Jordan Journal of Mathematics and Statistics (JJMS), 16(1), 2023, pp 165 - 185
DOI: https://doi.org/10.47013/16.1.11

KRASNER (m, n)-HYPERRING OF FRACTIONS

M. ANBARLOEI

Abstract. The formation of rings of fractions and the associated process of localization are the most important technical tools in commutative algebra. Krasner (m, n)-hyperrings are a generalization of (m, n)-rings. Let R be a commutative Krasner (m, n)-hyperring. The aim of this research work is to introduce the concept of hyperring of fractions generated by R and then investigate the basic properties such hyperrings.

1. Introduction

The notion of Krasner hyperrings was introduced by Krasner for the first time in [17]. Also, we can see some properties on Krasner hyperrings in [22] and [26]. In [11], Davvaz and Vougiouklis defined the notion of n-ary hypergroups which is a generalization of hypergroups in the sense of Marty. The concept of (m, n)-ary hyperrings was introduced in [23]. Davvaz and et. al. introduced Krasner (m, n)-hyperrings as a generalization of (m, n)-rings and studied some results in this context in [24]. We can see some important hyperideals of the Krasner (m, n)-hyperrings in [1] and [14]. Also, Ostadhadi and Davvaz studied the isomorphism theorems of ring theory and Krasner hyperring theory which are derived in the context of Krasner (m, n)-hyperrings in [27]. Ameri and Norouzi introduced in [1] the notions of n-ary prime and n-ary primary hyperideals in a Krasner (m, n)-hyperring and proved some results in this respect. The notion of n-ary 2-absorbing hyperideals in a Krasner (m, n)-hyperring as a generalization of the n-ary prime hyperideals was introduced in

2010 *Mathematics Subject Classification.* 20N20, 19Y99, 20N15.
Key words and phrases. n-ary prime hyperideal, n-ary multiplicative subset, Krasner (m, n)-hyperring.

Received: Feb. 23, 2022 Accepted: Jun. 7, 2022 .

[3]. To unify the concepts of the prime and primary hyperideals under one frame, the notion of δ-primary hyperideals was defined in Krasner (m,n)-hyperrings [4]. The formation of rings of fractions and the associated process of localization are the most important technical tools in commutative algebra. They correspond in the algebra-geometric picture to concentratining attention on an open set or near a point, and the importance of these notions should be self-evident. Procesi and Rota in [28] have studied ring of fractions in Krasner hyperrings.

In this paper, we aim to define the notion of a hyperring of fractions of Krasner (m,n)-hyperrings and provide several properties of them. The paper is organized as follows. In section 2, we have given some basic definitions and results of n-ary hyperstructures which we need to develop our paper. In section 3, we have constructed the Krasner (m,n)-hyperring of fractions. In section 4, we have studied the hyperideals of Krasner (m,n)-hyperring of fractions. In section 5, we have investigated construction of qutient Krasner (m,n)-hyperring of fractions.

2. Preliminaries

In this section we recall some definitions and results about n-ary hyperstructures which we need to develop our paper.

A mapping $f : H^n \longrightarrow P^*(H)$ is called an n-ary hyperoperation, where $P^*(H)$ is the set of all the non-empty subsets of H. An algebraic system (H, f), where f is an n-ary hyperoperation defined on H, is called an n-ary hypergroupoid.

We shall use the following abbreviated notation:

The sequence $x_i, x_{i+1}, ..., x_j$ will be denoted by x_i^j. For $j < i$, x_i^j is the empty symbol. In this convention $f(x_1, ..., x_i, y_{i+1}, ..., y_j, z_{j+1}, ..., z_n)$ will be written as $f(x_1^i, y_{i+1}^j, z_{j+1}^n)$. In the case when $y_{i+1} = ... = y_j = y$ the last expression will be written in the form $f(x_1^i, y^{(j-i)}, z_{j+1}^n)$. For non-empty subsets $A_1, ..., A_n$ of H we define

$$f(A_1^n) = f(A_1, ..., A_n) = \bigcup\{f(x_1^n) \mid x_i \in A_i, i = 1, ..., n\}.$$

An n-ary hyperoperation f is called associative if

$$f(x_1^{i-1}, f(x_i^{n+i-1}), x_{n+i}^{2n-1}) = f(x_1^{j-1}, f(x_j^{n+j-1}), x_{n+j}^{2n-1}),$$

hold for every $1 \leq i < j \leq n$ and all $x_1, x_2, ..., x_{2n-1} \in H$. An n-ary hypergroupoid with the associative n-ary hyperoperation is called an n-ary semihypergroup. An

n-ary hypergroupoid (H, f) in which the equation $b \in f(a_1^{i-1}, x_i, a_{i+1}^n)$ has a solution $x_i \in H$ for every $a_1^{i-1}, a_{i+1}^n, b \in H$ and $1 \leq i \leq n$, is called an n-ary quasihypergroup, when (H, f) is an n-ary semihypergroup, (H, f) is called an n-ary hypergroup. An n-ary hypergroupoid (H, f) is commutative if for all $\sigma \in \mathbb{S}_n$, the group of all permutations of $\{1, 2, 3, ..., n\}$, and for every $a_1^n \in H$ we have $f(a_1, ..., a_n) = f(a_{\sigma(1)}, ..., a_{\sigma(n)})$. If an $a_1^n \in H$ we denote $a_{\sigma(1)}^{\sigma(n)}$ as the $(a_{\sigma(1)}, ..., a_{\sigma(n)})$. We assume throughout this paper that all Krasner (m, n)-hyperrings are commutative. If f is an n-ary hyperoperation and $t = l(n-1)+1$, then t-ary hyperoperation $f_{(l)}$ is given by

$$f_{(l)}(x_1^{l(n-1)+1}) = f(f(..., f(f(x_1^n), x_{n+1}^{2n-1}), ...), x_{(l-1)(n-1)+1}^{l(n-1)+1}).$$

Definition 2.1. [24] Let (H, f) be an n-ary hypergroup and B be a non-empty subset of H. B is called an n-ary subhypergroup of (H, f), if $f(x_1^n) \subseteq B$ for $x_1^n \in B$, and the equation $b \in f(b_1^{i-1}, x_i, b_{i+1}^n)$ has a solution $x_i \in B$ for every $b_1^{i-1}, b_{i+1}^n, b \in B$ and $1 \leq i \leq n$. An element $e \in H$ is called a scalar neutral element if $x = f(e^{(i-1)}, x, e^{(n-i)})$, for every $1 \leq i \leq n$ and for every $x \in H$.

An element 0 of an n-ary semihypergroup (H, g) is called a zero element if for every $x_2^n \in H$ we have $g(0, x_2^n) = g(x_2, 0, x_3^n) = ... = g(x_2^n, 0) = 0$. If 0 and $0'$are two zero elements, then $0 = g(0', 0^{(n-1)}) = 0'$ and so the zero element is unique.

Definition 2.2. [18] Let (H, f) be a n-ary hypergroup. (H, f) is called a canonical n-ary hypergroup if

(1) there exists a unique $e \in H$, such that $f(x, e^{(n-1)}) = x$ for every $x \in H$;

(2) for all $x \in H$ there exists a unique $x^{-1} \in H$, such that $e \in f(x, x^{-1}, e^{(n-2)})$;

(3) if $x \in f(x_1^n)$, then $x_i \in f(x, x^{-1}, ..., x_{i-1}^{-1}, x_{i+1}^{-1}, ..., x_n^{-1})$ for all i .

We say that e is the scalar identity of (H, f) and x^{-1} is the inverse of x. Notice that $e^{-1} = e$

Definition 2.3. [24] A Krasner (m, n)-hyperring is an algebraic hyperstructure (R, f, g) which satisfies the following axioms:

(1) (R, f) is a canonical m-ary hypergroup;

(2) (R, g) is an n-ary semigroup;

(3) the n-ary operation g is distributive with respect to the m-ary hyperoperation f

, i.e.,

$$g(a_1^{i-1}, f(x_1^m), a_{i+1}^n) = f(g(a_1^{i-1}, x_1, a_{i+1}^n), ..., g(a_1^{i-1}, x_m, a_{i+1}^n))$$

for every $a_1^{i-1}, a_{i+1}^n, x_1^m \in R$ and $1 \leq i \leq n$;

(4) 0 is a zero element (absorbing element) of the n-ary operation g, i.e.,

$$g(0, x_2^n) = g(x_2, 0, x_3^n) = ... = g(x_2^n, 0) = 0$$

for every $x_2^n \in R$.

We denote the Krasner (m,n)-hyperring (R, f, g) simply by R. We say that R is with scalar identity if there exists an element 1 such that $x = g(x, 1^{(n-1)})$ for all $x \in R$. In this paper, we assume that R is with scalar identity.

A non-empty subset S of R is said to be a subhyperring of R if (S, f, g) is a Krasner (m, n)-hyperring. Let I be a non-empty subset of R, we say that I is a hyperideal of R if (I, f) is an m-ary subhypergroup of (R, f) and $g(x_1^{i-1}, I, x_{i+1}^n) \subseteq I$, for every $x_1^n \in R$ and $1 \leq i \leq n$.

Definition 2.4. [1] A proper hyperideal I of a Krasner (m, n)-hyperring R is said to be an n-ary prime hyperideal if for hyperideals $I_1, ..., I_n$ of R, $g(I_1^n) \subseteq I$ implies that $I_1 \subseteq I$ or $I_2 \subseteq I$ or ...or $I_n \subseteq I$.

Lemma 2.1. *A proper hyperideal I of a Krasner (m, n)-hyperring R is an n-ary prime hyperideal if for all $x_1^n \in R$, $g(x_1^n) \in I$ implies that $x_1 \in I$ or ... or $x_n \in I$. (Lemma 4.5 in* [1]*)*

Definition 2.5. [1] Let R be a Krasner (m, n)-hyperring. A non-empty subset S of R is called n-ary multiplicative, if $g(s_1^n) \in S$ for $s_1, ..., s_n \in S$.

In this paper, we assume that $1 \in S$.

Definition 2.6. [1] A Krasner (m, n)-hyperring R is said to be an n-ary hyperintegral domain, if R is a commutative Krasner (m, n)-hyperring and $g(x_1^n) = 0$ implies that $x_1 = 0$ or $x_2 = 0$ or ... or $x_n = 0$ for all x_1^n.

Definition 2.7. [1] Let R be a Krasner (m, n)-hyperring. An element $x \in R$ is said to be invertible if there exists $y \in R$ with $1 = g(x, y, 1^{(n-2)})$.

Definition 2.8. [24] Let (R_1, f_1, g_1) and (R_2, f_2, g_2) be two Krasner (m,n)-hyperrings. A mapping $\phi : R_1 \longrightarrow R_2$ is called a homomorphism if for all $x_1^m \in R_1$ and $y_1^n \in R_1$ we have

$$\phi(f_1(x_1, ..., x_m)) = f_2(\phi(x_1), ..., \phi(x_m))$$
$$\phi(g_1(y_1, ..., y_n)) = g_2(\phi(y_1), ..., \phi(y_n)).$$

3. Krasner (m,n)-hypering of fractions

Let R be any Krasner (m,n)-hyperring and let S be an n-ary multiplicative subset of R such that $1 \in S$. We shall construct the Krasner (m,n)-hyperring of fractions $S^{-1}R$. We define a relation $\sim$ on $R \times S$ by $(r,s) \sim (r',s')$ if and only if there exists some $s \in S$ such that

$$0 \in g(s, f(g(r, s', 1^{(n-2)}), g(r', s, 1^{(n-2)}), 0^{(m-2)}), 1^{(n-2)}).$$

Theorem 3.1. The relation $\sim$ is an equivalence relation on $R \times S$.

Proof. Clearly, $\sim$ is reflexive and symmetric. Suppose that $(r_1, s_1) \sim (r_2, s_2)$ and $(r_2, s_2) \sim (r_3, s_3)$. Then there exist $s \in S$ such that

$$0 \in g(s, f(g(r_1, s_2, 1^{(n-2)}), -g(r_2, s_1, 1^{(n-2)}), 0^{(m-2)}), 1^{(n-2)})$$

and

$$0 \in g(s', f(g(r_2, s_3, 1^{(n-2)}), -g(r_3, s_2, 1^{(n-2)}), 0^{(m-2)}), 1^{(n-2)}).$$

Since

$$\begin{aligned} 0 &\in g(s, f(g(r_1, s_2, 1^{(n-2)}), -g(r_2, s_1, 1^{(n-2)}), 0^{(m-2)}), 1^{(n-2)}) \\ &= f(g(s, r_1, s_2, 1^{(n-3)}), -g(s, r_2, s_1, 1^{(n-2)}), 0^{(m-2)}), \end{aligned}$$

we get $g(s, r_2, s_1, 1^{(n-2)}) \in f(g(s, r_1, s_2, 1^{(n-3)}), 0^{(m-1)})$.

Thus we have

$$\begin{aligned} 0 &= g(g(s, s_1, 1^{(n-2)}), 0^{(n-1)}) \\ &\in g(g(s, s_1, 1^{(n-2)}), g(s', f(g(r_2, s_3, 1^{(n-2)}), -g(r_3, s_2, 1^{(n-2)}), 0^{(m-2)}), 1^{(n-2)}), \\ &\quad 1^{(n-2)}) \\ &= g(g(s, s_1, 1^{(n-2)}), f(g(s', r_2, s_3, 1^{(n-3)}), -g(s', r_3, s_2, 1^{(n-3)}), 0^{(m-2)}), 1^{(n-2)}) \\ &= f(g(s, s_1, s', r_2, s_3, 1^{(n-5)}), -g(s, s_1, s', r_3, s_2, 1^{(n-5)}), 0^{(m-2)}) \\ &= f(g(s', g(s, r_2, s_1, 1^{(n-3)}), s_3), -g(s, s_1, s', r_3, s_2, 1^{(n-3)}), 0^{(m-3)}) \\ &\subseteq f(g(s', f(g(s, r_1, s_2, 1^{(n-3)}), 0^{(m-1)}), s_3, 1^{(n-3)}), -g(s, s_1, s', r_3, s_2, 1^{(n-3)}), \\ &\quad 0^{(m-3)}) \end{aligned}$$

$= f(g(s, s', s_2, r_1, s_3), -g(s, s', s_2, r_3, s_1), 0^{(m-2)})$
$= g(g(s, s', s_2, 1^{(n-3)}), f(g(r_1, s_3, 1^{(n-2)}), -g(r_3, s_1, 1^{(n-2)}), 0^{(m-2)}), 1^{(n-2)}).$

Since $g(s, s', s_2, 1^{(n-3)}) \in S$, then $(r_1, s_1) \sim (r_3, s_3)$. Consequently, $\sim$ is transitive. □

We denote the equivalence class of (a, s) with $\frac{r}{s}$ and let $S^{-1}R$ denote the set of all equivalence classes. We endow the set $S^{-1}R$ with a Krasner (m, n)-hyperring structure, by defining the m-ary hyperoperation F and the n-ary operation G as follows:

$F(\frac{r_1}{s_1}, ..., \frac{r_m}{s_m}) = \frac{f(g(r_1, s_2^m, 1^{(n-m)}), g(s_1, r_2, s_3^m, 1^{(n-m)}), ..., g(s_1^{m-1}, r_m, 1^{(n-m)})}{g(s_1^m, 1^{(n-m)})}$

$= \{\frac{r}{s} \mid r \in f(g(r_1, s_2^m, 1^{(n-m)}), g(s_1, r_2, s_3^m, 1^{(n-m)}), ..., g(s_1^{m-1}, r_m, 1^{(n-m)}),\ s = g(s_1^m)\}$

$G(\frac{r_1}{s_1}, ..., \frac{r_n}{s_n}) = \frac{g(r_1^n)}{g(s_1^n)}.$

We need to show that F and G are well defined. If $\frac{r_1}{s_1} = \frac{r'_1}{s'_1}$, $\frac{r_2}{s_2} = \frac{r'_2}{s'_2}, ..., \frac{r_m}{s_m} = \frac{r'_m}{s'_m}$, then there exist $t_1, ..., t_m \in S$ such that

$$0 \in g(t_1, f(g(r_1, s'_1, 1^{(n-2)}), -g(r'_1, s_1, 1^{(n-2)}), 0^{m-2}), 1^{(n-2)}) \quad (1)$$

$$0 \in g(t_2, f(g(r_2, s'_2, 1^{(n-2)}), -g(r'_2, s_2, 1^{(n-2)}), 0^{m-2}), 1^{(n-2)}) \quad (2)$$

$$\vdots$$

$$0 \in g(t_m, f(g(r_m, s'_m, 1^{(n-2)}), -g(r'_m, s_m, 1^{(n-2)}), 0^{m-2}), 1^{(n-2)}). \quad (m)$$

g-producting (1) by $g(g(t_2^m, 1^{(n-m+1)}), g(1^{(n-m+1)}, s_2^m), g(1^{(n-m+1)}, {s'}_2^m), 1^{(n-3)})$,
(2) by $g(g(t_1, 1^{(n-m+1)}, t_3^m), (s_1, 1^{(n-m+1)}, s_3^m), g(s'_1, 1^{(n-m+1)}, {s'}_3^m), 1^{(n-3)})$
$\vdots$
(m) by $g(g(t_1^{m-1}, 1^{(n-m+1)}), g(s_1^{m-1}, 1^{(n-m+1)}), g({s'}_1^{m-1}, 1^{(n-m+1)}), 1^{(n-3)})$.

Thus we get

$0 \in g(g(t_1^m, 1^{(n-m)}), f(g(g({s'}_1^m, 1^{(n-m)}), g(r_1, s_2^m, 1^{(n-m)}), 1^{(n-2)}),$
$\quad -g((g(s_1^m, 1^{(n-m)}), g(r'_1, {s'}_2^m, 1^{(n-m)}), 1^{(n-2)}), 0^{(m-2)}), 1^{(n-2)})$

$0 \in g(g(t_1^m, 1^{(n-m)}), f(g(g({s'}_1^m, 1^{(n-m)}), g(r_1, s_1, s_3^m, 1^{(n-m)}), 1^{(n-2)}),$
$\quad -g(g(s_1^m, 1^{(n-m)}), g(r'_2.s'_1, {s'}_3^m, 1^{(n-m)}), 1^{(n-2)}), 0^{(m-2)}), 1^{(n-2)})$

$\vdots$

$0 \in g(g(t_1^m, 1^{(n-m)}), f(g({s'}_1^m, 1^{(n-m)}), g(r_m, s_1^{m-1}, 1^{(n-m)}), 1^{(n-2)}),$
$\quad -g(g(s_1^m, 1^{(n-m)}), g(r'_m, {s'}_1^{m-1}, 1^{(n-m)}), 1^{(n-2)}), 0^{(m-2)}), 1^{(n-2)}).$

Now, we have

$$0 \in f(f(g(g(t_1^m, 1^{(n-m)}), g(s'^m_1, 1^{(n-m)}), g(r_1, s_2^m, 1^{(n-m)}), 1^{(n-3)}),$$
$$g(g(t_1^m, 1^{(n-m)}), g(s'^m_1, 1^{(n-m)}), g(r_1, s_1, s_3^m, 1^{(n-m)}), 1^{(n-3)}),$$
$$\cdots,$$
$$g(g(t_1^m, 1^{(n-m)}), g(s'^m_1, 1^{(n-m)}), g(r_m, s_1^{m-1}, 1^{(n-m)}), 1^{(n-3)})),$$
$$-f(g(g(t_1^m, 1^{(n-m)}), g(s_1^m, 1^{(n-m)}), g(r'_1, s'^m_2, 1^{(n-m+1)}), 1^{(n-3)}),$$
$$g(g(t_1^m, 1^{(n-m)}), g(s_1^m, 1^{(n-m)}), g(r'_2.s'_1, s'^m_3, 1^{(n-m)}), 1^{(n-3)}),$$
$$\cdots,$$
$$g(g(t_1^m, 1^{(n-m)}), g(s_1^m, 1^{(n-m)}), g(r'_m, s'^{m-1}_1, 1^{(n-m)}), 1^{(n-3)}), 0^{(m-2)}).$$

We put $t = g(t_1^m, 1^{(n-m)}), s = g(s_1^m, 1^{(n-m)}))$ and $s' = g(s'^m_1, 1^{(n-m)})$.

Therefore we have

$$0 \in f(g(t, g(s', f(g(r_1, s_2^m, 1^{(n-m)}), g(r'_2.s'_1, s'^m_3, 1^{(n-m)}), \cdots, g(r_m, s_1^{m-1}, 1^{(n-m)}),$$
$$1^{(n-2)}), -g(t, g(s, f(g(r'_1, s'^m_2, 1^{(n-m)}), g(r'_2.s'_1, s'^m_3, 1^{(n-m)}), \cdots, g(r'_m, s'^{m-1}_1,$$
$$1^{(n-m)}), 1^{(n-2)}, 0^{(m-2)}).$$

Thus $F(\frac{r_1}{s_1}, ..., \frac{r_m}{s_m}) = F(\frac{r'_1}{s'_1}, ..., \frac{r'_m}{s'_m})$, i. e., F is well defined.

Now, suppose that $\frac{r_1}{s_1} = \frac{r'_1}{s'_1}$, $\frac{r_2}{s_2} = \frac{r'_2}{s'_2}, ..., \frac{r_n}{s_n} = \frac{r'_n}{s'_n}$, then there exist $t_1, ..., t_n \in S$ such that

$$0 \in g(t_1, f(g(r_1, s'_1, 1^{(n-2)}), -g(r'_1, s_1, 1^{(n-2)}), 0^{(m-2)}), 1^{(n-2)})$$
$$0 \in g(t_2, f(g(r_2, s'_2, 1^{(n-2)}), -g(r'_2, s_2, 1^{(n-2)}), 0^{(m-2)}), 1^{(n-2)})$$
$$\vdots$$
$$0 \in g(t_n, f(g(r_n, s'_n, 1^{(n-2)}), -g(r'_n, s'_n, 1^{(n-2)}), 0^{(m-2)}), 1^{(n-2)}).$$

Then we conclude that

$$0 \in f(g(g(t_1, r_1, s'_1, 1^{(n-3)}), g(t_2, r_2, s'_2, 1^{(n-3)}), ..., g(t_n, r_n, s'_n, 1^{(n-3)}), 1^{(n-m)}),$$
$$-g(g(t_1, r'_1, s_1, 1^{(n-3)}), g(t_2, r'_2, s_2, 1^{(n-3)}), ..., g(t_n, r'_n, s'_n, 1^{(n-3)}), 1^{(n-m)}), 0^{(m-2)}).$$

It means

$$0 \in f(g(g(t_1^n), g(r_1^n), g(s'^n_1), 1^{(n-3)}), -g(g(t_1^n), g(r'^n_1), g(s_1^n), 1^{(n-3)}), 0^{(m-2)}).$$

Put $t = g(t_1^n)$. We have

$$0 \in f(g(t, g(r_1^n), g(s'^n_1), 1^{(n-3)}), -g(t, g(r'^n_1), g(s_1^n), 1^{(n-3)}), 0^{(m-2)})$$

and so

$$0 \in g(t, f(g(g(r_1^n), g(s'^n_1), 1^{(n-2)}), -g(g(t, g(r'^n_1), g(s_1^n), 1^{(n-2)}), 0^{(m-2)}), 1^{(n-2)}).$$

It implies that $\frac{g(r_1^n)}{g(s_1^n)} = \frac{g(r'^n_1)}{g(s'^n_1)}$ and so $G(\frac{r_1}{s_1}, ..., \frac{r_n}{s_n}) = G(\frac{r'_1}{s'_1}, ..., \frac{r'_n}{s'_n})$, i. e., G is well defined.

Lemma 3.1. Let R be a Krasner (m,n)-hyperring and S be an n-ary multiplicative subset of R with $1 \in S$. Then:

1) For all $s \in S$, $\frac{0}{1} = \frac{0}{s} = 0_{S^{-1}R}$.

2) $\frac{r}{s} = 0_{S^{-1}R}$, for $r \in R, s \in S$ if and only if there exists $t \in S$ such that $g(t, r, 1^{n-2}) = 0$.

3) For all $s \in S$, $\frac{s}{s} = \frac{1}{1} = 1_{S^{-1}R}$.

4) $\frac{g(r,s^{(m-1)},1^{(n-m)})}{g(s',s^{(m-1)},1^{(n-m)})} = \frac{g(r,1^{(n-1)})}{g(s',1^{(n-1)})}$, for $r \in R$ and $s, s' \in S$.

Proof. (1) Let $t \in S$. Then for all $s \in S$ we have

$$\begin{aligned} 0 &= g(t, s, 0, 1^{(n-3)}) \\ &= g(t, g(0, s, 1^{(n-2)}), 1^{(n-2)}) \\ &= g(t, f(g(0, s, 1^{(n-2)}), 0^{(m-1)}), 1^{(n-2)}) \\ &= g(t, f(g(0, s, 1^{(n-2)}), -g(1, 0, 1^{(n-2)}), 0^{(m-2)}), 1^{(n-2)}). \end{aligned}$$

Then we conclude that $\frac{0}{1} = \frac{0}{s} = 0_{S^{-1}R}$. Now, we show that $\frac{0}{1} = 0_{S^{-1}R}$. Let $r \in R$ and $s \in S$. Then

$$\begin{aligned} F(\tfrac{r}{s}, \tfrac{0}{1}^{(m-1)}) &= \{\tfrac{u}{v} \mid u \in f(g(r, 1^{(n-1)}), g(0, s, 1^{(n-2)})^{(m-1)}), s = g(s, 1^{n-1})\} \\ &= \{\tfrac{u}{v} \mid u \in f(g(r, 1^{(n-1)}), 0^{(m-1)}), v = g(s, 1^{n-1})\} \\ &= \{\tfrac{u}{v} \mid u \in f(r, 0^{(m-1)}), s = g(s, 1^{n-1})\} \\ &= \{\tfrac{u}{v} \mid u = r, s = g(s, 1^{n-1})\}. \end{aligned}$$

Thus $F(\frac{r}{s}, \frac{0}{1}^{(m-1)}) = \frac{r}{s}$. Consequently $\frac{0}{1} = 0_{S^{-1}R}$.

(2) $(\Longrightarrow)$: Let $\frac{r}{s} = 0_{S^{-1}R}$ for $r \in R, s \in S$. By (1), we have $\frac{r}{s} = \frac{0}{1}$. Hence there exists $t \in S$ such that

$$0 \in g(t, f(g(r, 1^{n-1}), -g(0, s, 1^{(n-2)}), 0^{(m-2)}), 1^{(n-2)}).$$

Therefore $0 \in g(t, f(r, 0^{(m-1)}), 1^{(n-2)})$. It means $g(t, r, 1^{(n-2)}) = 0$.

$(\Longleftarrow)$: Let $g(t, r, 1^{(n-2)}) = 0$ for some $t \in S$. Then $0 = g(t, f(r, 0^{(m-1)}), 1^{(n-2)})$. Since $r = g(r, 1^{n-1})$ and $0 = g(0, s, 1^{(n-2)})$, we get

$$0 = g(t, f(g(r, 1^{n-1}), g(0, s, 1^{(n-2)}), 0^{(m-2)}), 1^{(n-2)}).$$

Then $\frac{r}{s} = \frac{0}{1}$ and so $\frac{r}{s} = 0_{S^{-1}R}$, by (1).

(3) Let $s \in S$. It is clear that $0 = g(0, 1^{(n-1)})$. Then we get

$$0 = g(1, f(g(s, 1, 0^{(n-2)}), -g(1, s, 1^{(n-2)}), 0^{(m-2)}), 1^{(n-2)}).$$

It means $\frac{s}{s} = \frac{1}{1}$. Now, we show that $\frac{1}{1} = 1_{S^{-1}R}$. Let $r \in R$ and $s \in S$. Then we have

$G(\frac{r}{s}, \frac{1}{1}^{(n-1)}) = \frac{g(r,1^{(n-1)})}{g(s,1^{(n-1)})} = \frac{r}{s}$.

This implies that $\frac{1}{1} = 1_{S^{-1}R}$.

(4) Let $r \in R$ and $s, s' \in S$. Clearly,

$$\begin{aligned} F(\tfrac{r}{s'}, \tfrac{0}{s}^{(m-1)}) &= \tfrac{f(g(r,s^{(m-1)},1^{(n-m)}),g(s',0,s^{(m-2)},1^{(n-m)})^{(m-1)})}{g(s',s,1^{(n-2)})} \\ &= \tfrac{f(g(r,s^{(m-1)},1^{(n-m)}),0^{(m-1)})}{g(s',s^{(m-1)},1^{(n-m)})} \\ &= \tfrac{g(r,s^{(m-1)},1^{(n-m)})}{g(s',s^{(m-1)},1^{(n-m)})}. \end{aligned}$$

On the other hand,

$$\begin{aligned} F(\tfrac{r}{s'}, \tfrac{0}{1}^{(m-1)}) &= \tfrac{f(g(r,1^{(n-1)}),g(s',0,1^{(n-2)})^{(m-1)})}{g(s',1^{(n-1)})} \\ &= \tfrac{f(g(r,1^{(n-1)}),0^{(m-1)})}{g(s',1^{(n-1)})} \\ &= \tfrac{g(r,1^{(n-1)})}{g(s',1^{(n-1)})}. \end{aligned}$$

□

Definition 3.1. Let R be a Krasner (m,n)-hyperring and S be an n-ary multiplicative subset of R with $1 \in S$. The mapping $\phi : R \longrightarrow S^{-1}R$, defined by $r \longrightarrow \frac{r}{1}$, is called natural map.

Theorem 3.2. The natural map ϕ is a homomorphism of Krasner (m,n)-hyperring.

Proof. Let R be a Krasner (m,n)-hyperring and S be an n-ary multiplicative subset of R with $1 \in S$. For all $r_1^m \in R$, we get

$$\begin{aligned} \phi(f(r_1^m)) &= \tfrac{f(r_1^m)}{1} \\ &= \tfrac{f(g(r_1,1^{(n-1)}),g(r_2,1^{(n-1)}),...,g(r_m,1^{(n-1)}))}{g(1^{(m)},1^{(n-m)})} \\ &= \{\tfrac{r}{1} \mid r \in f(g(r_1, 1^{(n-1)}), g(r_2, 1^{(n-1)}), ..., g(r_m, 1^{(n-1)})\} \\ &= F(\tfrac{r_1}{1}, ..., \tfrac{r_m}{1}) \\ &= F(\phi(r_1), ..., \phi(r_m)). \end{aligned}$$

Also, for all $r_1^n \in R$, we have

$$\begin{aligned} \phi(g(r_1^n)) &= \tfrac{g(r_1^n)}{1} \\ &= \tfrac{g(r_1^n)}{g(1^{(n)})} \\ &= G(\tfrac{r_1}{1}, ..., \tfrac{r_1}{1}) \\ &= G(\phi(r_1), ..., \phi(r_n)). \end{aligned}$$

□

Theorem 3.3. Let $\frac{r}{s}$ be an nonzero element of $S^{-1}R$. Then

1) For all $s \in S$, $\phi(s)$ is an invertible element of $S^{-1}R$.

2) If $\phi(r) = 0$, then there exists $t \in S$ such that $g(t, r, 1^{(n-2)}) = 0$.

3) $\frac{r}{s} = G(\phi(r), \phi(s)^{-1}, \frac{1}{1}^{(n-2)})$, for all $\frac{r}{s} \in S^{-1}R$.

Proof. (1) Let $s \in S$. Then we have

$$\begin{aligned} G(\tfrac{s}{1}, \tfrac{1}{s}, \tfrac{1}{1}^{(n-2)}) &= \tfrac{g(s,1^{(n-1)})}{g(1,s,1^{(n-2)})} \\ &= \tfrac{g(s,1^{(n-1)})}{g(s,1^{(n-1)})} \\ &= \tfrac{1}{1} \qquad \text{by Lemma 3.1 (3)} \\ &= 1_{S^{-1}R}. \end{aligned}$$

(2) It is clear by 3.1 (2).

(3) Let $\frac{r}{s} \in S^{-1}R$. Then

$$\begin{aligned} \tfrac{r}{s} &= \tfrac{g(r,1^{(n-1)})}{g(s,1^{(n-1)})} \\ &= G(\tfrac{r}{1}, \tfrac{1}{s}, \tfrac{1}{1}^{(n-2)}) \\ &= G(\phi(r), \phi(s)^{-1}, \tfrac{1}{1}^{(n-2)}). \end{aligned}$$

□

Theorem 3.4. Let (R_1, f_1, h_1) and (R_2, f_2, g_2) be two Krasner (m, n)-hyperrings and S be an n-ary multiplicative subset of R_1 with $1 \in S$. Let $k : R_1 \longrightarrow R_2$ be a homomorphism such that for each $s \in S$, $k(s)$ is an invertible element of R_2. Then there exists an unique homomorphism $h : S^{-1}R_1 \longrightarrow R_2$ such that $ho\phi = k$.

Proof. Let (R_1, f_1, h_1), (R_2, f_2, g_2) and $(S^{-1}R_1, G, F)$ be Krasner (m, n)-hyperrings such that S is an n-ary multiplicative subset of R_1 and $1 \in S$. Define mapping h from $S^{-1}R_1$ to R_2 as follows:

$$h(\tfrac{r}{s}) = g_2(k(a), k(s)^{-1}, 1^{(n-2)}).$$

We need to show that h is well defined. Let $\frac{r_1}{s_1} = \frac{r'}{s'}$. Then there exists $t \in S$ such that

$$\begin{aligned} 0 &\in g_1(t, f_1(g_1(r, s', 1^{(n-2)}), -g_1(r', s, 1^{(n-2)}), 0^{(m-2)}), 1^{(n-2)}). \\ &= f_1(g_1(t, r, s', 1^{(n-2)}), -g_1(t, r', s, 1^{(n-2)}), 0^{(m-2)}). \end{aligned}$$

Hence

$$\begin{aligned} 0 &\in k(f_1(g_1(t, r, s', 1^{(n-2)}), -g_1(t, r', s, 1^{(n-2)}), 0^{(m-2)}) \\ &= f_2(k(g_1(t, r, s', 1^{(n-2)}), k(-g_1(t, r', s, 1^{(n-2)}), k(0)^{(m-2)}) \\ &= f_2(k(g_1(g_1(t, 1^{(n-1)}), g_1(r, 1^{(n-1)}), g_1(s', 1^{(n-1)}), 1^{(n-3)})), \\ &\quad k(-g_1(g_1(t, 1^{(n-1)}), g_1(r', 1^{(n-1)}), g_1(s, 1^{(n-1)}), 1^{(n-3)})), k(0)^{(m-2)}) \\ &= f_2(g_2(k(g_1(t, 1^{(n-1)})), k(g_1(r, 1^{(n-1)})), k(g_1(s', 1^{(n-1)})), 1^{(n-3)}), \\ &\quad -g_2(k(g_1(t, 1^{(n-1)})), k(g_1(r', 1^{(n-1)})), k(g_1(s, 1^{(n-1)})), 1^{(n-3)})), k(0)^{(m-2)}) \\ &= f_2(g_2(k(t), k(r)), k(s'), 1^{(n-3)}), \\ &\quad -g_2(k(t), k(r'), k(s), 1^{(n-3)})), 0^{(m-2)}) \end{aligned}$$

$$= f_2(g_2(k(t), g_2(k(r)), k(s'), 1^{(n-2)}), 1^{(n-2)}),$$
$$-g_2(k(t), g_2(k(r'), k(s), 1^{(n-2)}), 1^{(n-2)}), 0^{(m-2)})$$
$$= g_2(k(t), f_2(g_2(k(r)), k(s'), 1^{(n-2)}),$$
$$-g_2(k(r'), k(s), 1^{(n-2)})), 0^{(m-2)}), 1^{(n-2)}).$$

Since $k(t)$, $k(s)$ and $k(s')$ are invertible elements in R_2, we get

$$0 = g_2(k(t)^{-1}, k(s)^{-1}, k(s')^{-1}, 1^{(n-4)}, 0)$$
$$\in g_2(g_2(k(t)^{-1}, k(s)^{-1}, k(s')^{-1}, 1^{(n-3)}), k(t), f_2(g_2(k(r), k(s'), 1^{(n-2)}),$$
$$-g_2(k(r'), k(s), 1^{(n-2)})), 0^{(m-2)}), 1^{(n-3)})$$
$$= g_2(g_2(k(t)^{-1}, k(t), 1^{(n-2)}), g_2(k(s)^{-1}, k(s')^{-1}, 1^{(n-2)}), f_2(g_2(k(r), k(s'), 1^{(n-2)}),$$
$$-g_2(k(r'), k(s), 1^{(n-2)})), 0^{(m-2)}), 1^{(n-3)})$$
$$= g_2(1, g_2(k(s)^{-1}, k(s')^{-1}, 1^{(n-2)}), f_2(g_2(k(r), k(s'), 1^{(n-2)}),$$
$$-g_2(k(r'), k(s), 1^{(n-2)})), 0^{(m-2)}), 1^{(n-3)})$$
$$= f_2(g_2(g_2(k(s)^{-1}, k(s')^{-1}, 1^{(n-2)}), g_2(k(r), k(s'), 1^{(n-2)}), 1^{(n-2)})$$
$$-g_2(g_2(k(s)^{-1}, k(s')^{-1}, 1^{(n-2)}), g_2(k(r'), k(s), 1^{(n-2)}), 1^{(n-2)}), 0^{(m-2)})$$
$$= f_2(g_2(g_2(k(s')^{-1}, k(s'), 1^{(n-2)}), g_2(k(r), k(s)^{-1}, 1^{(n-2)}), 1^{(n-2)})$$
$$-g_2(g_2(k(s), k(s)^{-1}, 1^{(n-2)}), g_2(k(r'), k(s')^{-1}, 1^{(n-2)}), 1^{(n-2)}), 0^{(m-2)})$$
$$= f_2(g_2(k(r), k(s)^{-1}, 1^{(n-2)}) - g_2(k(r'), k(s')^{-1}, 1^{(n-2)}), 0^{(m-2)})$$
$$= f_2(h(\tfrac{r}{s}), h(\tfrac{r'}{s'}), 0^{(m-2)}).$$

Then we coclude that $h(\frac{r}{s}) = h(\frac{r'}{s'})$.

We must show that the mapping h is an homomorphism. Let $r_1^m \in R_1$ and $s_1^m \in S$. Then we get

$$h(F(\tfrac{r_1}{s_1}, ..., \tfrac{r_m}{s_m})$$
$$= h(\tfrac{f_1(g_1(r_1, s_2^m, 1^{(n-m)}), g_1(s_1, r_2, s_3^m, 1^{(n-m)}), ..., g_1(s_1^{m-1}, r_m, 1^{(n-m)})}{g_1(s_1^m, 1^{(n-m)})})$$
$$= g_2(k(f_1(g_1(r_1, s_2^m, 1^{(n-m)}), ..., g_1(s_1^{m-1}, r_m, 1^{(n-m)})), k(g_1(s_1^m, 1^{(n-m)}))^{-1}, 1^{(n-2)})$$
$$= g_2(f_2(k(g_1(r_1, s_2^m, 1^{(n-m)}), ..., k(g_1(s_1^{m-1}, r_m, 1^{(n-m)})), k(g_1(s_1^m, 1^{(n-m)}))^{-1}, 1^{(n-2)})$$
$$= g_2(f_2(g_2((k(r_1), k(s_2), ..., k(s_m), k(1)^{(n-m)}), ..., g_2(k(s_1), ..., k(s_{m-1}), k(r_m),$$
$$k(1)^{(n-m)}), g_2(k(s_1)^{-1}, ..., k(s_m)^{-1}, k(1)^{(n-m)})), 1^{(n-2)})$$
$$= f_2(g_2(g_2(k(s_1)^{-1}, ..., k(s_m)^{-1}, k(1)^{(n-m)}), g_2(k(r_1), k(s_2), ..., k(s_m),$$
$$k(1)^{(n-m}), 1^{(n-2)})), ..., g_2(g_2(k(s_1)^{-1}, ..., k(s_m)^{-1}, k(1)^{(n-m)}), g_2(k(s_1), ...,$$
$$k(s_{m-1}), k(r_m), k(1)^{(n-m)})), 1^{(n-2)}))$$
$$= f_2(g_2(k(r_1), k(s_1)^{-1}, 1^{(n-2)}), ..., g_2(k(r_m), k(s_m)^{-1}, 1^{(n-2)}))$$

$= f_2(h(\frac{r_1}{s_1}), ..., h(\frac{r_m}{s_m}))$.

Also, we have

$$h(G(\frac{r_1}{s_1}, ..., \frac{r_n}{s_n}) = h(\frac{g_1(r_1^n)}{g_1(s_1^n)})$$
$$= g_2(k(g_1(r_1^n)), k(g_1(s_1^n))^{-1}, 1^{(n-2)})$$
$$= g_2(g_2(k(r_1), ..., k(r_n)), g_2(k(s_1)^{-1}, ..., k(s_n)^{-1}), 1^{(n-2)})$$
$$= g_2(g_2(k(r_1), k(s_1)^{-1}), 1^{(n-2)}), ..., g_2(k(r_n), k(s_n)^{-1}), 1^{(n-2)}))$$
$$= g_2(h(\frac{r_1}{s_1}), ..., h(\frac{r_n}{s_n}))$$

for $r_1^n \in R_1$ and $s_1^n \in S$. Consequently, h is a homomorphism. Now, suppose that h' is another homomorphism frome $S^{-1}R_1$ to R_2 with $h'o\phi = k$. Then we obtain

$$h(\frac{r}{s}) = h(G(\frac{r}{1}, \frac{1}{s}, \frac{1}{1}^{(n-2)}))$$
$$= g_2(h(\frac{r}{1}), h(\frac{1}{s}), h(\frac{1}{1})^{(n-2)})$$
$$= g_2(h(\phi(r)), h(\phi(s)^{-1}), 1^{(n-2)})$$
$$= g_2(h(\phi(r)), (h(\phi(s))^{-1}, 1^{(n-2)})$$
$$= g_2(k(r), k(s)^{-1}, 1^{(n-2)})$$
$$= g_2(h'(\phi(r)), (h'(\phi(s))^{-1}, 1^{(n-2)})$$
$$= g_2(h'(\frac{r}{1})), (h'(\frac{s}{1}))^{-1}, 1^{(n-2)})$$
$$= g_2(h'(\frac{r}{1})), h'(\frac{1}{s}), 1^{(n-2)})$$
$$= h'(G(\frac{r}{1}, \frac{1}{s}, \frac{1}{1}^{(n-2)}))$$
$$= h'(\frac{r}{s}).$$

for every $\frac{r}{s} \in S^{-1}R$. It implies that the homomorphism h is unique. Thus the proof is completed. □

Corollary 3.1. Let (R_1, f_1, h_1) and (R_2, f_2, g_2) be two Krasner (m, n)-hyperrings and S be an n-ary multiplicative subset of R_1 with $1 \in S$. Let $k : R_1 \longrightarrow R_2$ be a homomorphism such that

i) $k(s)$ is an invertible element of R_2 for each $s \in S$.

ii) $k(r_1) = 0$ for $r_1 \in R_1$ implies that $g_1(t, r_1, 1^{(n-2)}) = 0$, for some $t \in S$.

iii) for each $r_2 \in R_2$, $r_2 = g_2(k(r_1), k(s)^{-1}, 1^{(n-2)})$ where $r_1 \in R_1$ and $s \in S$.

Then there exists an unique isomomorphism $h : S^{-1}R_1 \longrightarrow R_2$ such that $ho\phi = k$.

Proof. By using an argument similar to that in the proof of Theorem 3.4, one can easily complete the proof. □

Theorem 3.5. If R is an n-ary hyperintegral domain, then $S^{-1}R$ is an n-ary hyperintegral domain.

Proof. Let $G(\frac{r_1}{s_1},...,\frac{r_1}{s_1}) = 0_{S^{-1}R}$ for $r_1^n \in R$ and $s_1^n \in S$. Thus $\frac{g(a_1^n)}{g(s_1^n)} = 0_{S^{-1}R}$. By Lemma 3.1 (2), we have $g(t, g(a_1^n), 1^{(n-2)}) = 0$ for some $t \in S$. Since R is an n-ary hyperintegral domain and $t \neq 0$, we have $g(a_1^n) = 0$ which implies $a_1 = 0$ or $a_2 = 0$ or ... or $a_n = 0$. Hence we get $\frac{a_1}{s_1} = 0_{S^{-1}R}$ or $\frac{a_2}{s_2} = 0_{S^{-1}R}$ or ... or $\frac{a_n}{s_n} = 0_{S^{-1}R}$. Thus $S^{-1}R$ is an n-ary hyperintegral domain. □

Theorem 3.6. Let R be an n-ary hyperintegral domain and $S = R - \{0\}$. Then each nonzero element of $S^{-1}R$ is invertible.

Proof. Let $\frac{r}{s}$ be an nonzero element of $S^{-1}R$. Since $r \neq 0$, then $r \in S$ and so $\frac{s}{r} \in S^{-1}R$. Thus $G(\frac{r}{s}, \frac{s}{r}, \frac{1}{1}^{(n-2)}) = \frac{g(r,s,1^{(n-2)})}{g(s,r,1^{(n-2)})} = \frac{1}{1} = 1_{S^{-1}R}$, by Lemma 3.1 (3). □

4. Hyperideals in Krasner (m,n)-hyperring of fractions

Let I be a hyperideal of Krasner (m,n)-hyperring R and S be an n-ary multiplicative subset of R with $1 \in S$, then we can define that $S^{-1}I = \{\frac{a}{s} \mid a \in I, s \in S\}$, which is a hyperideal of $S^{-1}R$.

Theorem 4.1. Let R be a Krasner (m,n)-hyperring and S be an n-ary multiplicative subset of R with $1 \in S$. Let I be a hyperideal of R. Then $I \cap S \neq \varnothing$ if and only if $S^{-1}I = S^{-1}R$.

Proof. $(\Longrightarrow)$: Let $a \in I \cap S$. Then $\frac{1}{1} = \frac{a}{a} \in S^{-1}I$. Since I is a hyperideal of R, we have $G(\frac{1}{1}, \frac{r}{s}, \frac{1}{1}^{(n-2)}) \in S^{-1}I$ for all $\frac{r}{s} \in S^{-1}R$. Since $G(\frac{1}{1}, \frac{r}{s}, \frac{1}{1}^{(n-2)}) = \frac{g(1,r,1^{(n-2)})}{g(1,s,1^{(n-2)})} = \frac{r}{s}$, then $\frac{r}{s} \in S^{-1}I$. Thus $S^{-1}I = S^{-1}R$.
$(\Longleftarrow)$: By the homomorphism $\phi : R \longrightarrow S^{-1}R$, it implies that $\phi(1) = \frac{1}{1}$. Since $S^{-1}I = S^{-1}R$ and $\phi(1) \in S^{-1}R$, then $\phi(1) \in S^{-1}I$. Hence, there exist $a \in I$, $s \in S$ such that $\frac{1}{1} = \phi(1) = \frac{a}{s}$. So, there exists $t \in S$ such that

$$\begin{aligned}0 &\in g(t, f(g(a,1,1^{(n-2)}), -g(1,s,1^{(n-2)}), 0^{(m-2)}), 1^{(n-2)})\\ &= g(t, f(g(a,1^{(n-1)}), -g(s,1^{(n-1)}), 0^{(m-2)}), 1^{(n-2)})\\ &= f(g(t, g(a,1^{(n-1)}), 1^{(n-2)}), g(t, -g(s,1^{(n-1)}), 1^{(n-2)}), 0^{(m-2)})\end{aligned}$$

$= f(g(t, a, 1^{(n-2)}), -g(t, s, 1^{(n-2)}), 0^{(m-2)})$.

Since $g(t, a, 1^{(n-2)}) \in I$, then $g(t, s, 1^{(n-2)}) \in I$. Also, since S is an n-ary multiplicative subset of R, then $g(t, s, 1^{(n-2)}) \in S$. Consequently, $I \cap S \neq \varnothing$. □

If $(a, s) \in S^{-1}I$ we don't get necessarily $a \in I$, maybe $(a, s) = (a', s)$ such that $a' \in I$ but $a \notin I$.

Theorem 4.2. Let R be a Krasner (m, n)-hyperring and S be an n-ary multiplicative subset of R with $1 \in S$. Then every hyperideal of $S^{-1}R$ is an extended hyperideal.

Proof. Suppose that J is a hyperideal of $S^{-1}R$. Put $B = \{r \in R \mid \exists s \in S; \frac{r}{s} \in J\}$. Easily, it is proved that B is a hyperideal of R. We show that $B^e = S^{-1}B = J$. Let $\frac{r}{s} \in J$. Then $r \in B$ and so $\frac{r}{s} \in S^{-1}B$ which means $J \subseteq S^{-1}B$. Now, assume that $\frac{b}{s} \in S^{-1}B$. Then there exist $b' \in B$ and $s' \in S$ such that $\frac{b}{s} = \frac{b'}{s'}$. It means there exists $t \in S$ such that $0 \in g(t, f(g(b, s', 1^{(n-2)}), -g(b', s, 1^{(n-2)}), 0^{(m-2)}), 1^{(n-2)})$
$= f(g(t, b, s', 1^{(n-3)}), -g(t, b', s, 1^{(n-3)}), 0^{(m-2)}), 1^{(n-2)})$.

Since $g(t, b', s, 1^{(n-3)}) \in B$, then $g(g(t, s', 1^{(n-2)}, b, 1^{(n-2)}) = g(t, b, s', 1^{(n-3)}) \in B$. Put $t' = g(t, s', 1^{(n-2)}$. Therefore we have $g({t'}^{(m-1)}, b, 1^{(n-m)}) \in B$. Hence there exists $t'' \in S$ such that $\frac{g({t'}^{(m-1)}, b, 1^{(n-m)})}{t''} \in J$ and so $\frac{g({t'}^{(m-1)}, b, 1^{(n-m)})}{g({t''}^{(m-1)}, 1^{(n-m+1)})} \in J$. Then we have

$$G(\frac{g({t''}^{(m-1)}, 1^{(n-m+1)})}{g(t', s, 1^{(n-2)})}, \frac{g({t'}^{(m-1)}, b, 1^{(n-m)})}{g({t''}^{(m-1)}, 1^{(n-m+1)})}) = \frac{g(g(t'', t', 1^{(n-2)})^{(m-1)}, b, 1^{(n-m)})}{g(g(t'', t', 1^{(n-2)})^{(m-1)}, s, 1^{(n-m)})} = \frac{b}{s} \in J.$$

This means $S^{-1}B \subseteq J$. Consequently, $S^{-1}B = J$. □

Let R be a Krasner (m, n)-hyperring. Then the hyperideal M of R is said to be maximal if for every hyperideal I of R, $M \subseteq I \subseteq R$ implies that $I = M$ or $I = R$ [1].

Lemma 4.1. Let R be a Krasner (m, n)-hyperring such that M is a hyperideal of R. If each $x \in R - M$ is invertible, then M is a maximal hyperideal of R.

Proof. The proof is similar to ordinary algebra. □

Theorem 4.3. Let R be a Krasner (m, n)-hyperring and P be an n-ary prime hyperideal of R. If $S = R - P$, then $M = \{\frac{a}{s} \mid a \in P, s \in S\}$ is the only maximal hyperideal of $S^{-1}R$.

Proof. Clearly, $S = R - P$ is an n-ary multiplicative subset of R. Let $\frac{a_1}{s_1}, ..., \frac{a_m}{s_m} \in M$ such that $a_1^m \in P$ and $s_1^m \in S$. Then

$$F(\frac{a_1}{s_1},...,\frac{a_m}{s_m})=\frac{f(g(a_1,s_2^m,1^{(n-m)}),g(s_1,a_2,s_3^m,1^{(n-m)}),...,g(s_1^{m-1},a_m,1^{(n-m)}))}{g(s_1^m,1^{(n-m)})}.$$

Since $a_1^m\in P$, then $g(a_1,s_2^m,1^{(n-m)}),g(s_1,a_2,s_3^m,1^{(n-m)}),...,g(s_1^{m-1},a_m,1^{(n-m)})\in P$ and so $f(g(a_1,s_2^m,1^{(n-m)}),g(s_1,a_2,s_3^m,1^{(n-m)}),...,g(s_1^{m-1},a_m,1^{(n-m)}))\subseteq P$. Thus we conclude that $F(\frac{a_1}{s_1},...,\frac{a_m}{s_m})\subseteq M$.

Clearly, if $\frac{a}{r}\in M$, then $-\frac{a}{r}=\frac{-a}{r}\in M$. Also, since $0\in P$,then $0_{R_P}=\frac{0}{s}\in M$ for all $s\in S$. Hence (M,F) is a canonical n-ary hypergroup.

Now, let $r_1^n\in R$, $s_1^n\in S$ and $k\in\{1,...,n\}$. Then

$$\begin{aligned}G(\frac{r_1}{s_1},...,\frac{r_{k-1}}{s_{k-1}},M,\frac{r_{k+1}}{s_{k+1}},...,\frac{r_n}{s_n}) &= \bigcup\{G(\frac{r_1}{s_1},...,\frac{r_{k-1}}{s_{k-1}},\frac{a}{s},\frac{r_{k+1}}{s_{k+1}},...,\frac{r_n}{s_n})\mid \frac{a}{s}\in M\}\\ &= \bigcup\{\frac{g(r_1^{k-1},a,r_{k-1}^n)}{g(s_1^{k-1},s,s_{k-1}^n)}\mid a\in P, s\in S\}.\end{aligned}$$

Since $a\in P$, then $g(r_1^{k-1},a,r_{k-1}^n)\in P$ and so $G(\frac{r_1}{s_1},...,\frac{r_{k-1}}{s_{k-1}},M,\frac{r_{k+1}}{s_{k+1}},...,\frac{r_n}{s_n})\subseteq M$. Thus, (M,F,G) is a hyperideal of R_P.

Suppose that $1_{R_P}=\frac{1}{1}\in M$. Then there exist $a\in P$ and $s\in S$ such that $\frac{1}{1}=\frac{a}{s}$. It implies that there exists $t\in S$ such that

$$\begin{aligned}0 &\in g(t,f(g(a,1,1^{(n-2)}),-g(1,s,1^{(n-2)}),0^{(m-2)}),1^{(n-2)})\\ &= f(g(t,a,1^{(n-2)}),-g(t,s,1^{(n-2)}),0^{(m-2)}).\end{aligned}$$

Since $g(t,a,1^{(n-2)})\in P$, then $g(t,s,1^{(n-2)})\in P$. Since P is an n-ary prime hyperideal of R, then we obtain $t\in P$ or $s\in P$ which is a contradiction. Then M is a proper hyperideal of R.

Now, suppose that $\gamma\in R_P-M$. It means $\gamma=\frac{r}{s}$ such that $r\in R-P$ and $s\in S$. Then $r\in S$ and so $\frac{s}{r}\in M$. Hence $\frac{1}{1}=G(\frac{r}{s},\frac{s}{r},\frac{1}{1}^{(n-2)})\in M$ which is a contradiction. Consequently, M is the only maximal hyperideal of R_P. □

Theorem 4.4. Let R be a Krasner (m,n)-hyperring and S be an n-ary multiplicative subset of R with $1\in S$. If P is an n-ary prime hyperideal of R with $P\cap S=\varnothing$, then $S^{-1}P$ is an n-ary prime hyperideal of $S^{-1}R$.

Proof. Let $G(\frac{a_1}{s_1},...,\frac{a_n}{s_n})\in S^{-1}P$ for $\frac{a_1}{s_1},...,\frac{a_n}{s_n}\in S^{-1}R$. Then we have $\frac{g(a_1^n)}{g(s_1^n)}\in S^{-1}P$. It implies that there exists $t\in S$ such that $g(t,g(a_1^n),1^{(n-2)})\in P$. Since P is an n-ary prime hyperideal of R and $P\cap S=\varnothing$, then $g(a_1^n)\in P$ which means there exists $1\leq i\leq n$ with $a_i\in P$. Hence we conclude that $\frac{a_i}{s_i}\in S^{-1}P$ for some $1\leq i\leq n$. Thus $S^{-1}P$ is an n-ary prime hyperideal of $S^{-1}R$. □

Example 4.1. The set $R = \{0, 1, 2\}$ with the following 3-ary hyperoeration f and 3-ary operation g is a Krasner $(3, 3)$-hyperring such that f and g are commutative.

$$f(0,0,0) = 0, \quad f(0,0,1) = 1, \quad f(0,1,1) = 1, \quad f(1,1,1) = 1, \quad f(1,1,2) = R,$$

$$f(0,1,2) = R, \quad f(0,0,2) = 2, \quad f(0,2,2) = 2, \quad f(1,2,2) = R, \quad f(2,2,2) = 2,$$

$$g(1,1,1) = 1, \quad g(1,1,2) = g(1,2,2) = g(2,2,2) = 2,$$

and for $x_1, x_2 \in R$, $g(0, x_1, x_2) = 0$.

$S = \{1, 2\}$ is a 3-ary multiplicative subset of Krasner $(3, 3)$-hyperring (R, f, g) and hyperideal $P = \{0\}$ is a 3-ary prime hyperideal of R (see example 4.10 in [1]). Thus $S^{-1}P = \{\frac{0}{1}\}$ is a 3-ary prime hyperideal of $S^{-1}R$.

Let I be a hyperideal in a Krasner (m, n)-hyperring R with scalar identity. The radical (or nilradical) of I, denoted by $\sqrt{I}^{(m,n)}$ is the hyperideal $\bigcap P$, where the intersection is taken over all n-ary prime hyperideals P which contain I. If the set of all n-ary hyperideals containing I is empty, then $\sqrt{I}^{(m,n)}$ is defined to be R. Ameri and Norouzi showed that if $x \in \sqrt{I}^{(m,n)}$, then there exists $t \in \mathbb{N}$ such that $g(x^{(t)}, 1_R^{(n-t)}) \in I$ for $t \leq n$, or $g_{(l)}(x^{(t)}) \in I$ for $t = l(n-1)+1$ [1].

Lemma 4.2. Let R be a Krasner (m, n)-hyperring and S be an n-ary multiplicative subset of R with $1 \in S$. If I is an n-ary hyperideal of R, then $\sqrt{S^{-1}I}^{(m,n)} = S^{-1}\sqrt{I}^{(m,n)}$.

Proof. Let $\frac{a}{s} \in \sqrt{S^{-1}I}^{(m,n)}$. Then there exists $k \in \mathbb{N}$ with $G(\frac{a}{s}^{(k)}, \frac{1}{1}^{(n-k)}) \in S^{-1}I$ for $k \leq n$, or $G_{(l)}(\frac{a}{s}^{(k)}) \in S^{-1}I$ for $k = l(n-1)+1$. If $G(\frac{a}{s}^{(k)}, \frac{1}{1}^{(n-k)}) \in S^{-1}I$, then $\frac{g(a^{(k)}, 1^{(n-k)})}{g(a^{(k)}, 1^{(n-k)})} \in S^{-1}I$. Therefore $g(t, g(a^{(k)}, 1^{(n-k)}), 1^{(n-2)}) \in I$ for some $t \in S$ and so $g(g(t, a, 1^{(n-2)})^{(k)}, 1^{(n-k)}) = g(t^{(k)}, g(a^{(k)}, 1^{(n-k)}), 1^{(n-k-1)}) \in I$. It means $g(t, a, 1^{(n-2)}) \in \sqrt{I}^{(m,n)}$ and so $g(t^{(m-1)}, a, 1^{(n-m)}) \in \sqrt{I}^{(m,n)}$. Hence we get

$$\frac{g(t^{(m-1)}, a, 1^{(n-m)})}{g(t^{(m-1)}, s, 1^{(n-m)})} = \frac{a}{s} \in S^{-1}\sqrt{I}^{(m,n)},$$

by Lemma 3.1 (4). Similarly for the other case. Thus $\sqrt{S^{-1}I}^{(m,n)} \subseteq S^{-1}\sqrt{I}^{(m,n)}$. Now, let $\frac{a}{s} \in S^{-1}\sqrt{I}^{(m,n)}$. Then we conclude $g(t, a, 1^{(n-2)}) \in \sqrt{I}^{(m,n)}$ for some $t \in S$ and so $g(t^{(m-1)}, a, 1^{(n-m)}) \in \sqrt{I}^{(m,n)}$. It means that there exists $k \in \mathbb{N}$ with $g(g(t^{(m-1)}, a, 1^{(n-m)})^{(k)}, 1^{(n-k)}) \in I$ for $k \leq n$, or $g_{(l)}(g(t^{(m-1)}, a, 1^{(n-2)})^{(k)}) \in I$ for $k = l(n-1)+1$. If $g(g(t^{(m-1)}, a, 1^{(n-2)})^{(k)}, 1^{(n-k)}) \in I$, then we have

$$G(\frac{a}{s}^{(k)}, \frac{1}{1}^{(n-k)}) = G(\frac{g(t^{(m-1)},a,1^{(n-2)})}{g(t^{(m-1)},s,1^{(n-2)})}^{(k)}, \frac{1}{1}^{(n-k)})$$
$$= \frac{g(g(t^{(m-1)},a,1^{(n-2)})^{(k)},1^{(n-k)})}{g(g(t^{(m-1)},s,1^{(n-2)})^{(k)},1^{(n-k)})} \in S^{-1}I.$$

Therefore we get $\frac{a}{s} \in \sqrt{S^{-1}I}^{(m,n)}$. Similarly for the other case. Thus $S^{-1}\sqrt{I}^{(m,n)} \subseteq \sqrt{S^{-1}I}^{(m,n)}$. Consequently, $\sqrt{S^{-1}I}^{(m,n)} = S^{-1}\sqrt{I}^{(m,n)}$. □

A hyperideal $Q \neq R$ in a Krasner (m,n)-hyperring (R,f,g) with the scalar identity 1_R is said to be n-ary primary if $g(x_1^n) \in Q$ and $x_i \notin Q$ implies that $g(x_1^{i-1}, 1_R, x_{i+1}^n) \in \sqrt{Q}^{(m,n)}$ [1].

Theorem 4.5. Let R be a Krasner (m,n)-hyperring and S be an n-ary multiplicative subset of R with $1 \in S$. If P is an n-ary primary hyperideal of R with $P \cap S = \varnothing$, then $S^{-1}P$ is an n-ary primary hyperideal of $S^{-1}R$.

Proof. Let $\frac{a_1}{s_1}, ..., \frac{a_n}{s_n} \in S^{-1}R$ such that $G(\frac{a_1}{s_1}, ..., \frac{a_n}{s_n}) \in S^{-1}P$. Then we have $\frac{g(a_1^n)}{g(s_1^n)} \in S^{-1}P$. It implies that there exists $t \in S$ such that $g(t, g(a_1^n), 1^{(n-2)}) \in P$. Since P is an n-ary primary hyperideal of R, then there exist $1 \leq i \leq n$ such that at least one of the cases hold: $a_i \in P$, $g(a_1^{i-1}, 1, a_{i+1}^n) \in \sqrt{P}^{(m,n)}$, $t \in \sqrt{P}^{(m,n)}$ or $g(a_1^n) \in \sqrt{P}^{(m,n)}$. If $a_i \in P$, then $\frac{a_i}{s_i} \in S^{-1}P$ and we are done. If $g(a_1^{i-1}, 1, a_{i+1}^n) \in \sqrt{P}^{(m,n)}$, then $G(\frac{a_1}{s_1}, ..., \frac{a_{i-1}}{s_{i-1}}, \frac{1}{1}, \frac{a_{i+1}}{s_{i+1}}, ..., \frac{a_n}{s_n}) = \frac{g(a_1^{i-1},1,a_{i+1}^n)}{g(s_1^{i-1},1,s_{i+1}^n)} \in S^{-1}\sqrt{P}^{(m,n)} = \sqrt{S^{-1}P}^{(m,n)}$, by Lemma 4.2. If $t \in \sqrt{P}^{(m,n)}$, then $g(t^{(m-1)}, a_k, 1^{(m-n)}) \in \sqrt{P}^{(m,n)}$, for all $1 \leq k \leq n$. Therefore $\frac{g(t^{(m-1)},a_k,1^{(m-n)})}{g(t^{(m-1)},s_k,1^{(m-n)})} \in S^{-1}\sqrt{P}^{(m,n)} = \sqrt{S^{-1}P}^{(m,n)}$ and so $\frac{a_k}{s_k} \in \sqrt{S^{-1}P}^{(m,n)}$. Therefore for each $i \neq k$, $G(\frac{a_1}{s_1}, ..., \frac{a_{i-1}}{s_{i-1}}, \frac{1}{1}, \frac{a_{i+1}}{s_{i+1}}, ..., \frac{a_n}{s_n}) \in \sqrt{S^{-1}P}^{(m,n)}$. Let $g(a_1^n) \in \sqrt{P}^{(m,n)}$. Theorem 4.28. in [1] shows that $\sqrt{P}^{(m,n)}$ is an n-ary prime hyperideal of R. Hence there exists $1 \leq k \leq n$ such that $a_i \in \sqrt{P}^{(m,n)}$. It implies that $\frac{a_k}{s_k} \in S^{-1}\sqrt{P}^{(m,n)} = \sqrt{S^{-1}P}^{(m,n)}$. Therefore for each $i \neq k$, $G(\frac{a_1}{s_1}, ..., \frac{a_{i-1}}{s_{i-1}}, \frac{1}{1}, ..., \frac{a_{i+1}}{s_{i+1}}, \frac{a_n}{s_n}) \in \sqrt{S^{-1}P}^{(m,n)}$. Thus $S^{-1}P$ is an n-ary primary hyperideal of $S^{-1}R$. □

A proper hyperideal I of a Krasner (m,n)-hyperring (R,f,g) with the scalar identity 1_R is said to be n-ary 2-absorbing if for $x_1^n \in R$, $g(x_1^n) \in I$ implies that $g(x_i, x_j, 1_R^{(n-2)}) \in I$ for some $1 \leq i < j \leq n$ [3].

Theorem 4.6. Let R be a Krasner (m,n)-hyperring and S be an n-ary multiplicative subset of R with $1 \in S$. If P is an n-ary 2-absorbing hyperideal of R with $P \cap S = \varnothing$, then $S^{-1}P$ is an n-ary 2-absorbing hyperideal of $S^{-1}R$.

Proof. Let $G(\frac{a_1}{s_1},...,\frac{a_n}{s_n}) \in S^{-1}P$, for $\frac{a_1}{s_1},...,\frac{a_n}{s_n} \in S^{-1}R$. Then we have $\frac{g(a_1^n)}{g(s_1^n)} \in S^{-1}P$. It implies that there exists $t \in S$ such that $g(t, g(a_1^n), 1^{(n-2)}) \in P$. Since P is an n-ary 2-absorbing hyperideal of R, then there exist $1 \leq i < j \leq n$ such that $g(t, a_i, 1^{(n-2)}) \in P$ or $g(a_i, a_j, 1^{(n-2)}) \in P$. Hence we conclude that $\frac{a_i}{s_i} \in S^{-1}P$ for some $1 \leq i \leq n$. Thus $S^{-1}P$ is an n-ary prime hyperideal of $S^{-1}R$. If for some $1 \leq i \leq n$, $g(t, a_i, 1^{(n-2)}) \in P$, then $g(t^{(m-1)}, a_i, 1^{(n-m)}) \in P$ and so $\frac{g(t^{(m-1)}, a_i, 1^{(n-m)})}{g(t^{(m-1)}, s_i, 1^{(n-m)})} \in S^{-1}P$. Hence $\frac{a_i}{s_i} \in S^{-1}P$, by Lemma 3.1 (4). Therefore for every $1 \leq j \leq n$, $G(\frac{a_i}{s_i}, \frac{a_j}{s_j}, \frac{1}{1}^{(n-2)}) \in S^{-1}P$ and we are done. If $g(a_i, a_j, 1^{(n-2)}) \in P$, for some $1 \leq i < j \leq n$, then $\frac{g(a_i, a_j, 1^{(n-2)})}{g(s_i, s_j, 1^{(n-2)})} \in S^{-1}P$ which means $G(\frac{a_i}{s_i}, \frac{a_j}{s_j}, \frac{1}{1}^{(n-1)}) \in S^{-1}P$. Consequently, $S^{-1}P$ is an n-ary 2-absorbing hyperideal of $S^{-1}R$. □

5. qutient Krasner (m, n)-hyperring of fractions

Let R be a Krasner (m, n)-hyperring and I be a hyperideal of R. Then we consider the set R/I as follows:

$R/I = \{f(r, I, 0^{(m-2)}) \mid r \in R\}$.

Lemma 5.1. Let R be a Krasner (m, n)-hyperring and S be an n-ary multiplicative subset of R with $1 \in S$. Let I be a hyperideal of R such that $S \cap I = \varnothing$. Then $\bar{S} = \{f(s, I, 0^{(m-2)}) \mid s \in S\}$ is an n-ary multiplicative subset of R/I.

Proof. Let $f(s_1, I, 0^{(m-2)}), ..., f(s_n, I, 0^{(m-2)}) \in \bar{S}$, for $s_1^n \in S$. Then we have

$g(f(s_1, I, 0^{(m-2)}), ..., f(s_n, I, 0^{(m-2)})) = f(g(s_1^n), I, 0^{m-2)})$.

Since S is an n-ary multiplicative subset of R, then $g(s_1^n) \in S$. It implies that $g(f(s_1, I, 0^{(m-2)}), ..., f(s_n, I, 0^{(m-2)})) \in \bar{S}$. □

Theorem 5.1. Let R be a Krasner (m, n)-hyperring and S be an n-ary multiplicative subset of R with $1 \in S$. Let I be a hyperideal of R such that $S \cap I = \varnothing$. If $\bar{S} = \{f(s, I, 0^{(m-2)}) \mid s \in S\}$, then $\bar{S}^{-1}(R/I) \cong S^{-1}R/S^{-1}I$.

Proof. Define mapping $k : R/I \longrightarrow S^{-1}R/S^{-1}I$ as following:

$k(f(r, I, 0^{(m-2)}) = F(\frac{r}{1}, S^{-1}I, 0_{S^{-1}R}^{(m-2)})$.

It is easy to see the mapping is a homomorphism. Let $f(s, I, 0^{(m-2)}) \in \bar{S}$. Then $k(f(s, I, 0^{(m-2)})) = F(\frac{s}{1}, S^{-1}I, 0_{S^{-1}R}^{(m-2)})$. Since $F(\frac{1}{s}, S^{-1}I, 0_{S^{-1}R}^{(m-2)}) \in S^{-1}R/S^{-1}I$, then we obtain

$$G(F(\tfrac{s}{1},S^{-1}I,0^{(m-2)}_{S^{-1}R}),F(\tfrac{1}{s},S^{-1}I,0^{(m-2)}_{S^{-1}R}),F(\tfrac{1}{1},S^{-1}I,0^{(m-2)}_{S^{-1}R})^{(n-2)}))$$
$$=F(G(\tfrac{s}{1},\tfrac{1}{s},\tfrac{1}{1}^{(n-2)}),S^{-1}I,0^{(m-2)}_{S^{-1}R})$$
$$=F(\tfrac{1}{1},S^{-1}I,0^{(m-2)}_{S^{-1}R})).$$

Assume that $k(f(r,I,0^{(m-2)})=S^{-1}I$. Then we have $F(\frac{r}{1},S^{-1}I,0^{(m-2)}_{S^{-1}R})=S^{-1}I$. It means $\frac{r}{1}\in S^{-1}I$. Then there exists $t\in S$ such that $g(t,r,1^{(n-2)})\in I$. Clearly, $f(t,I,0^{(m-2)})\in\bar{S}$ and we have

$$g(f(t,I,0^{(m-2)},f(r,I,0^{(m-2)}),f(1,I,0^{(m-2)})^{(n-2)})$$
$$=f(g(t,r,1^{(n-2)}),I,0^{(m-2)})=I.$$

Now, suppose that $F(\frac{r}{s},S^{-1}I,0_{S^{-1}R})\in S^{-1}R/S^{-1}I$. Thus we have

$$F(\tfrac{r}{s},S^{-1}I,0_{S^{-1}R})=G(F(\tfrac{r}{1},S^{-1}I,0_{S^{-1}R}),F(\tfrac{1}{s},S^{-1}I,0_{S^{-1}R}),F(\tfrac{1}{1},S^{-1}I,0_{S^{-1}R})^{(n-2)})$$
$$=G(k(f(r,I,0^{(m-2)}),k(f(r,I,0^{(m-2)}),F(\tfrac{1}{1},S^{-1}I,0_{S^{-1}R})^{(n-2)}).$$

Hence, there exists an isomorphism from $\bar{S}^{-1}(R/I)$ to $S^{-1}R/S^{-1}I$, by Corollary 3.1. It means $\bar{S}^{-1}(R/I)\cong S^{-1}R/S^{-1}I$. □

Let P be an n-ary prime hyperideal of Krasner (m,n)-hyperring R. Put $S=R-P$. Then S is an n-ary multiplicative subset of R such that $1\in S$ and $0\notin S$. In this case, we denote $S^{-1}R=R_P$. Moreover, If $S^{-1}I$ is a hyperideal of R_P, then it is denoted by IR_P.

Example 5.1. Let R be a Krasner (m,n)-hyperring such that P is an n-ary prime hyperideal of R. Put $S=R-P$. Then $\bar{S}=\{f(s,P,0^{(n-2)})\ |s\in S\}=R/P-\{f(P,0^{(n-1)})\}$ is an n-ary multiplicative subset of R/P. By Theorem 4.6 in [1], R/P is an n-ary hyperintegral domain. Theorem 3.5 and 3.6 show that $\bar{S}^{-1}(R/P)$ is an n-ary hyperintegral domain and each nonzero element of $\bar{S}^{-1}(R/P)$ is invertible. Moreover, we have $\bar{S}^{-1}(R/P)\cong\frac{S^{-1}R}{S^{-1}P}=\frac{R_p}{PR_p}$, by Theorem 5.1.

Example 5.2. Let R be a Krasner (m,n)-hyperring such that P and Q are two n-ary prime hyperideals of R such that $Q\subseteq P$. Put $S=R-P$. Then $\bar{S}=\{f(s,Q,0^{(n-2)})\ |s\in S\}=R/Q-R/P$. It is clear that P/Q is an n-ary prime hyperideal of R/Q. Therefore $\bar{S}^{-1}(R/Q)=(R/Q)_{P/Q}$. By Theorem 5.1, we get $(R/Q)_{P/Q}\cong\frac{R_P}{QR_P}$.

Acknowledgement

I gratefully thank the referees for carefully reading the paper and for the suggestions that greatly improved the presentation of the paper.

References

[1] R. Ameri, M. Norouzi, Prime and primary hyperideals in Krasner (m,n)-hyperrings, *European Journal of Combinatorics*, **34** (2013), 379–390.

[2] S. Y. Akbiyik, Codes over the multiplicative hyperrings, *Turkish World Mathematical Society Journal of Applied and Engineering Mathematics*, **11(4)** (2021), 1260–1267.

[3] M. Anbarloei, n-ary 2-absorbing and 2-absorbing primary hyperideals in Krasner (m, n)-hyperideals , *Matematički Vesnik,* **71(3)** (2019), 250–262.

[4] M. Anbarloei, Unifing the prime and primary hyperideals under one frame in a Krasner (m,n)-hyperring, *Comm. Algebra*, **49** (2021), 3432–3446.

[5] S. Corsini, Prolegomena of hypergroup theory, *Second edition, Aviani editor, Italy,* (1993).

[6] S. Corsini, V. Leoreanu, Applications of hyperstructure theory, *Advances in Mathematics,* Kluwer Academic Publishers, **5** (2003).

[7] B. Davvaz, Weak algebraic hyperstructures as a model for interpretation of chemical reactions, *Iranian Journal of Mathematical Chemistry,* **7(2)** (2016), 267–283.

[8] A. Dehghan Nezhad, S. M. Moosavi Nejad, M. Nadjafikhah, B. Davvaz, A physical example of algebraic hyperstructures: Leptons, *Indian Journal of Physics*, **86 (11)** (2012), 1027–1032.

[9] B. Davvaz, A. Dehghan Nezhad, M. M. Heidari, Inheritance examples of algebraic hyperstructures, *Information Sciences*, **224** (2013), 180–187.

[10] B. Davvaz, T. Musavi, Codes over hyperrings, *Matematicki Vesnik*, **68 (1)** (2016), 26–38.

[11] B. Davvaz, T. Vougiouklis, n-Ary hypergroups, *Iranian Journal of Science and Technology, Transaction A*, **30 (A2)** (2006), 165– 174.

[12] B. Davvaz, V. Leoreanu-Fotea, Hyperring Theory and Applications, *International Academic Press, Palm Harbor, USA*, (2007).

[13] W. Dorente, V. Untersuchungen uber einen verallgemeinerten gruppenbegriff, *Math. Z.,* **29** (1928), 1–19.

[14] K. Hila, K. Naka, B. Davvaz, On (k,n)-absorbing hyperideals in Krasner (m,n)-hyperrings, *Quarterly Journal of Mathematics*, **69** (2018), 1035–1046.

[15] S. Hoskova-Mayerova, A. Maturo, Algebraic hyperstructures and social relations, *Italian Journal of Pure and Applied Mathematics,* **(39)** (2018), 701–709.

[16] E. Kasner, An extention of the group concept (reported by L. G. Weld), *Bull. Amer. Math. Soc.,* **10** (1904), 290–291.

[17] M. Krasner, A class of hyperrings and hyperfields, *International J. Math. and Math. Sci.,* **6** (1983), 307–312.

[18] V. Leoreanu, Canonical n-ary hypergroups, *Ital. J. Pure Appl. Math.,* **24** (2008).

[19] F. Marty, Sur une generalization de la notion de groupe, 8^{th} *Congress Math. Scandenaves, Stockholm,* (1934), 45–49.

[20] J. Mittas, Hyperstructures canoniques, *Mathematica Balkanica*, **2** (1972), 165–179.

[21] J. Mittas, Hyperstructures et certaines de leurs proprietes, *C. R. Acad. Sci. Paris Ser. A-B*, **269** (1969), A623–A626.

[22] S. Mirvakili, B. Davvaz, Applications of the $\alpha^{\star}$-relation to Krasner hyperrings, *Journal of Algebra*, **362** (2012), 145–156.

[23] S. Mirvakili, B. Davvaz, Constructions of (m, n)-hyperrings, *Matematicki Vesnik*, **67 (1)** (2015), 1–16.

[24] S. Mirvakili, B. Davvaz, Relations on Krasner (m, n)-hyperring, *European J. Combin*, **31** (2010), 790–802.

[25] S.M. Moosavi Nejad, M. Eslami Kalantari, A. Dehghan Nezhad, Extension of algebraic hyperstructures theory to the elementary particle physics and nuclear physics, *Iranian Journal of Physics Research*, **11 (4)** (2012), 429–434.

[26] S.,Omid, B., Davvaz, On ordered Krasner hyperrings, *Iranian Journal of Mathematical Sciences and Informatics*, **12 (2)** (2017), 35–49.

[27] S. Ostadhadi- Dehkordi, B. Davvaz, A note on isomorphism theorems of Krasner (m, n)- hyperrings, *Arabian Journal of Mathematics*, **5** (2016), 103–115.

[28] R. Procesi-Ciampi, R. Rota, The hyperring spectrum, *Riv. Mat. Pura Appl.,* **1** (1987), 71–80.

Department of Mathematics, Faculty of Sciences, Imam Khomeini International University, Qazvin, Iran.

Email address: m.anbarloei@sci.ikiu.ac.ir