: G and 2-out-of-4: G system in the series configuration under catastrophic failure using copula repair. Life Cycle Reliab Saf Eng., 10(2): 121-133.
[13] Raghav, D., Poonia, P. K., Gahlot, M., Monika Gahlot, Singh, V. V, Ayagi, H. I, and Abdullahi, A. H. (2020). Probabilistic analysis of a system consisting of two subsystems in the series configuration under copula repair approach. Journal of Korean Mathematical Educational Society: Series B Pure and Applied Mathematics, 27(3): 135- 155.
[14] Raghav, D., Poonia, P. K. and Singh, V. V. (2021). Probabilistic analysis of complex repairable system involving two subsystems in a series arrangement with (k- out- of- n): G operation Scheme and copula repair approach. Reliability Theory & Applications, 16(2): 1-18.
[15] Singh, V. V, Poonia, P. K, Labran, J and Abdullahi, I. (2021). Probabilistic analysis of a multistate warm standby k-out-of-n: G; system in a series configuration using copula Linguists. Reliability Theory & Applications, 162: 22-39.
[16] Singh, V. V, Ram, M and Rawal, D. K. (2013). Cost analysis of an engineering system involving subsystems in a series configuration. IEEE Transactions on Automation Science and Engineering, 10(4): 1124-1130.
[17] Singh, V. V and Ram, M. (2014). Multistate *k-out-of-n*-type system analysis. Mathematics in Engineering, Science, and Aerospace, 5(3): 281-292.
[18] Singh, V. V, Singh, S. B, Ram, M and Goel, C. K. (2013). Availability, MTTF and cost analysis of a system having two units in a series configuration with the controller. International Journal of System Assurance and Management, 4(4): 341-352.
[19] Singh, V. V., Lado, A. K., Yusuf, I. and Abdullahi, A. H. (2020). Probabilistic Assessment of Computer-Based Test (CBT) Network System Using Copula Linguistic Approach. Journal of Reliability and Statistical Studies, 13(2): 401-428.
[20] Singh, V. V., Rawal, D. K. and Kumar, P. (2020), Performance Assessment of Linux Operating System Employing Copula Distribution Approach. Life Cycle Reliability and Safety Engineering, 9(4): 389-403.
[21] Singh, V. V. and Rawal, D. K. (2011). Availability analysis of a system having two units in a series configuration with the controller and human failure under different repair policies. International J. of Scientific and Engineering Research, 2 (10) 1- 9.
[22] Sun, Y., Sun, T., Pecht, M. G. and Yu, C. (2018). Computing lifetime distribution and reliability for a system with outsourced components (A case study). IEEE Access, 6: 31359-31365.
[23] Yusuf, I., Yusuf, B., Babagana, M., Sani, B. and Lawan, A. M, (2018). Some reliability chrematistics of linear consecutive 2-out-of-4 supporting device for operation. Int. Journal of Engg & Technology, 7(1): 135-139.