Jordan Journal of Mathematics and Statistics (JJMS), 14(4), 2021, pp 691 - 706
DOI: https://doi.org/10.47013/14.4.6

# ALMOST $M$-PRECONTINUOUS FUNCTIONS IN BIMINIMAL STRUCTURE SPACES

ANJALU ALBIS BASUMATARY(1), DIGANTA JYOTI SARMA(2) AND BINOD CHANDRA TRIPATHY(3)

ABSTRACT. In this article, we define almost $M$-Precontinuous functions in biminimal structure spaces by using the concept of $M$-preopen sets. We have investigated some properties. We have proved some equivalent relations between some properties. We have studied the relationship of this type of functions with some other various existing functions together with $\delta$-open sets.

## 1. Introduction

The concept of minimal spaces has been introduced by Maki et al.[4]. Popa and Noiri [6] introduced the notion of $M$-continuous functions in minimal spaces and studied some of its properties. Min and Kim [5] explored the notion of $m$-preopen sets and $M$-Precontinuous functions in minimal spaces and obtained several characterisations. Boonpok [1] introduced the concept of biminimal structure spaces by taking two minimal structures on a non-empty set. Boonpok [2] introduced the idea of $M$-preopen sets and studied the notion of $M$-Precontinuous and weakly $M$-Precontinuous functions in biminimal structure spaces. It is found that Carpintero et al. [3] had introduced and characterized the concept of $m$-preopen sets and their related notions in

2010 *Mathematics Subject Classification.* 54A05; 54A10; 54C08.
*Key words and phrases.* Biminimal structure spaces; $M_{ij(X)}$-preopen; $M_{ij(X)}$-preclosed; $M_{ij(X)}$-$\delta$-open; $M_{ij(X)}$-regular open.

Received: June 12, 2020 Accepted: Sept. 24, 2020 .

biminimal structure spaces. Also, Phosri et al. [7] defined weakly $M$-Precontinuous functions on biminimal structure spaces in a different way. Fuzzy $m$-structures and $m$-open multifunctions in fuzzy bitopological space has been studied by Tripathy and Debnath [8].

## 2. Preliminaries

Throughout this article, $(X, M_{1(X)}, M_{2(X)})$ (respectively, $(X, M_{(X)})$) denotes a biminimal structure space (respectively, minimal space) with two minimal structures $M_{1(X)}$ and $M_{2(X)}$ (respectively, $M_{(X)}$) on a non-empty set $X$.

According to Maki et al. [4], a collection $M_{(X)}$ of a powerset $P(X)$ of a non-empty set $X$ is called a *minimal structure* (briefly $m$-structure) on $X$ if $\emptyset \in M_{(X)}$ and $X \in M_{(X)}$ and the space $(X, M_{(X)})$ is said to be a minimal space. Members of $M_{(X)}$ are called $M_{(X)}$-open sets and the complement of $M_{(X)}$-open sets are said to be $M_{(X)}$-closed sets. That is, for a subset $S$ of $X$, $S \in M_{(X)}$ means $S$ is $M_{(X)}$-open and $X \setminus S \in M_{(X)}$ means $S$ is $M_{(X)}$-closed. If $S \subset X$, then the $M_{(X)}$-closure and the $M_{(X)}$-interior of $S$ denoted by $M_{(X)}$-$Cl(S)$ and $M_{(X)}$-$Int(S)$ respectively are defined as $M_{(X)}$-$Cl(S) = \cap\{T : S \subset T, T \text{ is } M_{(X)}\text{-closed}\}$ and $M_{(X)}$-$Int(S) = \cup\{T : T \subset S, T \in M_{(X)}\}$.

The following results are due to Maki et al. [4]

**Lemma 2.1.** *Let $(X, M_{(X)})$ be a minimal space and $S \subset X$, then*
*(a) $M_{(X)}$-$Cl(X \setminus S) = X \setminus M_{(X)}$-$Cl(S)$ and $M_{(X)}$-$Int(X \setminus S) = X \setminus M_{(X)}$-$Int(S)$.*
*(b) $M_{(X)}$-$Int(S) \in M_{(X)}$ and $M_{(X)}$-$Cl(S)$ is $M_{(X)}$-closed.*
*(c) $S$ is $M_{(X)}$-closed if and only if $M_{(X)}$-$Cl(S) = S$ and $S \in M_{(X)}$ if and only if $M_{(X)}$-$Int(S) = S$.*
*(d) $S \subseteq M_{(X)}$-$Cl(S)$ and $M_{(X)}$-$Int(S) \subseteq S$.*

*(e)* $M_{(X)}$-$Cl(M_{(X)}$-$Cl(S)) = M_{(X)}$-$Cl(S)$ *and* $M_{(X)}$-$Int(M_{(X)}$- $Int(S)) = M_{(X)}$-$Int(S)$.

A point $a \in M_{(X)}$-$Cl(S)$ if and only if $T \cap S \neq \emptyset$, for every $T \in M_{(X)}$ containing $a$.
A subset $S$ of a minimal space $(X, M_{(X)})$ is $M$-preopen [5] if $S \subset M_{(X)}$-$Int(M_{(X)}$-$Cl(S))$.
A space $(X, M_{1(X)}, M_{2(X)})$ with two minimal structures $M_{1(X)}$ and $M_{2(X)}$ on a non-empty set $X$ is called a biminimal structure space [1].
According to Boonpok [2], a subset $S$ of a biminimal structure space $(X, M_{1(X)}, M_{2(X)})$ is said to be
(a) $M_{ij(X)}$-preopen if $S \subseteq M_{i(X)}$-$Int(M_{j(X)}$-$Cl(S))$ and $M_{ij(X)}$-preclosed if $X \setminus S$ is $M_{ij(X)}$-preopen.
(b) $M_{ij(X)}$-regular open if $S = M_{i(X)}$-$Int(M_{j(X)}$-$Cl(S))$.
(c) $M_{ij(X)}$-regular closed if $S = M_{i(X)}$-$Cl(M_{j(X)}$-$Int(S))$
where $i, j = 1, 2$ and $i \neq j$.

We denote the collection of all $M_{ij(X)}$-preopen, $M_{ij(X)}$-preclosed, $M_{ij(X)}$-regular open and $M_{ij(X)}$-regular closed sets of $X$ by $M_{ij(X)}$-$PO(X)$, $M_{ij(X)}$-$PC(X)$, $M_{ij(X)}$-$RO(X)$ and $M_{ij(X)}$-$RC(X)$ respectively.

A point $a$ in a biminimal structure space $(X, M_{1(X)}, M_{2(X)})$ is said to be (please refer to Carpintero et al. [3])
(a) [3] $M_{ij(X)}$-preinterior point of a subset $S$ of $X$ if there exists $T \in M_{ij(X)}$-$PO(X)$ such that $a \in T \subset S$.
(b) [3] $M_{ij(X)}$-precluster point of a subset $S$ of $X$ if $T \cap S \neq \emptyset$, for every $T \in M_{ij(X)}$-$PO(X)$ containing $a$.
The set of all $M_{ij(X)}$-preinterior points of $S$ is called $M_{ij(X)}$-preinterior of $S$ and is

denoted by $M_{ij(X)}$-$Int_p(S)$. Also, the set of all $M_{ij(X)}$-precluster points of $S$ is called $M_{ij(X)}$-preclosure of $S$ and it is denoted by $M_{ij(X)}$-$Cl_p(S)$.

The following results are due to Carpintero et al. [3]

**Lemma 2.2.** *Let $(X, M_{1(X)}, M_{2(X)})$ be a biminimal structure space and $S \subset X$. Then*
*(a) $M_{ij(X)}$-$Int_p(S) \in M_{ij(X)}$-$PO(X)$*
*(b) $M_{ij(X)}$-$Cl_p(S) \in M_{ij(X)}$-$PC(X)$*
*(c) $M_{ij(X)}$-$Int_p(S) = \cup\{T : T \subset S$ and $T \in M_{ij(X)}$-$PO(X)\}$*
*(d) $M_{ij(X)}$-$Cl_p(S) = \cap\{T : S \subset T$ and $T \in M_{ij(X)}$-$PC(X)\}$*
*(e) $M_{ij(X)}$-$Int_p(S)$ is the largest $M_{ij(X)}$-preopen set in $X$ contained in $S$.*
*(f) $M_{ij(X)}$-$Cl_p(S)$ is the smallest $M_{ij(X)}$-preclosed set in $X$ containing $S$.*
*(g) $S \in M_{ij(X)}$-$PO(X)$ if and only if $S = M_{ij(X)}$-$Int_p(S)$ and $S \in M_{ij(X)}$-$PC(X)$ if and only if $S = M_{ij(X)}$-$Cl_p(S)$.*

**Lemma 2.3.** *Let $(X, M_{1(X)}, M_{2(X)})$ be a biminimal structure space and $S \subset X$. Then*
*(a) A point $a \in M_{ij(X)}$-$Cl_p(S)$ if and only if $T \cap S \neq \emptyset$, for every $T \in M_{ij(X)}$-$PO(X)$ containing $a$.*
*(b) $X \setminus M_{ij(X)}$-$Cl_p(S) = M_{ij(X)}$-$Int_p(X \setminus S)$.*
*(c) $X \setminus M_{ij(X)}$-$Int_p(S) = M_{ij(X)}$-$Cl_p(X \setminus S)$.*

## 3. Almost $M_{ij}$-precontinuous function

**Definition 3.1.** A function $g : (X, M_{1(X)}, M_{2(X)}) \to (Y, M_{1(Y)}, M_{2(Y)})$ is said to be almost $M_{ij}$-precontinuous at a point $a \in X$ if for every $S \in M_{i(Y)}$ containing $g(a)$, there exists $T \in M_{ij(X)}$-$PO(X)$ containing $a$ such that $g(T) \subseteq M_{i(Y)}$-$Int(M_{j(Y)}$-$Cl(S))$.
If $g$ is almost $M_{ij}$-precontinuous at every point $a \in X$, then it is called almost $M_{ij}$-precontinuous.

**Theorem 3.1.** *Let* $g : (X, M_{1(X)}, M_{2(X)}) \to (Y, M_{1(Y)}, M_{2(Y)})$ *be a function. Then the following statements are equivalent:*

*(a)* $g$ *is almost* $M_{ij}$*-precontinuous.*

*(b)* $a \in M_{ij(X)}\text{-}Int_p(g^{-1}(M_{i(Y)}\text{-}Int(M_{j(Y)}\text{-}Cl(S))))$*, for every* $S \in M_{i(Y)}$ *containing* $g(a)$*, where* $a \in X$*.*

*(c)* $a \in M_{ij(X)}\text{-}Int_p(g^{-1}(S))$*, for every* $S \in M_{ij(Y)}\text{-}RO(Y)$ *containing* $g(a)$*, where* $a \in X$*.*

*(d) For every* $S \in M_{ij(Y)}\text{-}RO(Y)$ *containing* $g(a)$*, there exist* $T \in M_{ij(X)}\text{-}PO(X)$ *containing* $a$ *such that* $g(T) \subseteq S$*.*

*Proof.* (a) $\Rightarrow$ (b) Let $S \in M_{i(Y)}$ such that $g(a) \in S$. Since $g$ is almost $M_{ij}$-precontinuous, so there exists $T \in M_{ij(X)}\text{-}PO(X)$ containing $a$ such that $g(T) \subseteq M_{i(Y)}\text{-}Int(M_{j(Y)}\text{-}Cl(S))$. This implies $a \in T \subseteq g^{-1}(M_{i(Y)}\text{-}Int(M_{j(Y)}\text{-}Cl(S)))$. Since $T \in M_{ij(X)}\text{-}PO(X)$, so $a \in T = M_{ij(X)}\text{-}Int_p(T) \subseteq M_{ij(X)}\text{-}Int_p(g^{-1}(M_{i(Y)}\text{-}Int(M_{j(Y)}\text{-}Cl(S))))$. Hence, $a \in M_{ij(X)}\text{-}Int_p(g^{-1}(M_{i(Y)}\text{-}Int(M_{j(Y)}\text{-}Cl(S))))$.

(b) $\Rightarrow$ (c) Let $S \in M_{ij(Y)}\text{-}RO(Y)$ containing $g(a)$. So $S = M_{i(Y)}\text{-}Int(M_{j(Y)}\text{-}Cl(S))$. By (b), we have $a \in M_{ij(X)}\text{-}Int_p(g^{-1}(M_{i(Y)}\text{-}Int(M_{j(Y)}\text{-}Cl(S))))$ which implies that $a \in M_{ij(X)}\text{-}Int_p(g^{-1}(S))$.

(c) $\Rightarrow$ (d) Let $S \in M_{ij(Y)}\text{-}RO(Y)$ containing $g(a)$. By (c), $a \in M_{ij(X)}\text{-}Int_p(g^{-1}(S)) \subseteq g^{-1}(S)$. Since, $M_{ij(X)}\text{-}Int_p(g^{-1}(S)) \in M_{ij(X)}\text{-}PO(X)$ containing $a$, so if we take $T = M_{ij(X)}\text{-}Int_p(g^{-1}(S))$, then $T \in M_{ij(X)}\text{-}PO(X)$ such that $a \in T \subset g^{-1}(S)$. Hence $g(T) \subset S$.

(d) $\Rightarrow$ (a) Let $S \in M_{i(Y)}$ such that $g(a) \in S$. Then $g(a) \in S = M_{i(Y)}\text{-}Int(S) \subset M_{i(Y)}\text{-}Int(M_{j(Y)}\text{-}Cl(S))$. Since, $M_{i(Y)}\text{-}Int(M_{j(Y)}\text{-}Cl(S)) \in M_{ij(Y)}\text{-}RO(Y)$, so by (d), there exists $T \in M_{ij(X)}$-$PO(X)$ containing $a$ such that $g(T) \subset M_{i(Y)}$-$Int(M_{j(Y)}$-$Cl(S))$. Thus $g$ is almost $M_{ij}$-precontinuous function. □

**Theorem 3.2.** *Let $g : (X, M_{1(X)}, M_{2(X)}) \to (Y, M_{1(Y)}, M_{2(Y)})$ be a function. Then the following statements are equivalent:*

*(a) $g$ is almost $M_{ij}$-precontinuous.*

*(b) $g^{-1}(M_{i(Y)}$-$Int(M_{j(Y)}$-$Cl(S))) \in M_{ij(X)}$-$PO(X)$, for every $S \in M_{i(Y)}$.*

*(c) $g^{-1}(M_{i(Y)}$-$Cl(M_{j(Y)}$-$Int(T))) \in M_{ij(X)}$-$PC(X)$, for every $M_{i(Y)}$-closed set $T$ of $Y$.*

*(d) $g^{-1}(T) \in M_{ij(X)}$-$PC(X)$, for every $T \in M_{ij(Y)}$-$RC(Y)$.*

*(e) $g^{-1}(S) \in M_{ij(X)}$-$PO(X)$, for every $S \in M_{ij(Y)}$-$RO(Y)$.*

*Proof.* (a) ⇒ (b) Let $S \in M_{i(Y)}$ and let $a \in g^{-1}(M_{i(Y)}$-$Int(M_{j(Y)}$-$Cl(S)))$. This implies $g(a) \in M_{i(Y)}$-$Int(M_{j(Y)}$-$Cl(S))$. By (a), $g$ is almost $M_{ij}$-precontinuous and since $M_{i(Y)}$-$Int(M_{j(Y)}$-$Cl(S)) \in M_{ij(Y)}$-$RO(Y)$, so by Theorem 3.1, there exists $T \in M_{ij(X)}$-$PO(X)$ containing $a$ such that $g(T) \subset M_{i(Y)}$-$Int(M_{j(Y)}$-$Cl(S))$. This implies $a \in T \subset g^{-1}(M_{i(Y)}$-$Int(M_{j(Y)}$-$Cl(S)))$. Since $T \in M_{ij(X)}$-$PO(X)$, so $a \in T = M_{ij(X)}$-$Int_p(T) \subset M_{ij(X)}$-$Int_p(g^{-1}(M_{i(Y)}$-$Int(M_{j(Y)}$-$Cl(S)))) \subset g^{-1}(M_{i(Y)}$-$Int(M_{j(Y)}$-$Cl(S)))$. Hence, $g^{-1}(M_{i(Y)}$-$Int(M_{j(Y)}$-$Cl(S))) \in M_{ij(X)}$-$PO(X)$.

(b) ⇒ (c) Let $T$ be $M_{i(Y)}$-closed set in $Y$. Then $Y \setminus T \in M_{i(Y)}$. So by (b), we have $g^{-1}(M_{i(Y)}$-$Int(M_{j(Y)}$-$Cl(Y \setminus T))) \in M_{ij(X)}$-$PO(X)$. Now, $g^{-1}(M_{i(Y)}$-$Int(M_{j(Y)}$-$Cl(Y \setminus T))) = g^{-1}(M_{i(Y)}$-$Int(Y \setminus M_{j(Y)}$-$Int(T))) = g^{-1}(Y \setminus M_{i(Y)}$-$Cl(M_{j(Y)}$-$Int(T))) = g^{-1}(Y) \setminus g^{-1}(M_{i(Y)}$-$Cl(M_{j(Y)}$-$Int(T))) = X \setminus g^{-1}(M_{i(Y)}$-$Cl(M_{j(Y)}$-$Int(T)))$. Therefore, $g^{-1}(M_{i(Y)}$-$Int(M_{j(Y)}$-$Cl(Y \setminus T))) = X \setminus g^{-1}(M_{i(Y)}$-$Cl(M_{j(Y)}$-$Int(T))) \in M_{ij(X)}$-$PO(X)$. Hence, $g^{-1}(M_{i(Y)}$-$Cl(M_{j(Y)}$-$Int(T))) \in M_{ij(X)}$-$PC(X)$.

(c) ⇒ (d) Let $T \in M_{ij(Y)}$-$RC(Y)$. Then $T = M_{i(Y)}$-$Cl(M_{j(Y)}$-$Int(T))$. Also, $M_{i(Y)}$-$Cl(T) = M_{i(Y)}$-$Cl(M_{i(Y)}$-$Cl(M_{j(Y)}$-$Int(T))) = M_{i(Y)}$-$Cl(M_{j(Y)}$-$Int(T)) = T$. So, $T$ is $M_{i(Y)}$-closed in $Y$. By (c), $g^{-1}(M_{i(Y)}$-$Cl(M_{j(Y)}$-$Int(T))) \in M_{ij(X)}$-$PC(X)$. This implies that $g^{-1}(T) \in M_{ij(X)}$-$PC(X)$.

(d) $\Rightarrow$ (e) Let $S \in M_{ij(Y)}$-$RO(Y)$. Then $Y \setminus S \in M_{ij(Y)}$-$RC(Y)$. By (d), $g^{-1}(Y \setminus S) = X \setminus g^{-1}(S) \in M_{ij(X)}$-$PC(X)$. Hence, $g^{-1}(S) \in M_{ij(X)}$-$PO(X)$.

(e) $\Rightarrow$ (a) Let $a \in X$ and $S \in M_{ij(Y)}$-$RO(Y)$ containing $g(a)$. Then $a \in g^{-1}(S)$. By (e), we have $g^{-1}(S) \in M_{ij(X)}$-$PO(X)$. Now, $g(g^{-1}(S)) \subset S$. Thus by Theorem 3.1, $g$ is almost $M_{ij}$-precontinuous function. □

**Theorem 3.3.** *Let $g : (X, M_{1(X)}, M_{2(X)}) \to (Y, M_{1(Y)}, M_{2(Y)})$ be a function. Then the following statements are equivalent:*
*(a) $g$ is almost $M_{ij}$-precontinuous.*
*(b) $M_{ij(X)}$-$Cl_p(g^{-1}(M_{i(Y)}$-$Cl(M_{j(Y)}$-$Int(M_{i(Y)}$-$Cl(S))))) \subseteq g^{-1}(M_{i(Y)}$-$Cl(S))$, for every $S \subset Y$.*
*(c) $M_{ij(X)}$-$Cl_p(g^{-1}(M_{i(Y)}$-$Cl(M_{j(Y)}$-$Int(T)))) \subseteq g^{-1}(T)$, for every $T \in M_{ij(Y)}$-$RC(Y)$.*
*(d) $M_{ij(X)}$-$Cl_p(g^{-1}(M_{i(Y)}$-$Cl(F))) \subseteq g^{-1}(M_{i(Y)}$-$Cl(F))$, for every $F \in M_{j(Y)}$.*
*(e) $g^{-1}(F) \subseteq M_{ij(X)}$-$Int_p(g^{-1}(M_{i(Y)}$-$Int(M_{j(Y)}$-$Cl(F))))$, for every $F \in M_{i(Y)}$.*

*Proof.* (a) $\Rightarrow$ (b) Let $a \in X$ and $S \subset Y$. Suppose that, $a \in X \setminus g^{-1}(M_{i(Y)}$-$Cl(S))$. Therefore $g(a) \in Y \setminus M_{i(Y)}$-$Cl(S)$ and so by Lemma 2.1, there exists $F \in M_{i(Y)}$ containing $g(a)$ such that $F \cap S = \emptyset$, which implies $F \cap M_{i(Y)}$-$Cl(M_{j(Y)}$-$Int(M_{i(Y)}$-$Cl(S))) = \emptyset$. Thus, $M_{i(Y)}$-$Int(M_{j(Y)}$-$Cl(F)) \cap M_{i(Y)}$-$Cl(M_{j(Y)}$-$Int(M_{i(Y)}$-$Cl(S))) = \emptyset$. Since $g$ is almost $M_{ij}$-precontinuous, so there exists $W \in M_{ij(X)}$-$PO(X)$ containing $a$ such that $g(W) \subset M_{i(Y)}$-$Int(M_{j(Y)}$-$Cl(F))$. This implies $g(W) \cap M_{i(Y)}$-$Cl(M_{j(Y)}$-$Int(M_{i(Y)}$-$Cl(S))) = \emptyset$. That is, $W \cap g^{-1}(M_{i(Y)}$-$Cl(M_{j(Y)}$-$Int(M_{i(Y)}$-$Cl(S)))) = \emptyset$. Consequently, by Lemma 2.3, we have $a \in X \setminus M_{ij(X)}$-$Cl_p(g^{-1}(M_{i(Y)}$-$Cl(M_{j(Y)}$-$Int(M_{i(Y)}$-$Cl(S)))))$. Hence, $M_{ij(X)}$-$Cl_p(g^{-1}(M_{i(Y)}$-$Cl(M_{j(Y)}$-$Int(M_{i(Y)}$-$Cl(S))))) \subseteq g^{-1}(M_{i(Y)}$-$Cl(S))$.

(b) $\Rightarrow$ (c) Let $T \in M_{ij(Y)}$-$RC(Y)$. So $T = M_{i(Y)}$-$Cl(M_{j(Y)}$-$Int(T))$. Now, $M_{ij(X)}$-$Cl_p(g^{-1}(M_{i(Y)}$-$Cl(M_{j(Y)}$-$Int(T)))) = M_{ij(X)}$-$Cl_p(g^{-1}(M_{i(Y)}$-$Cl(M_{j(Y)}$-$Int(M_{i(Y)}$-$Cl(M_{j(Y)}$ -$Int(T)))))) \subseteq g^{-1}(M_{i(Y)}$-$Cl(M_{j(Y)}$-$Int(T))) = g^{-1}(T)$, by (b). Hence, $M_{ij(X)}$-$Cl_p(g^{-1}(M_{i(Y)}$-$Cl(M_{j(Y)}$-$Int(T)))) \subseteq g^{-1}(T)$.

(c) $\Rightarrow$ (d) Let $F \in M_{j(Y)}$. Then $M_{i(Y)}$-$Cl(F) \in M_{ij(Y)}$-$RC(Y)$. Therefore, by (c), we have $M_{ij(X)}$-$Cl_p(g^{-1}(M_{i(Y)}$-$Cl(F))) \subseteq M_{ij(X)}$-$Cl_p(g^{-1}(M_{i(Y)}$-$Cl(M_{j(Y)}$-$Int(M_{i(Y)}$-$Cl(F))))) \subseteq g^{-1}(M_{i(Y)}$-$Cl(F))$. Hence, $M_{ij(X)}$-$Cl_p(g^{-1}(M_{i(Y)}$-$Cl(F))) \subseteq g^{-1}(M_{i(Y)}$-$Cl(F))$.

(d) $\Rightarrow$ (e) Let $F \in M_{i(Y)}$. So $Y \setminus M_{j(Y)}$-$Cl(F) \in M_{j(Y)}$. Now by (d), we have $M_{ij(X)}$-$Cl_p(g^{-1}(M_{i(Y)}$-$Cl(Y \setminus M_{j(Y)}$-$Cl(F)))) \subseteq g^{-1}(M_{i(Y)}$-$Cl(Y \setminus M_{j(Y)}$-$Cl(F))) \Rightarrow$ $M_{ij(X)}$-$Cl_p(g^{-1}(Y \setminus M_{i(Y)}$-$Int(M_{j(Y)}$-$Cl(F)))) \subseteq g^{-1}(Y \setminus M_{i(Y)}$-$Int(M_{j(Y)}$-$Cl(F))) \Rightarrow$ $M_{ij(X)}$-$Cl_p(X \setminus g^{-1}(M_{i(Y)}$-$Int(M_{j(Y)}$-$Cl(F)))) \subseteq X \setminus g^{-1}(M_{i(Y)}$-$Int(M_{j(Y)}$-$Cl(F))) \Rightarrow$ $X \setminus M_{ij(X)}$-$Int_p(g^{-1}(M_{i(Y)}$-$Int(M_{j(Y)}$-$Cl(F)))) \subseteq X \setminus g^{-1}(M_{i(Y)}$-$Int(M_{j(Y)}$-$Cl(F))) \subseteq X \setminus g^{-1}(F)$. Hence $g^{-1}(F) \subseteq M_{ij(X)}$-$Int_p(g^{-1}(M_{i(Y)}$-$Int(M_{j(Y)}$-$Cl(F))))$.

(e) $\Rightarrow$ (a) Let $a \in X$ and $F \in M_{i(Y)}$ containing $g(a)$. Then $a \in g^{-1}(F) \subseteq M_{ij(X)}$-$Int_p(g^{-1}(M_{i(Y)}$-$Int(M_{j(Y)}$-$Cl(F))))$. Putting $W = M_{ij(X)}$-$Int_p(g^{-1}(M_{i(Y)}$-$Int(M_{j(Y)}$-$Cl(F))))$, then $W \in M_{ij(X)}$-$PO(X)$ containing $a$ and $W \subseteq g^{-1}(M_{i(Y)}$-$Int(M_{j(Y)}$-$Cl(F)))$. Thus $g(W) \subseteq M_{i(Y)}$-$Int(M_{j(Y)}$-$Cl(F))$ and hence $g$ is almost $M_{ij}$-precontinuous function. □

**Theorem 3.4.** *Let $g : (X, M_{1(X)}, M_{2(X)}) \to (Y, M_{1(Y)}, M_{2(Y)})$ be almost $M_{ij}$-precontinuous function and let $S \in M_{i(Y)} \cap M_{j(Y)}$. If for every $a \in X$, $a \in M_{ij(X)}$-$Cl_p(g^{-1}(S)) \setminus g^{-1}(S)$, then $g(a) \in M_{ij(Y)}$-$Cl_p(S)$.*

*Proof.* Let $a \in X$ and $a \in M_{ij(X)}$-$Cl_p(g^{-1}(S)) \setminus g^{-1}(S)$. Assume that, $g(a) \notin M_{ij(Y)}$-$Cl_p(S)$. Then by Lemma 2.3, there exists $T \in M_{ij(Y)}$-$PO(Y)$ containing $g(a)$ such that $T \cap S = \emptyset$. So, $M_{i(Y)}$-$Int(M_{j(Y)}$-$Cl(T)) \cap S = \emptyset$. Also, $M_{i(Y)}$-$Int(M_{j(Y)}$-$Cl(T)) \in M_{ij(Y)}$-$RO(Y)$ and since $g$ is almost $M_{ij}$-precontinuous, so by Theorem 3.1, there exists $W \in M_{ij(X)}$-$PO(X)$ containing $a$ such that $g(W) \subset M_{i(Y)}$-$Int(M_{j(Y)}$-$Cl(T))$ and so $g(W) \cap S = \emptyset$. Since $a \in M_{ij(X)}$-$Cl_p(g^{-1}(S))$ we have by Lemma 2.3, $W \cap g^{-1}(S) \neq \emptyset$, that is $g(W) \cap S \neq \emptyset$, which is a contradiction. Hence, $g(a) \in M_{ij(Y)}$-$Cl_p(S)$. □

**Definition 3.2.** A point $a$ in a biminimal structure space $(X, M_{1(X)}, M_{2(X)})$ is said to be $M_{ij(X)}$-$\delta$-cluster point of $P \subset X$ if $P \cap Q \neq \emptyset$, for every $Q \in M_{ij(X)}$-$RO(X)$ containing $a$. The set of all $M_{ij(X)}$-$\delta$-cluster points of $P$ is called $M_{ij(X)}$-$\delta$-closure of $P$ and may be denoted by $M_{ij(X)}$-$Cl_\delta(P)$. The subset $P$ of $X$ is called $M_{ij(X)}$-$\delta$-closed if the set of all $M_{ij(X)}$-$\delta$-cluster points of $P$ is a subset of $P$. Also, $P$ is $M_{ij(X)}$-$\delta$-open if $X \setminus P$ is $M_{ij(X)}$-$\delta$-closed. So, any subset of $(X, M_{1(X)}, M_{2(X)})$ is $M_{ij(X)}$-$\delta$-open if it can be expressed as the union of $M_{ij(X)}$-regular open sets of $X$.
We denote the set of all $M_{ij(X)}$-$\delta$-closed and $M_{ij(X)}$-$\delta$-open sets of $(X, M_{1(X)}, M_{2(X)})$ by $M_{ij(X)}$-$\delta C(X)$ and $M_{ij(X)}$-$\delta O(X)$ respectively.

**Theorem 3.5.** *Let $g : (X, M_{1(X)}, M_{2(X)}) \to (Y, M_{1(Y)}, M_{2(Y)})$ be a function. Then the following statements are equivalent:*
*(a) $g$ is almost $M_{ij}$-precontinuous.*
*(b) $g(M_{ij(X)}$-$Cl_p(S)) \subseteq M_{ij(Y)}$-$Cl_\delta(g(S))$, for every $S \subset X$.*
*(c) $M_{ij(X)}$-$Cl_p(g^{-1}(T)) \subseteq g^{-1}(M_{ij(Y)}$-$Cl_\delta(T))$, for every $T \subset Y$.*
*(d) $g^{-1}(S) \in M_{ij(X)}$-$PC(X)$, for every $S \in M_{ij(Y)}$-$\delta C(Y)$.*
*(e) $g^{-1}(T) \in M_{ij(X)}$-$PO(X)$, for every $T \in M_{ij(Y)}$-$\delta O(Y)$.*

*Proof.* (a) ⇒ (b) Let $a \in S$ and $S \subset X$. Also, let $F \in M_{i(Y)}$ containing $g(a)$. By (a), $g$ is almost $M_{ij}$-precontinuous, so there exists $W \in M_{ij(X)}$-$PO(X)$ containing $a$ such that $g(W) \subseteq M_{i(Y)}$-$Int(M_{j(Y)}$-$Cl(F))$. Let $a \in M_{ij(X)}$-$Cl_p(S)$, then by Lemma 2.3, $W \cap S \neq \emptyset$ and so $\emptyset \neq g(W) \cap g(S) \subseteq M_{i(Y)}$-$Int(M_{j(Y)}$-$Cl(F)) \cap g(S)$. Since we have $F \in M_{i(Y)}$, so $F \subseteq M_{i(Y)}$-$Int(M_{j(Y)}$-$Cl(F))$ and $M_{i(Y)}$-$Int(M_{j(Y)}$-$Cl(F)) \in M_{ij(Y)}$-$RO(Y)$. Hence $g(a) \in M_{ij(Y)}$-$Cl_\delta(g(S))$. Consequently, $a \in g^{-1}(M_{ij(Y)}$-$Cl_\delta(g(S)))$.

Thus $M_{ij(X)}$-$Cl_p(S) \subseteq g^{-1}(M_{ij(Y)}$-$Cl_\delta(g(S)))$. That is, $g(M_{ij(X)}$-$Cl_p(S)) \subseteq M_{ij(Y)}$-$Cl_\delta(g(S))$.
(b) $\Rightarrow$ (c) Let $T \subset Y$. Then $g^{-1}(T) \subset X$. By (b), we have $g(M_{ij(X)}$-$Cl_p(g^{-1}(T))) \subseteq M_{ij(Y)}$-$Cl_\delta(g(g^{-1}(T))) \subseteq M_{ij(Y)}$-$Cl_\delta(T) \Rightarrow M_{ij(X)}$-$Cl_p(g^{-1}(T)) \subseteq g^{-1}(M_{ij(Y)}$-$Cl_\delta(T))$.
(c) $\Rightarrow$ (d) Let $S \in M_{ij(Y)}$-$\delta C(Y)$. So by (c), $M_{ij(X)}$-$Cl_p(g^{-1}(S)) \subseteq g^{-1}(M_{ij(Y)}$-$Cl_\delta(S)) = g^{-1}(S)$. Also, $g^{-1}(S) \subseteq M_{ij(X)}$-$Cl_p(g^{-1}(S))$. Thus $g^{-1}(S) = M_{ij(X)}$-$Cl_p(g^{-1}(S))$ and hence $g^{-1}(S) \in M_{ij(X)}$-$PC(X)$.
(d) $\Rightarrow$ (e) Let $T \in M_{ij(Y)}$-$\delta O(Y)$. Then $Y \setminus T \in M_{ij(Y)}$-$\delta C(Y)$. By (d), we have $g^{-1}(Y \setminus T) = X \setminus g^{-1}(T) \in M_{ij(X)}$-$PC(X)$. Hence, $g^{-1}(T) \in M_{ij(X)}$-$PO(X)$.
(e) $\Rightarrow$ (a) Let $a \in X$ and $S \in M_{i(Y)}$ such that $g(a) \in S$. Then $M_{i(Y)}$-$Int(M_{j(Y)}$-$Cl(S)) \in M_{ij(Y)}$-$RO(Y)$ containing $g(a)$. Since, $M_{i(Y)}$-$Int(M_{j(Y)}$-$Cl(S)) \in M_{ij(Y)}$-$\delta O(Y)$, then by (e), we have $g^{-1}(M_{i(Y)}$-$Int(M_{j(Y)}$-$Cl(S))) \in M_{ij(X)}$-$PO(X)$. Since, $S \subseteq M_{i(Y)}$-$Int(M_{j(Y)}$-$Cl(S))$, so $g^{-1}(S) \subseteq g^{-1}(M_{i(Y)}$-$Int(M_{j(Y)}$-$Cl(S))) = M_{ij(X)}$-$Int_p(g^{-1}(M_{i(Y)}$-$Int(M_{j(Y)}$-$Cl(S))))$. Thus $g^{-1}(S) \subseteq M_{ij(X)}$-$Int_p(g^{-1}(M_{i(Y)}$-$Int(M_{j(Y)}$-$Cl(S))))$. Hence by Theorem 3.3, we have $g$ is almost $M_{ij}$-precontinuous. □

**Theorem 3.6.** *Let $g : (X, M_{1(X)}, M_{2(X)}) \to (Y, M_{1(Y)}, M_{2(Y)})$ be a function. Then the following statements are equivalent:*
*(a) $g$ is almost $M_{ij}$-precontinuous.*
*(b) For every $a \in X$ and every $S \in M_{ij(Y)}$-$\delta O(Y)$ containing $g(a)$, there exists $T \in M_{ij(X)}$-$PO(X)$ containing $a$ such that $g(T) \subset S$.*

*Proof.* (a) $\Rightarrow$ (b) Let $a \in X$ and $S \in M_{ij(Y)}$-$\delta O(Y)$ be such that $g(a) \in S$. Then there exists $W \in M_{i(Y)}$ containing $g(a)$ such that $W = M_{i(Y)}$-$Int(W) \subset M_{i(Y)}$-$Int(M_{j(Y)}$-$Cl(W)) \subset S$. Since, $M_{i(Y)}$-$Int(M_{j(Y)}$-$Cl(W)) \in M_{ij(Y)}$-$RO(Y)$ containing $g(a)$, then by Theorem 3.1, there exists $T \in M_{ij(X)}$-$PO(X)$ containing $a$ such that $g(T) \subset M_{i(Y)}$-$Int(M_{j(Y)}$-$Cl(W)) \subset S$.

(b) $\Rightarrow$ (a) Let $a \in X$ and every $S \in M_{i(Y)}$ containing $g(a)$. Then $M_{i(Y)}$-$Int(M_{j(Y)}$-$Cl(S)) \in M_{ij(Y)}$-$\delta O(Y)$ containing $g(a)$. By (b), there exists $T \in M_{ij(X)}$-$PO(X)$ containing $a$ such that $g(T) \subset M_{i(Y)}$-$Int(M_{j(Y)}$-$Cl(S))$. Hence $g$ is almost $M_{ij}$-precontinuous. □

**Definition 3.3.** A function $g : (X, M_{1(X)}, M_{2(X)}) \to (Y, M_{1(Y)}, M_{2(Y)})$ is $M_{ij}$-precontinuous if $g^{-1}(S) \in M_{ij(X)}$-$PO(X)$ for every $S \in M_{i(Y)}$ where $i, j = 1, 2$ and $i \neq j$.

**Remark 1.** *$M_{ij}$-precontinuity $\Rightarrow$ almost $M_{ij}$-precontinuity. However, the converse may not be true in general as shown in the following example.*

**Example 3.1.** *Let $X = \{p, q, r\} = Y$, $M_{1(X)} = \{\emptyset, \{p\}, \{q, r\}, X\}$, $M_{2(X)} = \{\emptyset, \{q\}, \{p, r\}, X\}$, $M_{1(Y)} = \{\emptyset, \{p\}, \{q\}, \{p, q\}, \{q, r\}, Y\}$, $M_{2(Y)} = \{\emptyset, \{p\}, Y\}$. Let $g : (X, M_{1(X)}, M_{2(X)}) \to (Y, M_{1(Y)}, M_{2(Y)})$ be the identity function. Then $g$ is almost $M_{12}$-precontinuous function but it is not $M_{12}$-precontinuous since $\{q\} \in M_{1(Y)}$ but $g^{-1}(\{q\}) \notin M_{12(X)}$-$PO(X)$.*

**Theorem 3.7.** *Let $g : (X, M_{1(X)}, M_{2(X)}) \to (Y, M_{1(Y)}, M_{2(Y)})$ be almost $M_{ij}$-precontinuous function satisfying $M_{ij(X)}$-$Int_p(g^{-1}(M_{j(Y)}$-$Cl(F))) \subset g^{-1}(F)$ for every $F \in M_{i(Y)}$, then $g$ is $M_{ij}$-precontinuous.*

*Proof.* Let $F \in M_{i(Y)}$ and $g$ be almost $M_{ij}$-precontinuous. Then by Theorem 3.3, we have $g^{-1}(F) \subset M_{ij(X)}$-$Int_p(g^{-1}(M_{i(Y)}$-$Int(M_{j(Y)}$-$Cl(F)))) \subset M_{ij(X)}$-$Int_p(g^{-1}(M_{j(Y)}$-$Cl(F)))$. Further by the given condition, $M_{ij(X)}$-$Int_p(g^{-1}(M_{j(Y)}$-$Cl(F))) \subset g^{-1}(F)$. So, $g^{-1}(F) = M_{ij(X)}$-$Int_p(g^{-1}(M_{j(Y)}$-$Cl(F)))$ and consequently by Lemma 2.2, $g^{-1}(F) \in M_{ij(X)}$-$PO(X)$. Hence $g$ is $M_{ij}$-precontinuous. □

**Definition 3.4.** A function $g : (X, M_{1(X)}, M_{2(X)}) \to (Y, M_{1(Y)}, M_{2(Y)})$ is said to be weakly $M_{ij}$-precontinuous at $a \in X$ if for every $S \in M_{i(Y)}$ containing $g(a)$, there exists $T \in M_{ij(X)}$-$PO(X)$ containing $a$ such that $g(T) \subset M_{j(Y)}$-$Cl(S)$. If $g$ is weakly $M_{ij}$-precontinuous at every point $a \in X$, then it is called weakly $M_{ij}$-precontinuous.

**Remark 2.** *Almost $M_{ij}$-precontinuity $\Rightarrow$ weakly $M_{ij}$-precontinuity. But the converse need not be true in general follows from the example given below.*

**Example 3.2.** *Let $X = \{p, q, r\} = Y$, $M_{1(X)} = \{\emptyset, \{p\}, \{q, r\}, X\}$, $M_{2(X)} = \{\emptyset, \{p, r\}, \{q, r\}, X\}$, $M_{1(Y)} = \{\emptyset, \{p\}, \{q\}, \{p, q\}, Y\}$, $M_{2(Y)} = \{\emptyset, \{p\}, Y\}$. Let $g : (X, M_{1(X)}, M_{2(X)}) \to (Y, M_{1(Y)}, M_{2(Y)})$ be the identity function. Then $g$ is weakly $M_{12}$-precontinuous function but it is not almost $M_{12}$-precontinuous, since for $\{q\} \in M_{1(Y)}$ containing $g(q)$ there does not exist $T \in M_{12(X)}$-$PO(X)$ containing $q$ such that $g(T) \subseteq M_{1(Y)}$-$Int(M_{2(Y)}$-$Cl(\{q\}))$.*

**Theorem 3.8.** *If $g : (X, M_{1(X)}, M_{2(X)}) \to (Y, M_{1(Y)}, M_{2(Y)})$ is weakly $M_{ij}$-precontinuous function which satisfies $g(T) \subset M_{i(Y)}$-$Int(M_{j(Y)}$-$Cl(g(T)))$ for every $T \in M_{ij(X)}$-$PO(X)$, then $g$ is almost $M_{ij}$-precontinuous.*

*Proof.* Let $a \in X$ and $S \in M_{i(Y)}$ containing $g(a)$. Since $g$ is weakly $M_{ij}$-precontinuous, so there exists $T \in M_{ij(X)}$-$PO(X)$ containing $a$ such that $g(T) \subset M_{j(Y)}$-$Cl(S)$. Also, $g(T) \subset M_{i(Y)}$-$Int(M_{j(Y)}$-$Cl(g(T))) \subset M_{i(Y)}$-$Int(M_{j(Y)}$-$Cl(M_{j(Y)}$-$Cl(S))) \subset M_{i(Y)}$-$Int(M_{j(Y)}$-$Cl(S))$. Hence, $g$ is almost $M_{ij}$-precontinuous. □

**Theorem 3.9.** *Let $g : (X, M_{1(X)}, M_{2(X)}) \to (Y, M_{1(Y)}, M_{2(Y)})$ be a function and $h : X \to X \times Y$ be a function defined by $h(a) = (a, g(a))$ for every $a \in X$. Then $g$ is almost $M_{ij}$-precontinuous if $h$ is almost $M_{ij}$-precontinuous.*

*Proof.* Let $h$ be almost $M_{ij}$-precontinuous and let $S \in M_{ij(Y)}$-$RO(Y)$ containing $g(a)$, where $a \in X$. Then $h(a) = (a, g(a)) \in X \times S$ and $X \times S \in M_{ij(X\times Y)}$-$RO(X \times Y)$. Since $h$ is almost $M_{ij}$-precontinuous, so there exists $T \in M_{ij(X)}$-$PO(X)$ containing $a$ such that $h(T) \subseteq X \times Y$. Then we get $g(T) \subseteq S$. Now, by Theorem 3.1, we have $g$ is almost $M_{ij}$-precontinuous. □

**Theorem 3.10.** *Let* $h : (X, M_{1(X)}, M_{2(X)}) \to (Y, M_{1(Y)}, M_{2(Y)})$ *be almost* $M_{ij}$*-precontinuous and* $R \in M_{ij(X\times Y)}$-$\delta C(X \times Y)$*. If* $P_{X\times Y\to X}$ *is the projection of* $X \times Y$ *onto* $X$ *and* $G_h$ *is the graph of* $h$*, then* $P_{X\times Y\to X}(R \cap G_h) \in M_{ij(X)}$-$PC(X)$*.*

*Proof.* Let $a \in M_{ij(X)}$-$Cl_p(P_{X\times Y\to X}(R \cap G_h))$ and $R \in M_{ij(X\times Y)}$-$\delta C(X \times Y)$. Also let $S \in M_{i(X)}$ such that $a \in S$ and $T \in M_{i(Y)}$ such that $h(a) \in T$. Since, $h$ is almost $M_{ij}$-precontinuous, so by Theorem 3.3, we have $a \in h^{-1}(T) \subseteq M_{ij(X)}$-$Int_p(h^{-1}(M_{i(Y)}$-$Int(M_{j(Y)}$-$Cl(T))))$ and $S \cap M_{ij(X)}$-$Int_p(h^{-1}(M_{i(Y)}$-$Int(M_{j(Y)}$-$Cl(T)))) \in M_{ij(X)}$-$PO(X)$ which contains $a$. Also, since $a \in M_{ij(X)}$-$Cl_p(P_{X\times Y\to X}(R\cap G_h))$, so $S\cap M_{ij(X)}$-$Int_p(h^{-1}(M_{i(Y)}$-$Int(M_{j(Y)}$-$Cl(T))))\cap P_{X\times Y\to X}(R\cap G_h)$ contains one point, say, $b \in X$ which implies $(b, h(b)) \in R$ and $h(b) \in M_{i(Y)}$-$Int(M_{j(Y)}$-$Cl(T))$. Therefore, $\emptyset \neq (S \times (M_{i(Y)}$-$Int(M_{j(Y)}$-$Cl(T)))) \cap R \subseteq M_{i(Y)}$-$Int(M_{j(Y)}$-$Cl(S \times T)) \cap R$. Thus $(a, h(a)) \in M_{ij(X\times Y)}$-$Cl_\delta(R)$. Since $R \in M_{ij(X\times Y)}$-$\delta C(X \times Y)$, so $(a, h(a)) \in R \cap G_h$ and $a \in P_{X\times Y\to X}(R\cap G_h)$. Therefore, $M_{ij(X)}$-$Cl_p(P_{X\times Y\to X}(R\cap G_h)) \subseteq P_{X\times Y\to X}(R\cap G_h)$. Consequently, $P_{X\times Y\to X}(R \cap G_h) \in M_{ij(X)}$-$PC(X)$. □

**Definition 3.5.** A biminimal structure space $(X, M_{1(X)}, M_{2(X)})$ is said to be $M_{ij(X)}$-semi regular if for every $a \in X$ and for every $S \in M_{i(X)}$, there exists $T \in M_{i(X)}$ containing $a$ such that $a \in T \subseteq M_{i(X)}$-$Int(M_{j(X)}$-$Cl(T)) \subseteq S$.

**Theorem 3.11.** *If* $g : (X, M_{1(X)}, M_{2(X)}) \to (Y, M_{1(Y)}, M_{2(Y)})$ *is almost* $M_{ij}$*-precontinuous and* $(Y, M_{1(Y)}, M_{2(Y)})$ *is* $M_{ij(Y)}$*-semi regular, then* $g$ *is* $M_{ij}$*-precontinuous.*

*Proof.* Let $S \in M_{i(Y)}$ such that $g(a) \in S$, where $a \in X$. Then $a \in g^{-1}(S)$. Since $(Y, M_{1(Y)}, M_{2(Y)})$ is $M_{ij(Y)}$-semi regular, so there exists $T \in M_{i(Y)}$ such that $g(a) \in T \subseteq M_{i(Y)}$-$Int(M_{j(Y)}$-$Cl(T)) \subseteq S$. Also, $g$ is almost $M_{ij}$-precontinuous, so there exists $R \in M_{ij(X)}$-$PO(X)$ containing $a$ such that $g(R) \subseteq M_{i(Y)}$-$Int(M_{j(Y)}$-$Cl(T)) \subseteq$

$S$. So, $a \in R = M_{ij(X)}$-$Int_p(R) \subseteq M_{ij(X)}$-$Int_p(g^{-1}(S))$. Thus $g^{-1}(S) \subseteq M_{ij(X)}$-$Int_p(g^{-1}(S))$. Hence, $g^{-1}(S) = M_{ij(X)}$-$Int_p(g^{-1}(S))$ and so $g^{-1}(S) \in M_{ij(X)}$-$PO(X)$. Consequently, $g$ is $M_{ij}$-precontinuous. □

**Definition 3.6.** [3] Let $f : (X, M_{1(X)}, M_{2(X)}) \to (Y, M_{1(Y)}, M_{2(Y)})$ be a function. Then the graph $G_f$ of this function $f$ is said to be $M_{ij(X\times Y)}$-preclosed in $X \times Y$ if for every $(a, b) \in (X \times Y) \setminus G_f$, there exists $S \in M_{ij(X)}$-$PO(X)$ containing $a$ and $T \in M_{i(Y)}$ containing $b$ such that $(S \times T) \cap G_f = \emptyset$.

**Lemma 3.1.** [3] *The graph $G_f$ of the function $f : (X, M_{1(X)}, M_{2(X)}) \to (Y, M_{1(Y)}, M_{2(Y)})$ is $M_{ij(X\times Y)}$-preclosed in $X\times Y$ if and only if for every $(a, b) \in (X\times Y)\setminus G_f$, there exists $S \in M_{ij(X)}$-$PO(X)$ containing $a$ and $T \in M_{i(Y)}$ containing $b$ such that $f(S)\cap T = \emptyset$.*

**Definition 3.7.** [3] A biminimal structure space $(X, M_{1(X)}, M_{2(X)})$ is said to be $M_{ij(X)}$-pre $T_2$ space if for every $a, b \in X$ such that $a \neq b$, there exists $S, T \in M_{ij(X)}$-$PO(X)$ containing $a$ and $b$ respectively such that $S \cap T = \emptyset$.

**Theorem 3.12.** *If a function $f : (X, M_{1(X)}, M_{2(X)}) \to (Y, M_{1(Y)}, M_{2(Y)})$ is almost $M_{ij}$-precontinuous, where the graph $G_f$ is $M_{ij(X\times Y)}$-preclosed in $X \times Y$ and $(Y, M_{1(Y)}, M_{2(Y)})$ is $M_{ij(Y)}$-semi regular, then $(X, M_{1(X)}, M_{2(X)})$ is said to be $M_{ij(X)}$-pre $T_2$ space.*

*Proof.* Let $a, b \in X$ such that $a \neq b$. Then $f(a) \neq f(b)$. Then $(a, f(b)) \in (X\times Y)\setminus G_f$. Since $G_f$ is $M_{ij(X\times Y)}$-preclosed in $X \times Y$, so by Lemma 3.1, there exists $S \in M_{ij(X)}$-$PO(X)$ containing $a$ and $T \in M_{i(Y)}$ containing $f(b)$ such that $f(S) \cap T = \emptyset$. Since $f$ is almost $M_{ij}$-precontinuous and $Y$ is $M_{ij(Y)}$-semi regular, so by Theorem 3.11, $f$ is $M_{ij}$-precontinuous. Therefore $f^{-1}(T) \in M_{ij(X)}$-$PO(X)$ containing $b$ such that $S \cap f^{-1}(T) = \emptyset$. Hence, $(X, M_{1(X)}, M_{2(X)})$ is $M_{ij(X)}$-pre $T_2$ space. □

**Definition 3.8.** A biminimal structure space $(X, M_{1(X)}, M_{2(X)})$ is said to be $M_{ij(X)}$-almost regular if for every $a \in X$ and for every $S \in M_{i(X)}$, there exists $T \in M_{i(X)}$ containing $a$ such that $a \in T \subseteq M_{j(X)}\text{-}Cl(T) \subseteq M_{i(X)}\text{-}Int(M_{j(X)}\text{-}Cl(S))$

**Theorem 3.13.** *If $f : (X, M_{1(X)}, M_{2(X)}) \to (Y, M_{1(Y)}, M_{2(Y)})$ be a function such that $(Y, M_{1(Y)}, M_{2(Y)})$ is $M_{ij(Y)}$-almost regular, then $g$ is almost $M_{ij}$-precontinuous if and only if $g$ is weakly $M_{ij}$-precontinuous.*

*Proof.* Let $g$ be weakly $M_{ij}$-precontinuous and $S \in M_{i(Y)}$ such that $a \in g^{-1}(S)$, where $a \in X$. Then $g(a) \in S$. Since $S \in M_{i(Y)}$ so $S \in M_{ij(Y)}\text{-}RO(Y)$ containing $g(a)$. Also, since $(Y, M_{1(Y)}, M_{2(Y)})$ is $M_{ij(Y)}$-almost regular so there exists $T \in M_{i(Y)}$ containing $g(a)$ such that $g(a) \in T \subseteq M_{j(Y)}\text{-}Cl(T) \subseteq M_{i(Y)}\text{-}Int(M_{j(Y)}\text{-}Cl(S))$ which implies that $a \in T \subseteq M_{j(Y)}\text{-}Cl(T) \subseteq S$. Now, $g$ is weakly $M_{ij}$-precontinuous, so there exists $R \in M_{ij(X)}\text{-}PO(X)$ containing $a$ such that $g(R) \subseteq M_{j(Y)}\text{-}Cl(T) \subseteq S$. Thus $R \subseteq g^{-1}(S)$ and $a \in R = M_{ij(X)}\text{-}Int_p(R) \subseteq M_{ij(X)}\text{-}Int_p(g^{-1}(S))$. Therefore, $g^{-1}(S) \subseteq M_{ij(X)}\text{-}Int_p(g^{-1}(S))$. Thus $g^{-1}(S) = M_{ij(X)}\text{-}Int_p(g^{-1}(S))$ and so $g^{-1}(S) \in M_{ij(X)}\text{-}PO(X)$. Now, by Theorem 3.2, we have $g$ is almost $M_{ij}$-precontinuous.
Conversely, it is obvious that every almost $M_{ij}$-precontinuous function is weakly $M_{ij}$-precontinuous. □

**Acknowledgement**

We would like to thank the editor and the referees for their valuable suggestions and comments those helped in improving the paper.

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(1) Department of Mathematics, Central Institute of Technology, Rangalikhata, Balagaon, Kokrajhar, Assam-783370, India.

and

Department of Mathematics, Tripura University, Suryamaninagar, Agartala 799022, India.

*Email address*: aa.basumatary@cit.ac.in, albisbasu@gmail.com

(2) (Corresponding Author) Department of Mathematics, Central Institute of Technology, Rangalikhata, Balagaon, Kokrajhar, Assam-783370, India.

*Email address*: dj.sarma@cit.ac.in, djs_math@rediffmail.com

(3) Department of Mathematics, Tripura University, Suryamaninagar, Agartala-799022, India.

*Email address*: tripathybc@yahoo.com,tripathybc@rediffmail.com