Jordan Journal of Mathematics and Statistics (JJMS), 15(3B), 2022, pp 615 - 627
DOI: https://doi.org/10.47013/15.3.14

ON REGULAR δ-PREOPEN SETS

J. B. TORANAGATTI[(1)] AND T. NOIRI[(2)]

Abstract. The aim of this paper is to introduce a new class of sets called regular δ-preopen sets in topological spaces.We characterize these sets and study some of their fundamental properties. Also, new decompositions of complete continuity and perfect continuity are obtained.

1. Introduction

In 1968,Veličko [20] introduced the concept of δ-open sets as a stronger form of open sets. In 1993, Raychaudhuri and Mukherjee [18] introduced the concept of δ-preopen sets as a generalization of δ-open sets. This paper deals with a new class of sets called regular δ-preopen sets. Some properties and characterizations of regular δ-preopen sets are established. Moreover, we obtain decomposition theorems of completely continuous functions and perfectly continuous functions.

Throughout this paper, (U, τ) and (V, η) (or simply U and V) represent topological spaces on which no separation axioms are assumed unless explicitly stated and $f : (U, \tau) \to (V, \eta)$ or simply $f : U \to V$ denotes a function f of a topological space U into a topological space V. Let N $\subseteq$ U, then int(N) and cl(N) denote the interior of N and the closure of U,respectively.

2. Preliminaries

Definition 2.1. [19] A set M $\subseteq$ U is called regular-closed if M=cl(int(M)) and regular-open if M = int(cl(M)).

2010 *Mathematics Subject Classification.* 54*A*05;54*A*08.
Key words and phrases. δ-preopen sets, δ-preclosed sets, regular δ-preopen sets, regular δ-preclosed sets, rδp-continuity.

Received: Feb. 18, 2021 Accepted: Jul. 6, 2021 .

Definition 2.2. [20] A subset M is said to be δ-open if for each p $\in$ M there exists a regular open set N such that p $\in$ N $\subset$ M. A point p $\in$ U is called a δ-cluster point of M if int(cl(G)) $\cap$ M $\neq \phi$ for every open set G containing p. The set of all δ-cluster points of M is called the δ-closure of M and is denoted by δ-cl(M). The set {p$\in$U : p $\in$ G $\subset$ M for some regular open set G of U} is called the δ-interior of M and is denoted by δ-int(M).

Definition 2.3. A set M $\subseteq$ U is called
(1) δ-preclosed [18] if cl(δ-int(M)) $\subseteq$ M and δ-preopen if M $\subseteq$ int(δ-cl(M)),
(2) a-closed [8] if cl(int(δ-cl(M))) $\subseteq$ M and a-open if M $\subseteq$ int(cl(δ-int(M))),
(3) δ-semiclosed [17] if int(δ-cl(M)) $\subseteq$ M and δ-semiopen if M $\subseteq$ cl(δ-int(M)),
(4) e^*-closed [11] if int(cl(δ-int(M))) $\subseteq$ M and e^*-open if M $\subseteq$ cl(int(δ-cl(M))),
(5) e-closed [7] if cl(δ-int(M)) $\cap$ int(δ-cl(M)) $\subseteq$ M and e-open if M $\subseteq$ cl(δ-int(M)) $\cup$ int(δ-cl(M)).

The class of closed(resp.,regular open,δ-preopen, δ-preclosed, δ-semiopen,δ-semiclosed, e^*-open,e^*-closed,e-open,e-closed and clopen) sets of (U, τ) is denoted by C(U) (resp., RO(U), δPO(U),δPC(U), δSO(U),δSC(U), e^*O(U),e^*C(U),eO(U),eC(U) and CO(U)).

Definition 2.4. [1] A subet M of a space U is called δ-semiregular if it is both δ-semiopen and δ-semiclosed.

Definition 2.5. [9] A subet M of a space U is said to be δ-dense if δ-cl(M) = U.

Definition 2.6. [7, 17, 18] For any topological space (U,τ) and M $\subseteq$U, the e-closure,δ-semi closure,δ-preclosure and δ-preinteriour of M are denoted and defined as follows:
(1) e-cl(M) = $\cap${F $\subseteq$ U : F $\in$eC(U), M $\subseteq$ F}.
(2) δ-scl(M) = $\cap${F $\subseteq$ U : F $\in\delta$SC(U), M $\subseteq$ F}.
(3) δ-pcl(M) = $\cap${F $\subseteq$ U : F $\in\delta$PC(U), M $\subseteq$ F}.
(4) δ-pint(M) = $\cup${G $\subseteq$ U : G $\in\delta$PO(U), M $\supseteq$ G}.

Theorem 2.1. [18] *Let M be a subset of a space (U,τ), then δ-pcl(M) = M $\cup$ cl(δ-int(M)) and δ-pint(M) = M $\cap$ int(δ-cl(M)).*

Theorem 2.2. [7] *Let M be a subset of a space (U,τ), then*

(a) δ-pint(δ-pcl(N)) = δ-pint((e-cl(M)).

(b) e-cl(M) = δ-pcl(M) $\cap$ δ-scl(M).

(c) δ-pint(δ-pcl(M)) = δ-pcl(M) $\cap$ int(δ-cl(M).

(d) int(δ-cl(M)) = δ-int(δ-scl(M)) = δ-scl(δ-pint(M)).

Definition 2.7. A function $f : (U, \tau) \to (V, \eta)$ is called

(1) perfectly continuous [15] if f^{-1}(N) is clopen in (U, τ) for every N $\in \eta$,

(2) contra-super-continuous [12] if f^{-1}(N) is δ-closed in (U, τ) for every N $\in \eta$,

(3) RC-continuous [4] if f^{-1}(N) is regular closed in (U, τ) for every N $\in \eta$,

(4) completely continuous [2] if f^{-1}(N) is regular open in (U, τ) for every N $\in \eta$,

(5) super-continuous [14] if f^{-1}(N) is δ-open in (U, τ) for every N $\in \eta$,

(6) contra continuous [3] if f^{-1}(N) is closed in (U, τ) for every N $\in \eta$,

(7) δ-semiregular-continuous if f^{-1}(N) is δ-semiregular in (U, τ) for every N $\in \eta$,

(8) a-continuous [8] if f^{-1}(N) is a-open in (U, τ) for every N $\in \eta$,

(9) δ-semicontinuous [5] if f^{-1}(N) is δ-semiopen in (U, τ) for every N $\in \eta$,

(10) e-continuous [7] if f^{-1}(N) is e-open in (U, τ) for every N $\in \eta$,

(11) δ-almost continuous [18] if f^{-1}(N) is δ-preopen in (U, τ) for every N $\in \eta$,

(12) e^*-continuous [11] if f^{-1}(N) is e^*-open in (U, τ) for every N $\in \eta$,

(13) contra e^*-continuous [10] if f^{-1}(N) is e^*-closed in (U, τ) for every N $\in \eta$,

(14) contra δ-semicontinuous [6] if f^{-1}(N) is δ-semiclosed in (U, τ) for every N $\in \eta$.

Lemma 2.1. [20] *For a subset M of a space (U,τ),the following properties are equivalent:*

(a) M is clopen;

(b) M is δ-open and δ-closed;

(c) M is regular-open and regular-closed.

Definition 2.8. [13] A space (U, τ) is called δ-partition if δO(U) = C(U).

3. Regular δ-preopen sets

Definition 3.1. A subset N of a space (U,τ) is said to be regular δ-preopen if N = δ-pint(δ-pcl(N)). The complement of a regular δ-preopen set is called regular δ-preclosed.

Clearly, N is regular δ-preclosed if and only if N = δ-pcl(δ-pint(N)).

The collection of all regular δ-preopen (resp. regular δ-preclosed) sets of (U, τ) will be denoted by RδPO(U) (resp. RδPC(U)).

Theorem 3.1. *Let (U, τ) be a topological space and M, N $\subseteq$ U. Then the following hold:*

(i) If M $\subseteq$ N, then δ-pint(δ-pcl(M) $\subseteq$ δ-pint(δ-pcl(N)).

(ii) If M $\in$ $\delta PO(U)$, then M $\subseteq$ δ-pint(δ-pcl(M)).

(iii) If M $\in$ $\delta PC(U)$, then δ-pcl(δ-pint(M)) $\subseteq$ M.

(iv) δ-pint(δ-pcl(N)) is regular δ-preopen.

(v) If M $\in$ $\delta PC(U)$, then δ-pint(M) is regular δ-preopen.

(vi) If M $\in$ $\delta PO(U)$, then δ-pcl(M) is regular δ-preclosed.

Proof. *(i) Clear.*

(ii) Let M $\in$ $\delta PO(U)$. Since M $\subseteq$ δ-pcl(M), then M $\subseteq$ δ-pint(δ-pcl(M).

(iii) Let M $\in$ $\delta PC(U)$. Since δ-pint(M) $\subseteq$ M, then δ-pcl(δ-pint(M) $\subseteq$ M.

(iv) We have δ-pint(δ-pcl(δ-pint(δ-pcl(M)) $\subseteq$ δ-pint(δ-pcl(δ-pcl(M)) = δ-pint(δ-pcl(M) and δ-pint(δ-pcl(δ-pint(δ-pcl(M))) $\supseteq$ δ-pint(δ-pint(δ-pcl(M)) = δ-pint(δ-pcl(M). Hence δ-pint(δ-pcl(δ-pint(δ-pcl(M))) = δ-pint(δ-pcl(M).

(v) Suppose that M $\in$ $\delta PC(U)$. By (iii), δ-pint(δ-pcl(δ-pint(M)) $\subseteq$ δ-pint(M). On the other hand, we have δ-pint(M) $\subseteq$ δ-pcl(δ-pint(M) and hence δ-pint(M) $\subseteq$ δ-pint(δ-pcl(δ-pint(M)). Therefore δ-pint(δ-pcl(δ-pint(M)) = δ-pint(M).

This shows that δ-pint(M) is a regular δ-preopen set.

(vi) Similar to (v).

Theorem 3.2. *Let (U,τ) be a topological space and N $\subseteq$ U. Then*

(i) If N is a regular δ-preopen set, then it is δ-preopen .

(ii) If N is a regular δ-preopen set, then it is e-open and hence e^*-open.

(iii) If N is a regular δ-preopen set, then it is e-closed and hence e^*-closed.

Proof. *(i) and (ii) are obvious.*

(iii) Let N be regular δ-preopen, then N = δ-pint(δ-pcl(N)). By (i) and Theorem 2.2[(c) and (d)], we have N = δ-pcl(N) $\cap$ δ-scl(δ-pint(N)) = δ-pcl(N) $\cap$ δ-scl(N) = e-cl(N). Thus N is e-closed.

Remark 1. *By the following example, we show that every δ-preopen set need not be regular δ-preopen.*

Example 3.1. *Let $U = \{p,q,r,s\}$ and $\tau = \{U, \phi, \{p\}, \{q\}, \{p,q\}, \{p,r\}, \{p,q,r\}\}$. Then $\{p,q\}$ is a δ-preopen set but $\{p,q\} \notin R\delta PO(U)$.*

Theorem 3.3. *In a δ-partition space (U, τ), a subset M of U is δ-preopen if and only if it is regular δ-preopen.*

Remark 2. *The class of regular δ-preopen sets is not closed under finite union as well as finite intersection. It will be shown in the following example.*

Example 3.2. *Consider (U,τ) as in Example 3.1. Let $A = \{p\}$ and $B = \{q\}$, then A and B are regular δ-preopen sets but $A \cup B = \{p,q\} \notin R\delta PO(U)$. Moreover, $C = \{p,q,s\}$ and $D = \{q,r,s\}$ are regular δ-preopen sets but $C \cap D = \{q,s\} \notin R\delta PO(U)$.*

Theorem 3.4. *Let $N \in \delta PC(U)$. Then N is regular δ-preopen if and only if it is δ-preopen.*

Proof. *Necessity: Obvious from Theorem 3.2(i).*

Sufficiency: Let N be δ-preopen. Then by hypothesis, we have $N = \delta$-pint(N) and $N = \delta$-pcl(N). Therefore, δ-pint(δ-pcl(N)) $= \delta$-pint(N) $= N$.

Theorem 3.5. *A subset $N \subseteq U$ is regular δ-preopen if and only if N is e-closed and δ-preopen.*

Proof. *Necessity: It follows from Theorem 3.2[(i)and (iii)].*

Sufficiency: Let N be both e-closed and δ-preopen. Then $N =$ e-cl(N) and $N = \delta$-pint(N). By Theorem 2.2(a), δ-pint(δ-pcl(N)) $= \delta$-pint(e-cl(N)) $= \delta$-pint(N) $= N$. Hence N is regular δ-preopen.

Theorem 3.6. *For a subset M of a space (U, τ), the following properties are equivalent:*

(a) M is regular δ-preopen;

(b) $M = \delta$-pcl(M) $\cap$ int(δ-cl(M);

(c) $M = \delta$-pcl(M) $\cap$ δ-int(δ-scl(M));

(d) $M = \delta$-pcl(M) $\cap$ δ-scl(δ-pint(M));

(e) M = [M ∪ cl(δ-int(M))] ∩ int(δ-cl(M);

(f) M = δ-pint(e-cl(M)).

Proof. It follows from Theorems 2.1 and 2.2.

Remark 3. *Let M ⊆ U, then int(δ-cl(M)) is regular open in (U, τ).*

Definition 3.2. A space (U,τ) is called δ-submaximal if every δ-dense subset of U is δ-open

Theorem 3.7. *Let (U,τ) be a topological space, then the following properties are equivalent:*

(1) (U, τ) is δ-submaximal;

(2) Every δ-preopen set is δ-open.

Proof. (1) ⟶(2): Let N ⊆ U be a δ-preopen set. Then N ⊆ int(δ-cl(N) = M, say. This implies δ-cl(M) = δ-cl(N) and hence δ-cl((U-M) ∪ N) = δ-cl(U-M) ∪ δ-cl(N) = δ-cl(U-M) ∪ δ-cl(M) = U and thus (U-M) ∪ N is δ-dense in U. By (1), (U-M) ∪ N is δ-open. Now N = ((U-M) ∪ N) ∩ M and N is the intersection of two δ-open sets and hence N is δ-open.

(2) ⟶(1): Let M be a δ-dense subset of U. Then int(δ-cl(M)) = U, then M ⊆ int(δ-cl(M)) and M is δ-preopen. By (2), M is δ-open.

Theorem 3.8. *If a space (U,τ) is δ-submaximal, then any finite intersection of δ-preopen sets is δ-preopen.*

Proof. It follows from the fact that δO(X) is closed under finite intersection.

Theorem 3.9. *If a space (U,τ) is δ-submaximal, then any finite intersection of regular δ-preopen sets is regular δ-preopen.*

Proof. Let $\{G_i$: *i=1,2,...,n*$\}$ *be a finite family of regular δ-preopen sets. Since the space (U, τ) is δ-submaximal, then by Theorem 3.8, we have* $\bigcap_{i=1}^{n} G_i \in \delta PO(U)$*. By Theorem 3.1(ii),* $\bigcap_{i=1}^{n} G_i \subseteq$ *δ-pint(δ-pcl(*$\bigcap_{i=1}^{n} G_i$*)). Now,for each i, we have* $\bigcap_{i=1}^{n} G_i \subseteq G_i$ *and thus δ-pint(δ-pcl(*$\bigcap_{i=1}^{n} G_i$*))* ⊆ *δ-pint(δ-pcl(*G_i*))* = G_i *as δ-pint(δ-pcl(*G_i*))* = G_i*. Therefore, δ-pint(δ-pcl(*$\bigcap_{i=1}^{n} G_i$*))* ⊆ $\bigcap_{i=1}^{n} G_i$*, in consequence,* $\bigcap_{i=1}^{n} G_i \in R\delta PO(U)$*.*

Recall that a subset M of a space (U, τ) is called δ-preclopen if it is δ-preclosed and δ-preopen

Theorem 3.10. *Every δ-preclopen set is regular δ-preopen but not conversely.*
Proof. Let N be δ-preclopen, then N = δ-pint(N) = δ-pcl(N). Therefore, δ-pint(δ-pcl(N)) = δ-pint(N) = N.

Example 3.3. *In Example 3.1, the set $\{q\}$ is regular δ-preopen but it is not δ-preclopen.*

Definition 3.3. A space (U,τ) is called extremally δ-predisconnected if the δ-preclosure of every δ-preopen subset of U is δ-preopen.

Theorem 3.11. *Let (U,τ) be a topological space, then the following are equivalent:*
(1) (U,τ) is extremally δ-predisconnected;
(2) Every regular δ-preopen set is δ-preclopen.
Proof. (1) $\longrightarrow$ (2): Let M be a regular δ-preopen set, then M = δ-pint(δ-pcl(M)) = δ-pcl(M). Hence M is δ-preclosed and combined with Theorem 3.2(i), we have M is δ-preclopen.
(2) $\longrightarrow$ (1): Let $M \in \delta PO(X)$. Then by Theorem 3.1(vi), δ-pcl(M) is a regular δ-preclosed set which is δ-preclopen by (2). Hence δ-pcl(M) is δ-preopen.

Lemma 3.1. *If $M \subseteq U$ is open, then int(cl(M)) = int(δ-cl(M)).*
Proof. It is known in Lemma 2 of [20] that cl(M) = δ-cl(M) for every open subset M of U. Therefore, we have int(cl(M)) = int(δ-cl(M)).

Remark 4. *By the following example, we show that int(cl(M)) $\neq$ int(δ-cl(M)), in general.*

Example 3.4. *Let (U,τ) be a space as in Example 3.5. Consider M = $\{r,s\}$. Then δ-cl(M) = $\{p,r,s\}$ and cl(M) = $\{r,s\}$. Therefore int(cl(M)) = $\phi \neq \{p,r\}$ = int(δ-cl(M)).*

Lemma 3.2. *A subset M of a space (U,τ) is regular open if and only if M = int(cl(M)) = int(δ-cl(M)).*

Theorem 3.12. *Every regular open set is regular δ-preopen.*

Proof. Let M be regular open. Then M = int(cl(M)) = int(δ-cl(M)). By Theorem 2.2, δ-pint(δ-pcl(M)) = δ-pcl(M) ∩ int(δ-cl(M)) = δ-pcl(M) ∩ M = M. This shows that M is regular δ-preopen.

Definition 3.4. A subset N of a space (U,τ) is called q*-set if int(δ-cl(N)) ⊆ cl(δ-int(N)) and the family of q^*-sets of (U, τ) is denoted by q*O(U).

Theorem 3.13. *Every δ-semiopen set is q^*-set but not conversely.*

Proof. Let M be δ-semiopen, then by Lemma 3.1 of [16], int(δ-cl(M)) ⊆ cl(δ-int(M)). Hence M is q^*-set.

Example 3.5. *In Example 3.1, the set {s} is q^*-set but it is not δ-semiopen.*

Theorem 3.14. *Every δ-semiclosed set is q^*-set but not conversely.*

Proof. Let M be δ-semiclosed, then int(δ-cl(M)) ⊆ M. Therefore int(δ-cl(M)) ⊆ cl(δ-int(M)). Hence M is q^*-set.

Example 3.6. *In Example 3.5, the set {p,q,r} is q^*-set but it is not δ-semiclosed.*

Corollary 3.1. *Every δ-semi-regular set is q^*-set.*

Remark 5. *The above discussions can be summarized in the following diagram:*

DIAGRAM I

$$\begin{array}{ccccccccc} \text{regular open} & \to & \delta\text{-open} & \to & a\text{-open} & \to & \delta\text{-semiopen} & \to & q^*\text{-set} \\ \downarrow & & & & \downarrow & & \downarrow & & \\ \text{regular } \delta\text{-preopen} & \longrightarrow & & & \delta\text{-preopen} & \longrightarrow & e\text{-open} & \longrightarrow & e^*\text{-open} \end{array}$$

Remark 6. *The notions of q^*-sets and regular δ-preopen (hence δ-preopen, e-open, e^*-open) sets are independent of each other.*

Example 3.7. *Let (U, τ) be a space as in Example 3.1. Then {s} is q^*-set but not a e^*-open set and the set {p,q,s} is regular δ-preopen but it is not q^*-set.*

Theorem 3.15. *A subset M of a space (U,τ) is δ-semiopen if and only if it is both e-open and q^*-set.*

Theorem 3.16. *For a subset M of a space (U,τ), the following properties are equivalent:*

(i) M is regular open;

(ii) M is regular δ-preopen and δ-semi-regular;

(iii) M is regular δ-preopen and q^*-set.

Proof. *(i) $\longrightarrow$ (ii): Clear.*

(ii) $\longrightarrow$ (iii): It follows from Corollary 3.1.

(iii) $\longrightarrow$ (i): Let M be regular δ-preopen and q^*-set. Then, by Theorems 2.1 and 2.2, we obtain

$$\begin{aligned} M &= \delta\text{-}pint(\delta\text{-}pcl(M)) \\ &= (M \cup cl(\delta\text{-}int(M))) \cap int(\delta\text{-}cl(M)) \\ &= (M \cap int(\delta\text{-}cl(M))) \cup (cl(\delta\text{-}int(M)) \cap int(\delta\text{-}cl(M))) \\ &= (M \cap int(\delta\text{-}cl(M)) \cup int(\delta\text{-}cl(M)) \\ &= int(\delta\text{-}cl(M)). \end{aligned}$$

Therefore, $M = int(\delta\text{-}cl(M)) = int(cl(M))$. Hence M is regular open.

Theorem 3.17. *For a subset M of a space (U,τ), the following properties are equivalent:*

(i) M is regular open;

(ii) M is δ-open and regular δ-preopen;

(iii) M is a-open and regular δ-preopen;

(iv) M is a-open and e^*-closed.

Proof. *(i) $\longrightarrow$(ii) and (ii)$\longrightarrow$(iii) are obvious.*

(iii)$\longrightarrow$(iv): It follows from Theorem 3.2(iii).

(iv)$\longleftrightarrow$(i): It is shown in Theorem 3 of [9].

Corollary 3.2. *For a subset M of a space (U, τ), the following properties are equivalent:*

(1) M is regular open;

(2) M is δ-semiopen and regular δ-preopen;

(3) M is δ-semiclosed and regular δ-preopen;

(4) M is δ-semi-regular and regular δ-preopen;

(5) M is q^*-set and regular δ-preopen.

Theorem 3.18. *For a subset M of a space (U,τ), the following properties are equivalent:*

(1) M is clopen;

(2) M is regular δ-preopen and δ-closed.

Proof. *(1)⟶(2): It follows from Lemma 2.1 and Theorem 3.12.*

(2)⟶(1): Let M be regular δ-preopen and δ-closed. By Theorem 2.2(c), we have $M = \delta\text{-}pcl(M) \cap int(\delta\text{-}cl(M)) = \delta\text{-}pcl(M) \cap \delta\text{-}int(\delta\text{-}cl(M)) = \delta\text{-}pcl(M) \cap \delta\text{-}int(M) = \delta\text{-}int(M)$. *Therefore M is δ-open.Then by Lemma 2.1,M is clopen.*

4. Decompositions of complete continuity

In this section, the notion of regular δ-preopen-continuity is introduced and the decompositions of complete continuity are discussed.

Definition 4.1. A function $f : (U, \tau) \to (V, \eta)$ is said to be

(1) regular δ-pre continuous (briefly, rδp-continuous) if f^{-1}(N) is regular δ-preopen in (U, τ) for each N $\in \eta$,

(2) q*-continuous if f^{-1}(N) is q*-set in X for each N $\in \eta$.

Remark 7. *By Diagram I,we have the following diagram:*

DIAGRAM II

complete continuity→super-continuity→a-continuity→δ-semicontinuity → q*-continuity

↓ ↙ ↙

rδp-continuity ⟶ δ-almost continuity ⟶ e-continuity ⟶ e*-continuity

Theorem 4.1. *For a function $f : (U, \tau) \to (V, \eta)$, the following properties are equivalent:*

(1) f is completely continuous;

(2) f is super continuous and rδp-continuous;

(3) f is a-continuous and rδp-continuous;

(4) f is a-continuous and contra e*-continuous;

(5) f is δ-semicontinuous and rδp-continuous;

(6) f is contra δ-semicontinuous and rδp-continuous;

(7) f is δ-semiregular-continuous and rδp-continuous;

(8) f is q^*-continuous and rδp-continuous.

Proof. This is an immediate consequence of Theorem 3.17 and Corollary 3.2.

Remark 8. *The following properties are shown by Example 4.1 (below).*

(1) rδp-continuity and super continuity (hence a-continuity, δ-semicontinuity, q^*-continuity) are independent of each other.

(2) rδp-continuity and δ-semiregular-continuity (hence contra δ-semicontinuity) are independent of each other.

Example 4.1. *Let (U,τ) be a space as in Example 3.1 and let $\eta = \{U, \phi, \{p\}, \{q\}, \{p,q\}, \{p,q,r\}\}$*

(1) Define $f : (U,\tau) \to (V,\eta)$ by f(p) = f(r) = p , f(q) = q and f(s) = s. Clearly f is super continuous but for $\{p,q\} \in \eta$, $f^{-1}(\{p,q\}) = \{p,q,r\} \notin R\delta PO(U)$. Therefore f is not r$\delta$p-continuous. Define $g : (U,\tau) \to (V,\eta)$ by g(p) = q, g(q) = g(r) = g(s) = p. Then g is rδp-continuous but for $\{p\} \in \eta$, $g^{-1}(\{p\}) = \{q,r,s\} \notin q^*O(U)$. Therefore g is not q^*-continuous.

(2) Define $f : (U,\tau) \to (V,\eta)$ by f(p) = f(r) = f(s) = q and f(q) = p. Clearly f is δ-semiregular-continuous but for $\{q\} \in \eta$, $f^{-1}(\{q\}) = \{p,r,s\} \notin R\delta PO(U)$. Therefore f is not r$\delta$p-continuous. Define $g : (U,\tau) \to (V,\eta)$ by g(p) = g(q) = g(s) =p, g(r) = q. Then g is rδp-continuous but for $\{p\} \in \eta$, $g^{-1}(\{p\}) = \{p,q,s\} \notin \delta SC(U)$. Therefore g is not contra δ-semicontinuous.

5. Decompositions of perfect continuity

In this section, the decompositions of perfect continuity are obtained.

Theorem 5.1. *For a function $f : (U,\tau) \to (V,\eta)$, the following are equivalent:*

(i) f is perfectly continuous;

(ii) f is super continuous and contra super continuous;

(iii) f is completely continuous and RC-continuous;

(iv) f is rδp-continuous and contra super continuous.

Proof. It is a direct consequence of Theorem 3.18 and Lemma 2.1

Remark 9. *As shown by the following examples, $r\delta p$-continuity and contra super continuity are independent of each other.*

Example 5.1. *Let (U,τ) be a space as in Example 3.1 and let $\eta = \{U, \phi, \{p\}, \{q\}, \{p,q\}, \{p,q,r\}\}$. Define $f : (U,\tau) \to (V,\eta)$ by $f(p) = f(r) = f(s) = p$ and $f(q) = r$. Then f is contra super continuous but it is not $r\delta p$-continuous since $\{p\} \in \eta$, $f^{-1}(\{p\}) = \{p,r,s\} \notin R\delta PO(U)$. Define $g : (U,\tau) \to (V,\eta)$ by $g(p) = q$, $g(q) = g(r) = g(s) = p$. Then g is $r\delta p$-continuous but it is not contra super continuous since $\{p\} \in \eta$, $g^{-1}(\{p\}) = \{q,r,s\} \notin \delta C(U)$.*

Acknowledgement

We would like to thank the referees for their valuable suggestions.

References

[1] Z. A. Ameen, B. A. Asaad ,R. A. Muhammed, On superclasses of δ-open sets in topological spaces, *Int. J. Appl. Math.*, **32**(2019), 259–277.

[2] S. P. Arya , R. Gupta, On Strongly continuous mappings, *Kyungpook Math. J.* **14**(1974), 131–143.

[3] J. Dontchev, Contra-continuous functions and strongly S-closed spaces, *Int. J. Math. Math. Sci.*,**19(2)**(1996), 303–310.

[4] J. Dontchev and T. Noiri,Contra-semicontinuous functions, *Math. Pannonica*, **10**(1999), 159–168.

[5] E. Ekici and G. Navalagi,δ-semicontinuous functions,*Mathematical Forum*,**17**(2004-2005), 29–42.

[6] E. Ekici, On a weaker form of RC-continuity, *Analele Universiătii de Vest din Timisoara: Seria Matematică-Informatică*,**42**, fasc.,1(2004),79–91.

[7] E. Ekici,On e-open sets, DP^*-sets and DPE^*-sets and decompositions of continuity, *Arab. J. Sci. Eng.*, **33**(2008), 269–282.

[8] E. Ekici, On a-open sets, A^*-sets and decompositions of continuity and supercontinuity, *Ann. Univ. Sci. Budapest. Eotvos Sect. Math.*, **51**(2008), 39–51.

[9] E. Ekici,A note on a-open sets and e^*-sets, *Filomat*,**22(1)**(2008), 89–96.

[10] E. Ekici,New forms of contra-continuity,*Carpathian J. Math.*, **24(1)**(2008), 37–45.

[11] E. Ekici, On e^*-open sets and $(D,S)^*$-sets and decompositions of continuous functions, *Math. Moravica*,**13**(2009),29–36.

[12] S. Jafari and T. Noiri,Contra-super-continuous functions,*Ann. Univ. Sci. Budapest. Eotvos Sect. Math.* **42**(1999), 27–34.

[13] J. K. Kohli and D. Singh,δ-perfectly continuous functions, *Demonstr. Math.*, **42(1)**(2009), 221–231.

[14] B. M. Munshi and D. S. Bassan, Super-continuous mappings, *Indian J. Pure Appl. Math.*, **13(2)**(1982), 229–236.

[15] T. Noiri,Super-continuity and some strong forms of continuity,*Indian J. Pure Appl. Math.*, **15(3)**(1984), 241–250.

[16] T. Noiri, Remarks on δ-semi-open sets and δ-preopen sets, *Demonstr. Math.*, **36**(4)(2003), 1007–1020.

[17] J. H. Park, B. Y. Lee and M. J. Son,On δ-semiopen sets in topological space, *J. Indian Acad. Math.*, **19(1)**(1997), 59–67.

[18] S. Raychaudhuri and M. N. Mukherjee,On δ-almost continuity and δ-preopen sets,*Bull. Inst. Math. Acad. Sinica*, **21(4)**(1993),357–366.

[19] M. Stone,Application of the theory of boolean rings to general topology, *Trans. Amer. Math. Soc.*, **41**(1937), 371–381.

[20] N.V.Veličko,H-closed topological spaces,*Amer. Math. Soc. Transl.*, **78(2)**(1968),103–118.

(1) (Corresponding Author) Department of Mathematics, Karnatak University's Karnatak Arts College, Dharwad-580 001 India

Email address: jagadeeshbt2000@gmail.com

(2) Shiokita-cho, Hinagu, Yatsushiro-shi, Kumamoto-ken, 869-5142 Japan

Email address: t.noiri@nifty.com