

## SEVERAL RESULTS ON SUM DIVISOR CORDIAL GRAPH

A. LOURDUSAMY <sup>(1)</sup> AND F. PATRICK <sup>(2)</sup>

**ABSTRACT.** A sum divisor cordial labeling of a graph  $G$  with vertex set  $V$  is a bijection  $f$  from  $V$  to  $\{1, 2, \dots, |V(G)|\}$  such that an edge  $uv$  is assigned the label 1 if 2 divides  $f(u) + f(v)$  and 0 otherwise; and the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. A graph with a sum divisor cordial labeling is called a sum divisor cordial graph. In this paper, we prove that every transformed tree admits sum divisor cordial labeling. Also, we investigate the sum divisor cordial labeling of the graph obtained by identifying the vertex of graphs. Finally, we discuss the sum divisor cordial labeling of splitting graph and middle graph.

### 1. INTRODUCTION

Graphs considered here are finite, undirected and simple. The vertex set and the edge set of a graph are denoted by  $V(G)$  and  $E(G)$ , respectively. We follow the basic notations and terminology of graph theory as in [4]. A labeling of a graph is a map that carries the graph elements to the set of numbers, usually to the set of non-negative or positive integers. If the domain is the set of vertices, then the labeling is called vertex labeling. If the domain is the set of edges, then the labeling is called edge labeling. If the labels are assigned to both vertices and edges then the labeling is called total labeling. A detailed survey of graph labeling is available in [3]. The concept of cordial labeling was introduced by Cahit in [2].

Lourdusamy et al. introduced the concept of sum divisor cordial labeling in [6].

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**Definition 1.1.** Let  $G = (V(G), E(G))$  be a simple graph and  $f : V(G) \rightarrow \{1, 2, \dots, |V(G)|\}$  be a bijection function. For each edge  $uv$ , assign the label 1 if  $2|(f(u) + f(v))|$  and the label 0 otherwise. The function  $f$  is called a sum divisor cordial (SDC) labeling if  $|e_f(1) - e_f(0)| \leq 1$ , where  $e_f(s)$  denotes the number of edges labeled with  $s$  ( $s = 0, 1$ ). A graph which admits a sum divisor cordial labeling is called a sum divisor cordial (SDC) graph.

In [6], Lourdusamy et al. proved that paths, combs, stars, complete bipartite,  $K_2 + mK_1$ , bistars, jewels, crowns, flowers, gears, subdivisions of stars, the graph obtained from  $K_{1,3}$  by attaching the root of  $K_{1,n}$  at each pendent vertex of  $K_{1,3}$ , and the square  $B_{n,n}$  are SDC graphs. Also they discussed the SDC labeling of star related graphs, path related graphs and cycle related graphs in [7, 8].

In [9], Lourdusamy et al. discussed the SDC labeling of transformed tree related graphs like  $T\hat{O}P_n, T\hat{O}C_n$  ( $n \equiv 1, 3, 0 \pmod{4}$ ),  $T\hat{O}K_{1,n}, T\hat{O}\bar{K}_n, T\hat{O}Q_n, T\tilde{O}C_n$  ( $n \equiv 1, 3, 0 \pmod{4}$ ) and  $T\tilde{O}Q_n$ .

In [12, 13, 14, 15], Sugumaran et al. investigated the behaviour of SDC labeling of swastiks, path unions of finite number of copies of swastiks, cycles of  $k$  copies of swastiks, when  $k$  is odd, jelly fish, Petersen graphs, theta graphs, the fusion of any two vertices in the cycle of swastiks, duplication of any vertex in the cycle of swastiks, the switchings of a central vertex in swastiks, the path unions of two copies of a swastik, the star graph of the theta graphs, the Herschel graph, the fusion of any two adjacent vertices of degree 3 in Herschel graphs, the duplication of any vertex of degree 3 in the Herschel graph, the switching of central vertex in Herschel graph, the path union of two copies of the Herschel graph,  $H$ -graph  $H_n$ , when  $n$  is odd,  $C_3 @ K_{1,n}, < F_n^1 \Delta F_n^2 >$  and open star of swastik graphs  $S(t.Sw_n)$ , when  $t$  is odd.

In [16, 17, 18, 19], Sugumaran et al. proved that the following graphs are SDC graphs:  $H$ -graph  $H_n$ , when  $n$  is even, duplication of all edges of the  $H$ -graph  $H_n$ , when  $n$  is even,  $H_n \odot K_1, P(r.H_n), C(r.H_n)$ , plus graphs, umbrella graphs, path unions of odd cycles, kites, complete binary trees, drums graph, twigs, fire crackers of the form  $P_n \odot S_n$ , where  $n$  is even, and the double arrow graph  $DA_m^n$ , where  $|m - n| \leq 1$  and  $n$  is even. Further results on SDC labeling are given in [1, 10].

In this paper, we prove that every transformed tree admits SDC labeling. Also, we investigate the SDC labeling of the graph obtained by identifying the vertex of graphs. Finally we discuss the SDC labeling of splitting graph of path, middle graph of path and splitting graph of cycle. We use the following definitions in the subsequent sections.

**Definition 1.2.** [5] Let  $T$  be a tree and  $u_0$  and  $v_0$  be two adjacent vertices in  $T$ . Let there be two pendant vertices  $u$  and  $v$  in  $T$  such that the length of  $u_0 - u$  path is equal to the length of  $v_0 - v$  path. If the edge  $u_0v_0$  is deleted from  $T$  and  $u, v$  are joined by an edge  $uv$ , then such a transformation of  $T$  is called an elementary parallel transformation (or an ept) and the edge  $u_0v_0$  is called transformable edge.

If  $T$  can be reduced to a path by the sequence of ept's, then  $T$  is called a  $T_p$ -tree (transformed tree) and such a sequence regarded as a composition of mappings (ept's) denoted by  $P$ , is called a parallel transformation of  $T$ . The path, the image of  $T$  under  $P$  is denoted by  $P(T)$ .

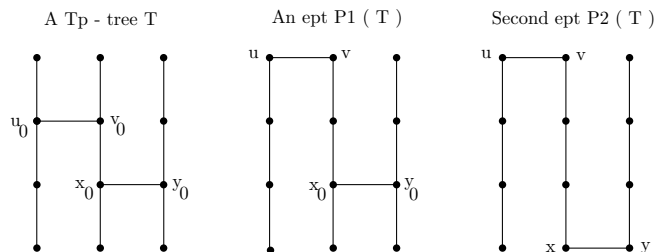


FIGURE 1

A  $T_p$ -tree and a sequence of two ept's reducing it to a path are shown in Figure 1.

**Definition 1.3.** [8] For a graph  $G$  the splitting graph  $S'(G)$  of a graph  $G$  is obtained by adding a new vertex  $v'$  corresponding to each vertex  $v$  of  $G$  such that  $N(v) = N(v')$ .

**Definition 1.4.** [11] The middle graph  $M(G)$  of a graph  $G$  is the graph whose vertex set is  $V(G) \cup E(G)$  and in which two vertices are adjacent if and only if either they are adjacent edges of  $G$  or one is a vertex of  $G$  and the other is an edge incident with it.

## 2. TRANSFORMED TREE

**Theorem 2.1.** *Every  $T_p$ -tree is a SDC graph.*

*Proof.* Let  $T$  be a  $T_p$ -tree with  $m$  vertices. By the definition of a transformed tree there exists a parallel transformation  $P$  of  $T$  such that for the path  $P(T)$ , we have,

$$(i) \ V(P(T)) = V(T) \quad \text{and}$$

$$(ii) \ E(P(T)) = (E(T) - E_d) \cup E_p,$$

where  $E_d$  is the set of edges deleted from  $T$  and  $E_p$  is the set of edges newly added through the sequence  $P = (P_1, P_2, \dots, P_r)$  of the *epts*  $P$  used to arrive at the path  $P(T)$ . Clearly,  $E_d$  and  $E_p$  have the same number of edges. Denote the vertices of  $P(T)$  successively as  $u_1, u_2, \dots, u_m$  starting from one pendant vertex of  $P(T)$  right up to the other.

Define  $g : V(T) \rightarrow \{1, 2, \dots, m\}$  as follows:

**Case 1 :**  $m$  is odd and  $1 \leq j \leq m$ .

$$g(u_j) = \begin{cases} j & \text{if } j \equiv 0, 1 \pmod{4} \\ j + 1 & \text{if } j \equiv 2 \pmod{4} \\ j - 1 & \text{if } j \equiv 3 \pmod{4}. \end{cases}$$

Let  $u_j u_k$  be a transformed edge in  $T$ ,  $1 \leq j < k \leq m$  and let  $P_1$  be the *ept* obtained by deleting the edge  $u_j u_k$  and adding the edge  $u_{j+t} u_{k-t}$  where  $t$  is the distance of  $u_j$  from  $u_{j+t}$  and the distance of  $u_k$  from  $u_{k-t}$ . Let  $P$  be a parallel transformation of  $T$  that contains  $P_1$  as one of the constituent *epts*.

Since  $u_{j+t} u_{k-t}$  is an edge in the path  $P(T)$ , it follows that  $j + t + 1 = k - t$  which implies that  $k = j + 2t + 1$ . Therefore,  $j$  and  $k$  are of opposite parity.

The induced edge label  $u_j u_k$  is given by

$$\begin{aligned} g^*(u_j u_k) &= g^*(u_j u_{j+2t+1}) \\ &= \begin{cases} 1 & \text{if } j \equiv 1, 3 \pmod{4} \text{ and } 1 \leq j \leq m \\ 0 & \text{if } j \equiv 2, 0 \pmod{4} \text{ and } 1 \leq j \leq m. \end{cases} \end{aligned}$$

The induced edge label  $u_{j+t}u_{k-t}$  is given by

$$\begin{aligned} g^*(u_{j+t}u_{k-t}) &= g^*(u_{j+t}u_{j+t+1}) \\ &= \begin{cases} 1 & \text{if } j \equiv 1, 3 \pmod{4} \text{ and } 1 \leq j \leq m \\ 0 & \text{if } j \equiv 2, 0 \pmod{4} \text{ and } 1 \leq j \leq m. \end{cases} \end{aligned}$$

Therefore,  $g^*(u_ju_k) = g^*(u_{j+t}u_{k-t})$ .

The induced edge labels are

$$g^*(u_ju_{j+1}) = \begin{cases} 1 & \text{if } j \text{ is odd and } 1 \leq j \leq m-1 \\ 0 & \text{if } j \text{ is even and } 1 \leq j \leq m-1. \end{cases}$$

**Case 2 :**  $m$  is even and  $1 \leq j \leq m$ .

$$g(u_j) = \begin{cases} j & \text{if } j \equiv 1, 2 \pmod{4} \\ j+1 & \text{if } j \equiv 3 \pmod{4} \\ j-1 & \text{if } j \equiv 0 \pmod{4}. \end{cases}$$

Let  $u_ju_k$  be a transformed edge in  $T$ ,  $1 \leq j < k \leq m$  and let  $P_1$  be the *ept* obtained by deleting the edge  $u_ju_k$  and adding the edge  $u_{j+t}u_{k-t}$  where  $t$  is the distance of  $u_j$  from  $u_{j+t}$  and the distance of  $u_k$  from  $u_{k-t}$ . Let  $P$  be a parallel transformation of  $T$  that contains  $P_1$  as one of the constituent *epts*.

Since  $u_{j+t}u_{k-t}$  is an edge in the path  $P(T)$ , it follows that  $j+t+1 = k-t$  which implies that  $k = j+2t+1$ . Therefore,  $j$  and  $k$  are of opposite parity.

The induced edge label  $u_ju_k$  is given by

$$\begin{aligned} g^*(u_ju_k) &= g^*(u_ju_{j+2t+1}) \\ &= \begin{cases} 0 & \text{if } j \equiv 1, 3 \pmod{4} \text{ and } 1 \leq j \leq m \\ 1 & \text{if } j \equiv 2, 0 \pmod{4} \text{ and } 1 \leq j \leq m. \end{cases} \end{aligned}$$

The induced edge label  $u_{j+t}u_{k-t}$  is given by

$$\begin{aligned} g^*(u_{j+t}u_{k-t}) &= g^*(u_{j+t}u_{j+t+1}) \\ &= \begin{cases} 0 & \text{if } j \equiv 1, 3 \pmod{4} \text{ and } 1 \leq j \leq m \\ 1 & \text{if } j \equiv 2, 0 \pmod{4} \text{ and } 1 \leq j \leq m. \end{cases} \end{aligned}$$

Therefore,  $g^*(u_j u_k) = g^*(u_{j+t} u_{k-t})$ .

The induced edge labels are

$$g^*(u_j u_{j+1}) = \begin{cases} 0 & \text{if } j \text{ is odd and } 1 \leq j \leq m-1 \\ 1 & \text{if } j \text{ is even and } 1 \leq j \leq m-1. \end{cases}$$

In the above two cases, we observe that  $|e_g(1) - e_g(0)| \leq 1$ . Hence,  $T_p$ -tree  $T$  admits a SDC labeling.  $\square$

### 3. IDENTIFYING THE VERTEX OF GRAPHS

**Lemma 3.1.** *Let  $g$  be a SDC labeling of a graph  $G$  of order  $p$  and size  $q$ . Let  $z \in V(G)$  be a vertex such that  $g(z) = 2$ . The graph obtained by identifying a vertex  $z$  in  $G$  and a vertex of degree  $m$  in  $K_{1,m}$  admits a SDC labeling if one of the following conditions holds:*

- (1)  $m$  is even,
- (2)  $m$  is odd and  $q$  is even,
- (3)  $m$  is odd,  $q$  is odd,  $p$  is even and  $e_g(1) = \frac{q+1}{2}$ ,
- (4)  $m$  is odd,  $q$  is odd,  $p$  is odd and  $e_g(1) = \frac{q-1}{2}$ .

*Proof.* Let  $g$  be a SDC labeling of a graph  $G$  of order  $p$  and size  $q$ . That is the vertices of  $G$  are labeled with numbers  $\{1, 2, \dots, p\}$  and  $|e_g(1) - e_g(0)| \leq 1$ .

Let  $z \in V(G)$  be such that  $g(z) = 2$ . Let us denote by  $H$  the graph obtained by identifying a vertex  $z$  in  $G$  and a vertex of degree  $m$  in  $K_{1,m}$ .

We define a vertex labeling  $h$  of  $H$  such that

$$h(v) = g(v), \quad v \in V(G);$$

$$h(x_j) = p + j, \quad j = 1, 2, \dots, m.$$

Thus for the induced edge labeling we get

$$h(uv) = g(vu), \quad v \in V(G);$$

for  $j = 1, 2, \dots, m$

$$h(zx_j) = 1 \quad \text{if } (p \text{ is odd and } j \text{ is odd}) \text{ or } (p \text{ is even and } j \text{ is even}).$$

$$h(zx_j) = 0 \text{ if } (p \text{ is odd and } j \text{ is even}) \text{ or } (p \text{ is even and } j \text{ is odd}).$$

Let us denote by  $e_h^*(k)$  number of edges  $zx_j$  labeled with  $k$ , where  $k = 0, 1$ . Then

$$\begin{aligned} |e_h(1) - e_h(0)| &= |(e_g(1) - e_h^*(1)) - (e_g(0) - e_h^*(0))| \\ &= |e_g(1) - e_g(0) + e_h^*(1) - e_h^*(0)| \\ &= \left| \frac{q}{2} - \frac{q}{2} + \frac{m}{2} - \frac{m}{2} \right| = 0, \text{ if } (m \text{ is even and } q \text{ is even}); \\ &= \left| \frac{q+1}{2} - \frac{q-1}{2} + \frac{m}{2} - \frac{m}{2} \right| = 1, \text{ if } (m \text{ is even, } q \text{ is odd and } e_g(1) = \frac{q+1}{2}); \\ &= \left| \frac{q-1}{2} - \frac{q+1}{2} + \frac{m}{2} - \frac{m}{2} \right| = 1, \text{ if } (m \text{ is even, } q \text{ is odd and } e_g(1) = \frac{q-1}{2}); \\ &= \left| \frac{q}{2} - \frac{q}{2} + \frac{m-1}{2} - \frac{m+1}{2} \right| = 1, \text{ if } (m \text{ is odd, } p \text{ is even and } q \text{ is even}); \\ &= \left| \frac{q}{2} - \frac{q}{2} + \frac{m+1}{2} - \frac{m-1}{2} \right| = 1, \text{ if } (m \text{ is odd, } p \text{ is odd and } q \text{ is even}); \\ &= \left| \frac{q+1}{2} - \frac{q-1}{2} + \frac{m-1}{2} - \frac{m+1}{2} \right| = 0, \text{ if } (m \text{ is odd, } p \text{ is even, } q \text{ is odd} \\ &\quad \text{and } e_g(1) = \frac{q+1}{2}); \\ &= \left| \frac{q-1}{2} - \frac{q+1}{2} + \frac{m+1}{2} - \frac{m-1}{2} \right| = 0, \text{ if } (m \text{ is odd, } p \text{ is odd, } q \text{ is odd} \\ &\quad \text{and } e_g(1) = \frac{q-1}{2}). \end{aligned}$$

In the above cases,  $|e_h(1) - e_h(0)| \leq 1$ . Hence the proof is complete.  $\square$

**Lemma 3.2.** *Let  $g$  be a SDC labeling of a graph  $G$  of order  $p$  and size  $q$ . Let  $z \in V(G)$  be a vertex such that  $g(z) = 1$ . The graph obtained by identifying a vertex  $z$  in  $G$  and a vertex of degree  $m$  in  $K_{1,m}$  admits a SDC labeling if one of the following conditions holds:*

- (1)  $m$  is even,
- (2)  $m$  is odd and  $q$  is even,
- (3)  $m$  is odd,  $q$  is odd,  $p$  is odd and  $e_g(1) = \frac{q+1}{2}$ ,
- (4)  $m$  is odd,  $q$  is odd,  $p$  is even and  $e_g(1) = \frac{q-1}{2}$ .

*Proof.* The proof is similar to that of Lemma 3.1.  $\square$

**Theorem 3.1.** *Let  $g$  be a SDC labeling of a graph  $G$  of order  $p$  and size  $q$ . Let  $z \in V(G)$  be a vertex such that  $g(z) = 2^r$ , where  $r = 0, 1, 2, \dots$ . The graph obtained by identifying a vertex  $z$  in  $G$  and a vertex of degree  $m$  in  $K_{1,m}$  admits a SDC labeling.*

*Proof.* The proof follows from Lemma 3.2 and Lemma 3.1.  $\square$

**Lemma 3.3.** *Let  $g$  be a SDC labeling of a graph  $G$  of order  $p$  and size  $q$ . Let  $z \in V(G)$  be a vertex such that  $g(z) = 2$ . The graph obtained by identifying a vertex  $z$  in  $G$  and*

a vertex of degree 1 in  $P_m$  admits a SDC labeling if one of the following conditions holds:

- (1)  $m$  is odd,
- (2)  $m$  is even and  $q$  is even,
- (3)  $m$  is even,  $q$  is odd,  $p$  is even and  $e_g(1) = \frac{q+1}{2}$ ,
- (4)  $m$  is even,  $q$  is odd,  $p$  is odd and  $e_g(1) = \frac{q-1}{2}$ .

*Proof.* Let  $g$  be a SDC labeling of a graph  $G$  of order  $p$  and size  $q$ . That is the vertices of  $G$  are labeled with numbers  $\{1, 2, \dots, p\}$  and  $|e_g(1) - e_g(0)| \leq 1$ .

Let  $z \in V(G)$  be such that  $g(z) = 2$ . Let us denote by  $H$  the graph obtained by identifying a vertex  $z$  in  $G$  and a vertex of degree 1 in  $P_m$ .

We define a vertex labeling  $h$  of  $H$  such that

$$h(v) = g(v), \quad v \in V(G);$$

If  $m$  is odd and  $p$  is odd, then define  $h(y_j)$  as

$$h(y_j) = \begin{cases} p+j-1 & \text{if } j \equiv 2, 3 \pmod{4} \text{ and } 2 \leq j \leq m \\ p+j & \text{if } j \equiv 0 \pmod{4} \text{ and } 2 \leq j \leq m \\ p+j-2 & \text{if } j \equiv 1 \pmod{4} \text{ and } 2 \leq j \leq m ; \end{cases}$$

If  $m$  is odd and  $p$  is even, then define  $h(y_j)$  as

$$h(y_j) = \begin{cases} p+j & \text{if } j \equiv 2 \pmod{4} \text{ and } 2 \leq j \leq m \\ p+j-2 & \text{if } j \equiv 3 \pmod{4} \text{ and } 2 \leq j \leq m \\ p+j-1 & \text{if } j \equiv 0, 1 \pmod{4} \text{ and } 2 \leq j \leq m ; \end{cases}$$

If  $m$  is even, then define  $h(y_j)$  as

$$h(y_j) = \begin{cases} p+j-1 & \text{if } j \equiv 2, 1 \pmod{4} \text{ and } 2 \leq j \leq m \\ p+j & \text{if } j \equiv 3 \pmod{4} \text{ and } 2 \leq j \leq m \\ p+j-2 & \text{if } j \equiv 0 \pmod{4} \text{ and } 2 \leq j \leq m. \end{cases}$$

Thus for the induced edge labeling we get

$$h(uv) = g(vu), \quad v \in V(G);$$



$$\begin{aligned}
h(zy_2) &= 0 \text{ if } (p \text{ is even and } m \text{ is even}); \\
h(zy_2) &= 1 \text{ if } (m \text{ is odd}) \text{ or } (p \text{ is odd and } m \text{ is even}); \\
h(y_{2j-1}y_{2j}) &= 1 \text{ if } m \text{ is odd and } 2 \leq j \leq \frac{m-1}{2}; \\
h(y_{2j-1}y_{2j}) &= 0 \text{ if } m \text{ is even and } 2 \leq j \leq \frac{m}{2}; \\
h(y_{2j}y_{2j+1}) &= 0 \text{ if } m \text{ is odd and } 1 \leq j \leq \frac{m-1}{2}; \\
h(y_{2j}y_{2j+1}) &= 1 \text{ if } m \text{ is even and } 1 \leq j \leq \frac{m-2}{2}.
\end{aligned}$$

Let us denote by  $e_h^*(k)$  number of edges  $zy_2, y_jy_{j+1}$  labeled with  $k$ , where  $k = 0, 1$ .

Then

$$\begin{aligned}
|e_h(1) - e_h(0)| &= |(e_g(1) - e_h^*(1)) - (e_g(0) - e_h^*(0))| \\
&= |e_g(1) - e_g(0) + e_h^*(1) - e_h^*(0)| \\
&= \left| \frac{q}{2} - \frac{q}{2} + \frac{m-1}{2} - \frac{m-1}{2} \right| = 0, \text{ if } (m \text{ is odd and } q \text{ is even}); \\
&= \left| \frac{q+1}{2} - \frac{q-1}{2} + \frac{m-1}{2} - \frac{m-1}{2} \right| = 1, \text{ if } (m \text{ is odd, } q \text{ is odd and} \\
&\quad e_g(1) = \frac{q+1}{2}); \\
&= \left| \frac{q-1}{2} - \frac{q+1}{2} + \frac{m-1}{2} - \frac{m-1}{2} \right| = 1, \text{ if } (m \text{ is odd, } q \text{ is odd and} \\
&\quad e_g(1) = \frac{q-1}{2}); \\
&= \left| \frac{q}{2} - \frac{q}{2} + \frac{m}{2} - \frac{m-2}{2} \right| = 1, \text{ if } (m \text{ is even, } q \text{ is even and } p \text{ is odd}); \\
&= \left| \frac{q}{2} - \frac{q}{2} + \frac{m-2}{2} - \frac{m}{2} \right| = 1, \text{ if } (m \text{ is even, } q \text{ is even and } p \text{ is even}); \\
&= \left| \frac{q+1}{2} - \frac{q-1}{2} + \frac{m-2}{2} - \frac{m}{2} \right| = 0, \text{ if } (m \text{ is even, } q \text{ is odd, } p \text{ is even and} \\
&\quad e_g(1) = \frac{q+1}{2}); \\
&= \left| \frac{q-1}{2} - \frac{q+1}{2} + \frac{m}{2} - \frac{m-2}{2} \right| = 0, \text{ if } (m \text{ is even, } q \text{ is odd, } p \text{ is odd and} \\
&\quad e_g(1) = \frac{q-1}{2}).
\end{aligned}$$

In the above cases,  $|e_h(1) - e_h(0)| \leq 1$ . Hence the proof is complete.  $\square$

**Lemma 3.4.** *Let  $g$  be a SDC labeling of a graph  $G$  of order  $p$  and size  $q$ . Let  $z \in V(G)$  be a vertex such that  $g(z) = 1$ . The graph obtained by identifying a vertex  $z$  in  $G$  and a vertex of degree 1 in  $P_m$  admits a SDC labeling if one of the following conditions holds:*

- (1)  $m$  is odd,
- (2)  $m$  is even and  $q$  is even,
- (3)  $m$  is even,  $q$  is odd,  $p$  is even and  $e_g(1) = \frac{q-1}{2}$ ,
- (4)  $m$  is even,  $q$  is odd,  $p$  is odd and  $e_g(1) = \frac{q+1}{2}$ .

*Proof.* The proof is similar to that of Lemma 3.3.  $\square$

**Theorem 3.2.** *Let  $g$  be a SDC labeling of a graph  $G$  of order  $p$  and size  $q$ . Let  $z \in V(G)$  be a vertex such that  $g(z) = 2^r$ , where  $r = 0, 1, 2, \dots$ . The graph obtained by identifying a vertex  $z$  in  $G$  and a vertex of degree 1 in  $P_m$  admits a SDC labeling.*

*Proof.* The proof follows from Lemma 3.4 and Lemma 3.3.  $\square$

**Lemma 3.5.** *Let  $g$  be a SDC labeling of a graph  $G$  of order  $p$  and size  $q$ . Let  $z \in V(G)$  be a vertex such that  $g(z) = 2$ . The graph obtained by identifying a vertex  $z$  in  $G$  and a vertex of degree  $m$  in  $F_m$  admits a SDC labeling if one of the following conditions holds:*

- (1)  $m$  is even and  $q$  is even,
- (2)  $m$  is even,  $q$  is odd and  $e_g(1) = \frac{q+1}{2}$ ,
- (3)  $m$  is odd and  $q$  is even,
- (4)  $m$  is odd,  $q$  is odd,  $p$  is even and  $e_g(1) = \frac{q+1}{2}$ ,
- (5)  $m$  is odd,  $q$  is odd,  $p$  is odd and  $e_g(1) = \frac{q-1}{2}$ .

*Proof.* Let  $g$  be a SDC labeling of a graph  $G$  of order  $p$  and size  $q$ . That is the vertices of  $G$  are labeled with numbers  $\{1, 2, \dots, p\}$  and  $|e_g(1) - e_g(0)| \leq 1$ .

Let  $z \in V(G)$  be such that  $g(z) = 2$ . Let us denote by  $H$  the graph obtained by identifying a vertex  $z$  in  $G$  and a vertex of degree  $m$  in  $F_m$ .

We define a vertex labeling  $h$  of  $H$  such that

$$h(v) = g(v), \quad v \in V(G);$$

If  $m$  is even, then define  $h(x_j)$  as

$$h(x_j) = \begin{cases} p+j & \text{if } j \equiv 1, 2 \pmod{4} \text{ and } 1 \leq j \leq m \\ p+j+1 & \text{if } j \equiv 3 \pmod{4} \text{ and } 1 \leq j \leq m \\ p+j-1 & \text{if } j \equiv 0 \pmod{4} \text{ and } 1 \leq j \leq m ; \end{cases}$$

If  $m$  is odd, then define  $h(x_j)$  as

$$h(x_j) = \begin{cases} p+j & \text{if } j \equiv 1, 0 \pmod{4} \text{ and } 1 \leq j \leq m \\ p+j+1 & \text{if } j \equiv 2 \pmod{4} \text{ and } 1 \leq j \leq m \\ p+j-1 & \text{if } j \equiv 3 \pmod{4} \text{ and } 1 \leq j \leq m. \end{cases}$$

Thus for the induced edge labeling we get

$$h(uv) = g(vu), \quad v \in V(G);$$

for  $1 \leq j \leq m$ ,

$$\begin{aligned} h(zx_j) = 1 & \quad \text{if } (p \text{ is odd, } m \text{ is even and } j \equiv 1, 0 \pmod{4}) \text{ or} \\ & \quad (p \text{ is even, } m \text{ is even and } j \equiv 2, 3 \pmod{4}) \text{ or} \\ & \quad (p \text{ is odd, } m \text{ is odd and } j \equiv 1, 2 \pmod{4}) \text{ or} \\ & \quad (p \text{ is even, } m \text{ is odd and } j \equiv 3, 0 \pmod{4}); \end{aligned}$$

$$\begin{aligned} h(zx_j) = 0 & \quad \text{if } (p \text{ is odd, } m \text{ is even and } j \equiv 2, 3 \pmod{4}) \text{ or} \\ & \quad (p \text{ is even, } m \text{ is even and } j \equiv 1, 0 \pmod{4}) \text{ or} \\ & \quad (p \text{ is odd, } m \text{ is odd and } j \equiv 3, 0 \pmod{4}) \text{ or} \\ & \quad (p \text{ is even, } m \text{ is odd and } j \equiv 1, 2 \pmod{4}); \end{aligned}$$

for  $1 \leq j \leq m-1$ ,

$$h(x_jx_{j+1}) = 0 \quad \text{if } (m \text{ is even and } j \text{ is odd}) \text{ or } (m \text{ is odd and } j \text{ is even});$$

$$h(x_jx_{j+1}) = 1 \quad \text{if } (m \text{ is even and } j \text{ is even}) \text{ or } (m \text{ is odd and } j \text{ is odd}).$$

Let us denote by  $e_h^*(k)$  number of edges  $zx_j, x_jx_{j+1}$  labeled with  $k$ , where  $k = 0, 1$ .

Then

$$\begin{aligned} |e_h(1) - e_h(0)| &= |(e_g(1) - e_h^*(1)) - (e_g(0) - e_h^*(0))| \\ &= |e_g(1) - e_g(0) + e_h^*(1) - e_h^*(0)| \\ &= \left| \frac{q}{2} - \frac{q}{2} + (m-1) - m \right| = 1, \text{ if } (m \text{ is even and } q \text{ is even}); \\ &= \left| \frac{q+1}{2} - \frac{q-1}{2} + (m-1) - m \right| = 0, \text{ if } (m \text{ is even, } q \text{ is odd and} \\ & \quad e_g(1) = \frac{q+1}{2}); \\ &= \left| \frac{q}{2} - \frac{q}{2} + m - (m-1) \right| = 1, \text{ if } (m \text{ is odd, } q \text{ is even and } p \text{ is odd}); \\ &= \left| \frac{q}{2} - \frac{q}{2} + (m-1) - m \right| = 1, \text{ if } (m \text{ is odd, } q \text{ is even and } p \text{ is even}); \end{aligned}$$

$$\begin{aligned}
&= \left| \frac{q+1}{2} - \frac{q-1}{2} + (m-1) - m \right| = 0, \text{ if } (m \text{ is odd, } q \text{ is odd, } p \text{ is even}) \\
&\quad \text{and } e_g(1) = \frac{q+1}{2}; \\
&= \left| \frac{q-1}{2} - \frac{q+1}{2} + m - (m-1) \right| = 0, \text{ if } (m \text{ is odd, } q \text{ is odd, } p \text{ is odd}) \\
&\quad \text{and } e_g(1) = \frac{q-1}{2}.
\end{aligned}$$

In the above cases,  $|e_h(1) - e_h(0)| \leq 1$ . Hence the proof is complete.  $\square$

**Lemma 3.6.** *Let  $g$  be a SDC labeling of a graph  $G$  of order  $p$  and size  $q$ . Let  $z \in V(G)$  be a vertex such that  $g(z) = 1$ . The graph obtained by identifying a vertex  $z$  in  $G$  and a vertex of degree  $m$  in  $F_m$  admits a SDC labeling if one of the following conditions holds:*

- (1)  $m$  is even and  $q$  is even,
- (2)  $m$  is even,  $q$  is odd and  $e_g(1) = \frac{q+1}{2}$ ,
- (3)  $m$  is odd and  $q$  is even,
- (4)  $m$  is odd,  $q$  is odd,  $p$  is odd and  $e_g(1) = \frac{q+1}{2}$ ,
- (5)  $m$  is odd,  $q$  is odd,  $p$  is even and  $e_g(1) = \frac{q-1}{2}$ .

*Proof.* Let  $g$  be a SDC labeling of a graph  $G$  of order  $p$  and size  $q$ . That is the vertices of  $G$  are labeled with numbers  $\{1, 2, \dots, p\}$  and  $|e_g(1) - e_g(0)| \leq 1$ .

Let  $z \in V(G)$  be such that  $g(z) = 1$ . Let us denote by  $H$  the graph obtained by identifying a vertex  $z$  in  $G$  and a vertex of degree  $m$  in  $F_m$ .

We define a vertex labeling  $h$  of  $H$  such that

$$h(v) = g(v), \quad v \in V(G);$$

If  $m$  is even, then define  $h(x_j)$  as

$$h(x_j) = \begin{cases} p+j & \text{if } j \equiv 1, 2 \pmod{4} \text{ and } 1 \leq j \leq m \\ p+j+1 & \text{if } j \equiv 3 \pmod{4} \text{ and } 1 \leq j \leq m \\ p+j-1 & \text{if } j \equiv 0 \pmod{4} \text{ and } 1 \leq j \leq m ; \end{cases}$$

If  $m$  is odd, then define  $h(x_j)$  as

$$h(x_j) = \begin{cases} p+j & \text{if } j=1 \\ p+j+1 & \text{if } j=2 \\ p+j-1 & \text{if } j=3 \\ p+j & \text{if } j \equiv 1, 2 \pmod{4} \text{ and } 4 \leq j \leq m \\ p+j+1 & \text{if } j \equiv 3 \pmod{4} \text{ and } 4 \leq j \leq m \\ p+j-1 & \text{if } j \equiv 0 \pmod{4} \text{ and } 4 \leq j \leq m. \end{cases}$$

Using the above labeling and similar method of Lemma 3.5, one can easily verify that the graph obtained by identifying a vertex  $z$  in  $G$  and a vertex of degree  $m$  in  $F_m$  admits a SDC labeling.  $\square$

**Theorem 3.3.** *Let  $g$  be a SDC labeling of a graph  $G$  of order  $p$  and size  $q$ . Let  $z \in V(G)$  be a vertex such that  $g(z) = 2^r$ , where  $r = 0, 1, 2, \dots$ . The graph obtained by identifying a vertex  $z$  in  $G$  and a vertex of degree  $m$  in  $F_m$  admits a SDC labeling.*

*Proof.* The proof follows from Lemma 3.6 and Lemma 3.5.  $\square$

**Conjecture 3.1.** *Let  $h$  be a SDC labeling of a graph  $H_1$  of order  $p$  and size  $q$ . Let  $z \in V(H_1)$  be a vertex such that  $h(z) = 2^r$ , where  $r = 0, 1, 2, \dots$ . The graph obtained by identifying a vertex  $z$  in  $H_1$  and any one of the vertices in any graph  $H_2$  admits a SDC labeling.*

#### 4. SPECIAL FAMILIES OF GRAPHS

**Theorem 4.1.** *The middle graph  $M(P_n)$  is SDC graph.*

*Proof.* Let  $v_1, v_2, \dots, v_n$  be the vertices of the path  $P_n$ . Let  $V(M(P_n)) = \{v_i : 1 \leq i \leq n\} \cup \{v'_i : 1 \leq i \leq n-1\}$  and  $E(M(P_n)) = \{v_i v'_i, v'_i v_{i+1} : 1 \leq i \leq n-1\} \cup \{v'_i v'_{i+1} : 1 \leq i \leq n-2\}$ . Therefore,  $M(P_n)$  is of order  $2n-1$  and size  $3n-4$ . Define  $g : V(M(P_n)) \rightarrow \{1, 2, \dots, 2n-1\}$  as follows:

**Case 1.**  $n$  is odd.

$$g(v_i) = \begin{cases} 2i-1 & \text{if } i \equiv 1, 3 \pmod{4} \text{ and } 1 \leq i \leq n \\ 2i & \text{if } i \equiv 2 \pmod{4} \text{ and } 1 \leq i \leq n \\ 2i-2 & \text{if } i \equiv 0 \pmod{4} \text{ and } 1 \leq i \leq n; \end{cases}$$

$$g(v'_i) = \begin{cases} 2i & \text{if } i \equiv 1, 0 \pmod{4} \text{ and } 1 \leq i \leq n-1 \\ 2i-1 & \text{if } i \equiv 2 \pmod{4} \text{ and } 1 \leq i \leq n-1 \\ 2i+1 & \text{if } i \equiv 3 \pmod{4} \text{ and } 1 \leq i \leq n-1. \end{cases}$$

Then the induced edge labels are as follows:

$$g^*(v_i v'_i) = \begin{cases} 0 & \text{if } i \equiv 1, 2 \pmod{4} \text{ and } 1 \leq i \leq n \\ 1 & \text{if } i \equiv 3, 0 \pmod{4} \text{ and } 1 \leq i \leq n; \end{cases}$$

$$g^*(v'_i v_{i+1}) = \begin{cases} 1 & \text{if } i \equiv 1, 2 \pmod{4} \text{ and } 1 \leq i \leq n-1 \\ 0 & \text{if } i \equiv 3, 0 \pmod{4} \text{ and } 1 \leq i \leq n-1; \end{cases}$$

$$g^*(v'_i v'_{i+1}) = \begin{cases} 0 & \text{if } i \equiv 1 \pmod{2} \text{ and } 1 \leq i \leq n-2 \\ 1 & \text{if } i \equiv 0 \pmod{2} \text{ and } 1 \leq i \leq n-2. \end{cases}$$

**Case 2.**  $n$  is even.

$$g(v_i) = \begin{cases} 1 & \text{if } i = 1 \\ 2i-2 & \text{if } i \equiv 1, 2 \pmod{4} \text{ and } 2 \leq i \leq n \\ 2i & \text{if } i \equiv 3 \pmod{4} \text{ and } 2 \leq i \leq n \\ 2i-1 & \text{if } i \equiv 0 \pmod{4} \text{ and } 2 \leq i \leq n; \end{cases}$$

$$g(v'_i) = \begin{cases} 2i+1 & \text{if } i \equiv 1, 0 \pmod{4} \text{ and } 1 \leq i \leq n-1 \\ 2i & \text{if } i \equiv 2 \pmod{4} \text{ and } 1 \leq i \leq n-1 \\ 2i-1 & \text{if } i \equiv 3 \pmod{4} \text{ and } 1 \leq i \leq n-1. \end{cases}$$

Then the induced edge labels are as follows:

$$g^*(v_1 v'_1) = 1;$$

$$g^*(v_i v'_i) = \begin{cases} 0 & \text{if } i \equiv 1 \pmod{2} \text{ and } 2 \leq i \leq n \\ 1 & \text{if } i \equiv 0 \pmod{2} \text{ and } 2 \leq i \leq n; \end{cases}$$

$$g^*(v'_i v_{i+1}) = \begin{cases} 0 & \text{if } i \equiv 1, 0 \pmod{4} \text{ and } 1 \leq i \leq n-1 \\ 1 & \text{if } i \equiv 2, 3 \pmod{4} \text{ and } 1 \leq i \leq n-1; \end{cases}$$

$$g^*(v'_i v'_{i+1}) = \begin{cases} 0 & \text{if } i \equiv 1, 2 \pmod{4} \text{ and } 1 \leq i \leq n-2 \\ 1 & \text{if } i \equiv 3, 0 \pmod{4} \text{ and } 1 \leq i \leq n-2. \end{cases}$$

In the above two cases, we observe that  $e_g(0) = \lceil \frac{3n-4}{2} \rceil$ ,  $e_g(1) = \lfloor \frac{3n-4}{2} \rfloor$  and  $|e_g(0) - e_g(1)| \leq 1$ . Hence  $M(P_n)$  is SDC graph.  $\square$

**Theorem 4.2.** *The splitting graph  $S'(P_n)$  is SDC graph.*

*Proof.* Let  $v_1, v_2, \dots, v_n$  be the vertices of the path  $P_n$ . Let  $V(S'(P_n)) = \{v_i, v'_i : 1 \leq i \leq n\}$  and  $E(S'(P_n)) = \{v_i v'_{i+1}, v'_i v_{i+1}, v_i v_{i+1} : 1 \leq i \leq n-1\}$ . Therefore,  $S'(P_n)$  is of order  $2n$  and size  $3n-3$ . Define  $g : V(S'(P_n)) \rightarrow \{1, 2, \dots, 2n\}$  as follows:

$$g(v_i) = \begin{cases} 2i-1 & \text{if } i \equiv 1, 2 \pmod{4} \text{ and } 1 \leq i \leq n \\ 2i & \text{if } i \equiv 3, 0 \pmod{4} \text{ and } 1 \leq i \leq n; \end{cases}$$

$$g(v'_i) = \begin{cases} 2i & \text{if } i \equiv 1, 2 \pmod{4} \text{ and } 1 \leq i \leq n \\ 2i-1 & \text{if } i \equiv 3, 0 \pmod{4} \text{ and } 1 \leq i \leq n. \end{cases}$$

Then the induced edge labels are as follows:

$$g^*(v_i v'_{i+1}) = \begin{cases} 0 & \text{if } i \equiv 1 \pmod{2} \text{ and } 1 \leq i \leq n-1 \\ 1 & \text{if } i \equiv 0 \pmod{2} \text{ and } 1 \leq i \leq n-1; \end{cases}$$

$$g^*(v'_i v_{i+1}) = \begin{cases} 0 & \text{if } i \equiv 1 \pmod{2} \text{ and } 1 \leq i \leq n-1 \\ 1 & \text{if } i \equiv 0 \pmod{2} \text{ and } 1 \leq i \leq n-1; \end{cases}$$

$$g^*(v_i v_{i+1}) = \begin{cases} 1 & \text{if } i \equiv 1 \pmod{2} \text{ and } 1 \leq i \leq n-1 \\ 0 & \text{if } i \equiv 0 \pmod{2} \text{ and } 1 \leq i \leq n-1. \end{cases}$$

We observe that  $e_g(0) = \lceil \frac{3n-3}{2} \rceil$ ,  $e_g(1) = \lfloor \frac{3n-3}{2} \rfloor$  and  $|e_g(0) - e_g(1)| \leq 1$ . Hence  $S'(P_n)$  is SDC graph.  $\square$

**Theorem 4.3.** *The splitting graph  $S'(C_n)$  is SDC graph if  $n \equiv 0, 1, 3 \pmod{4}$ .*

*Proof.* Let  $v'_1, v'_2, \dots, v'_n$  be the added vertices corresponding to  $v_1, v_2, \dots, v_n$  of the cycle  $C_n$ . Also,  $S'(C_n)$  is of order  $2n$  and size  $3n$ . Define  $g : V(S'(C_n)) \rightarrow \{1, 2, \dots, 2n\}$  as follows:

**Case 1.** For  $n = 3$ .

$g(v_1) = 1, g(v_2) = 3, g(v_3) = 2, g(v'_1) = 4, g(v'_2) = 6, g(v'_3) = 5$ . Then the induced edge labels are as follows:  $g^*(v_2 v_3) = g^*(v_3 v_1) = g^*(v'_2 v_1) = g^*(v'_1 v_2) = 0$  and

$g^*(v_1v_2) = g^*(v'_1v_3) = g^*(v'_3v_1) = g^*(v'_3v_2) = g^*(v'_2v_3) = 1$ . We observe that  $e_g(0) = 4$  and  $e_g(1) = 5$ .

**Case 2.** For  $n > 3$ .

**Subcase 2.1.**  $n \equiv 0 \pmod{4}$

For  $1 \leq i \leq n$ .

$$g(v_i) = \begin{cases} i & \text{if } i \equiv 1, 0 \pmod{4} \\ i + 1 & \text{if } i \equiv 2 \pmod{4} \\ i - 1 & \text{if } i \equiv 3 \pmod{4}; \end{cases}$$

$$g(v'_i) = g(v_n) + i.$$

Then the induced edge labels are as follows:

for  $1 \leq i \leq n - 1$ ,

$$g^*(v_iv_{i+1}) = \begin{cases} 1 & \text{if } i \text{ is odd} \\ 0 & \text{if } i \text{ is even;} \end{cases}$$

$$g^*(v'_iv_{i+1}) = \begin{cases} 1 & \text{if } i \equiv 1, 2 \pmod{4} \\ 0 & \text{if } i \equiv 3, 0 \pmod{4}; \end{cases}$$

$$g^*(v'_nv_1) = g^*(v_nv'_1) = g^*(v_nv_1) = 0;$$

$$g^*(v_iv'_{i+1}) = \begin{cases} 0 & \text{if } i \equiv 1, 0 \pmod{4} \\ 1 & \text{if } i \equiv 2, 3 \pmod{4}. \end{cases}$$

We observe that  $e_g(0) = e_g(1) = \frac{3n}{2}$ .

**Subcase 2.2.**  $n \equiv 1, 3 \pmod{4}$ .

We assign the labels to the vertices  $v_i, v'_i$  for  $1 \leq i \leq n$  as in Subcase 2.1.

Then the induced edge labels are as follows:

for  $1 \leq i \leq n - 1$ ,

$$g^*(v_iv_{i+1}) = \begin{cases} 1 & \text{if } i \text{ is odd} \\ 0 & \text{if } i \text{ is even;} \end{cases}$$

$$g^*(v_nv_1) = 0;$$

$$g^*(v'_iv_{i+1}) = \begin{cases} 0 & \text{if } i \equiv 1, 2 \pmod{4} \\ 1 & \text{if } i \equiv 3, 0 \pmod{4}; \end{cases}$$

$$g^*(v'_nv_1) = g^*(v_nv'_1) = 1;$$



$$g^*(v_i v'_{i+1}) = \begin{cases} 1 & \text{if } i \equiv 1, 0 \pmod{4} \\ 0 & \text{if } i \equiv 2, 3 \pmod{4}. \end{cases}$$

We observe that  $e_g(0) = \lceil \frac{3n}{2} \rceil$  and  $e_g(1) = \lfloor \frac{3n}{2} \rfloor$ .

In above two cases, we get  $|e_g(0) - e_g(1)| \leq 1$ . Hence  $S'(C_n)$  is a SDC graph.  $\square$

## CONCLUSION

In this paper, we have proved that every transformed tree admits sum divisor cordial labeling. Also, we have investigated the sum divisor cordial labeling of the graph obtained by identifying the vertex of graphs. Finally, we have discussed the sum divisor cordial labeling of splitting graphs and middle graphs.

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(1,2) DEPARTMENT OF MATHEMATICS, ST. XAVIER'S COLLEGE (AUTONOMOUS), PALAYAMKOTTAI-627002, TAMIL NADU, INDIA.

*Email address:* (1) lourdusamy15@gmail.com

*Email address:* (2) patrick881990@gmail.com