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ON SOME PROPERTIES OF WEAKLY PRIME SEMI-IDEALS OF POSETS

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Abstract. In this paper, we discuss some properties of direct product of weakly prime semi-ideals of $P_1 \times P_2$ where P_1 and P_2 are partially ordered sets (posets). We also find an equivalence condition for a semi-ideal to be weakly prime. Further we show that the semi-ideals are prime provided the direct product P of semi-ideals is a weakly prime semi-ideal of P, and the intersection of n distinct prime semi-ideals of P is a weakly (n+1)-prime semi-ideal of P.

1. Preliminaries

Throughout this paper, $(P, \leq)$ denotes a poset with zero element 0 and K denotes a proper semi-ideal of P. For $K \subseteq P$, let $(K)^l := \{r \in P : r \leq k \text{ for all } k \in K\}$ denotes the lower cone of K in P. For $K_1, K_2 \subseteq P$, we write $(K_1, K_2)^l$ instead of $(K_1 \cup K_2)^l$. If $K = \{d_1, ..., d_n\}$ is finite, then we use the notation $(d_1, ..., d_n)^l$ instead of $(\{d_1, ..., d_n\})^l$. Following [4], a non-empty subset K of P is a semi-ideal of P if $b' \in K$ and $u' \leq b'$, then $u' \in K$. A semi-ideal K of P is prime if for each $d_1, d_2 \in P$, $(d_1, d_2)^l \subseteq K$ implies $d_1 \in K$ or $d_2 \in K$. Following [8], the extension of K by $d_1 \in P$ is $(K : d_1) = \{d_2 \in P \ : \ (d_1, d_2)^l \subseteq K\}$. Clearly, $(K : d_1)$ is a semi-ideal of P. A semi-ideal K is said to be u-semi-ideal if, for $k_1, k_2 \in K$, $(k_1, k_2)^u \cap K \neq \{\phi\}$ [7].

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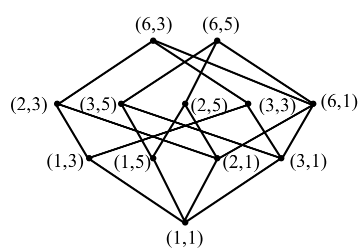
Many researchers studied various properties of a poset P (see [5], [8], [9]). Following [2], a semi-ideal K of P is weakly n-prime if for $d_1, d_2, ..., d_n \in P$, $(d_1, d_2, ..., d_n)^l \subseteq K$, then either $(d_2, ..., d_n)^l \subseteq K$ or $(d_1, d_3, ..., d_n)^l \subseteq K$, ..., or $(d_1, d_2, ..., d_{n-1})^l \subseteq K$. It is evident that every prime semi-ideal of P is weakly n-prime but a weakly n-prime semi-ideal of P is not necessary a prime semi-ideal of P.

For example, consider the set $X = \{u', v', w'\}$. Then $(P(X), \subseteq)$ is a poset. Here $K = \{\phi\}$ is weakly 3-prime semi-ideal of P but not prime as $(u', v')^l \subseteq K$ with $u', v' \notin K$.

Following [3], we defined the direct product of two posets P_1 and P_2 in [6] and studied various properties. The direct product of two posets P_1 and P_2 is defined as $P_1 \times P_2 = \{(u', v') : u' \in P_1,\ v' \in P_2\}$ such that $(u', v') \leq (u'', v'')$ in $P_1 \times P_2$ if $u' \leq u''$ in P_1 and $v' \leq v''$ in P_2. It is clear that the direct product of semi-ideals is a semi-ideal, and the direct product of prime semi-ideals need not be prime. Also, the direct product of two weakly 3-prime semi-ideals of posets need not be a weakly 3-prime semi-ideal as shown in Example 1.1.

Example 1.1. *Let* $P_1 = \{1, 2, 3, 6\}$ *and* $P_2 = \{1, 3, 5\}$ *be posets under the relation division. Then the Hasse diagram of* $P_1 \times P_2$ *is given below.*



Here $I_1 = \{1\}$ *and* $I_2 = \{1\}$ *are weakly 3-prime semi-ideals of* P_1 *and* P_2 *respectively, and* $I_1 \times I_2 = \{(1, 1)\}$ *is not weakly 3-prime as* $((2, 3), (3, 3), (6, 1))^l \subseteq I_1 \times I_2$*, but* $((2, 3), (3, 3))^l \nsubseteq I_1 \times I_2$, $((2, 3), (6, 1))^l \nsubseteq I_1 \times I_2$ *and* $((3, 3), (6, 1))^l \nsubseteq I_1 \times I_2$.

For basic terminology in poset we refer to [4].

2. Direct product of weakly 3-prime semi-ideals of posets

We study some properties of weakly prime semi-ideals of P. We also find an equivalence condition for a semi-ideal to be weakly prime. Further we show that the extension of a weakly 3-prime semi-ideal K by $x \in P\backslash K$ is a weakly 3-prime semi-ideal of P.

Theorem 2.1. [2] *If K is a weakly n-prime semi-ideal of P, then it is also weakly (n+1)-prime for all $n \geq 2$.*

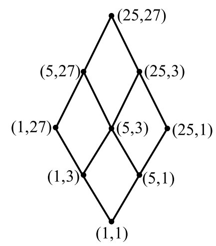
Theorem 2.2. [2] *Let K be a semi-ideal of P and $n \geq 3$. If $(K : x)$ is a weakly (n-1)-prime for every $x \in P\backslash K$, then K is a weakly n-prime semi-ideal of P.*

Lemma 2.1. *If K is a weakly 3-prime semi-ideal of P, then $(K : d_1)$ is a weakly 3-prime semi-ideal for each $d_1 \in P\backslash K$.*

Proof. Assume that $(s', t', w')^l \subseteq (K : d_1)$ for each $d_1 \in P\backslash K$ and $s', t', w' \in P$. Then $(s', t', w', d_1)^l \subseteq K$ and any one of the set $(s', t', w')^l$, $(s', t', d_1)^l$, $(s', w', d_1)^l$, $(t', w', d_1)^l$ is contained in K. If $(s', t', w')^l \subseteq K$, then we get either $(s', t')^l \subseteq K$ or $(t', w')^l \subseteq K$ or $(s', w')^l \subseteq K$ which gives $(K : d_1)$ is weakly 3-prime semi-ideal as $K \subseteq (K : d_1)$. If $(s', t', d_1)^l \subseteq K$, then we get either $(s', t')^l \subseteq K$ or $(t', d_1)^l \subseteq K$ or $(s', d_1)^l \subseteq K$ which gives $(K : d_1)$ is weakly 3-prime semi-ideal as $K \subseteq (K : d_1)$. If $(s', w', d_1)^l \subseteq K$, then either $(s', d_1)^l \subseteq K$ or $(d_1, w')^l \subseteq K$ or $(s', w')^l \subseteq K$ which gives $(K : d_1)$ is weakly 3-prime semi-ideal as $K \subseteq (K : d_1)$. If $(t', w', d_1)^l \subseteq K$, then either $(d_1, t')^l \subseteq K$ or $(t', w')^l \subseteq K$ or $(d_1, w')^l \subseteq K$ which gives $(K : d_1)$ is weakly 3-prime semi-ideal as $K \subseteq (K : d_1)$. □

The below example shows that the direct product of prime semi-ideals of a poset P need not be a prime semi-ideal of P.

Example 2.1. *Let $P_1 = \{1, 5, 25\}$ and $P_2 = \{1, 3, 27\}$. Then $(P_1, /)$ and $(P_2, /)$ are posets, and the Hasse diagram of $P_1 \times P_2$ is given below.*



Here $I' = \{1\}$ *and* $J' = \{1\}$ *are prime semi-ideals of* P_1 *and* P_2 *respectively, and* $I = I' \times J'$ *is a semi-ideal of* $P_1 \times P_2$ *but not prime as* $((1,3),(5,1))^l = \{(1,1)\} \subseteq I' \times J'$ *with* $(1,3) \nsubseteq I' \times J'$ *and* $(5,1) \nsubseteq I' \times J'$.

In general, the intersection of prime semi-ideals of P need not be prime but we can have the following.

Lemma 2.2. *Suppose that* $\{I_i\}_{i=1}^{n}$ *are distinct prime semi-ideals of* P. *Then* $\bigcap_{i=1}^{n} I_i$ *is a weakly (n+1)-prime semi-ideal of* P.

Proof. Clearly $\bigcap_{i=1}^{n} I_i$ is a semi-ideal of P. Let $(a_1, a_2, a_3, ..., a_{n+1})^l \subseteq \bigcap_{i=1}^{n} I_i$ for $a_1, ..., a_{n+1} \in P$. Then $a_i \in I_j$ for some $1 \leq i \leq n+1$, $1 \leq j \leq n$ which implies any one of $(a_1, a_2, a_3, ..., a_n)^l$, $(a_1, a_2, a_3, ..., a_{n-1}, a_{n+1})^l$, ..., $(a_2, a_3, ..., a_{n+1})^l$ is a subset of $\bigcap_{i=1}^{n} I_i$ and so $\bigcap_{i=1}^{n} I_i$ is a weakly (n+1)-prime semi-ideal of P. □

The following example shows that the intersection of n distinct prime semi-ideals need not be weakly n-prime semi-ideal of P.

Example 2.2. *Let* $X = \{a, b, c\}$ *be a set. Then* $(P(X), \subseteq)$ *is a poset. Here* $I_1 = \{\{\phi\}, \{a\}, \{b\}, \{a, b\}\}$, $I_2 = \{\{\phi\}, \{b\}, \{c\}, \{b, c\}\}$ *and* $I_3 = \{\{\phi\}, \{a\}, \{c\}, \{a, c\}\}$ *are prime semi-ideals of* P. *Also* $\bigcap_{i=1}^{3} I_i = \{\phi\}$ *is not weakly 3-prime semi-ideal as* $(\{a, b\}, \{b, c\}, \{a, c\})^l \subseteq \bigcap_{i=1}^{3} I_i$ *with* $(\{a, b\}, \{b, c\})^l \nsubseteq \bigcap_{i=1}^{3} I_i$, $(\{a, b\}, \{a, c\})^l \nsubseteq \bigcap_{i=1}^{3} I_i$ *and* $(\{b, c\}, \{a, c\})^l \nsubseteq \bigcap_{i=1}^{3} I_i$.

Theorem 2.3. *If I and J are weakly 3-prime semi-ideals of P and if $I \cup J$ is a u-semi-ideal of P, then the intersection and union of I and J is a weakly 3-prime semi-ideal of P.*

Proof. Let $I \cup J$ be a u-semi-ideal of P. We now claim that $I \cap J$ is a weakly 3-prime semi-ideal of P. Suppose $(a,b,c)^l \subseteq I \cap J$. If $(a,b)^l \subseteq I$ and $(a,b)^l \subseteq J$, then $I \cap J$ is a weakly 3-prime semi-ideal of P. If $(b,c)^l \subseteq I$ and $(b,c)^l \subseteq J$, then $I \cap J$ is a weakly 3-prime semi-ideal of P. If $(a,c)^l \subseteq I$ and $(a,c)^l \subseteq J$, then $I \cap J$ is a weakly 3-prime semi-ideal of P. Suppose $(a,b)^l \subseteq I \backslash J$ and $(b,c)^l \subseteq J \backslash I$. Then there exist $s \in (a,b)^l \backslash J$ and $t \in (b,c)^l \backslash I$. Since $I \cup J$ is a u-semi-ideal of P, we have $(s,t)^u \cap (I \cup J) \neq \phi$. Let $k \in (s,t)^u \cap (I \cup J)$. If $k \in I$, then $t \in I$, a contradiction. If $k \in J$, then $s \in J$, a contradiction. Similarly, we can also prove the other cases. Thus $I \cap J$ is a weakly 3-prime semi-ideal of P.

We now claim that $I \cup J$ is a weakly 3-prime semi-ideal of P. Suppose $(a,b,c)^l \subseteq I \cup J$. If $(a,b,c)^l \subseteq I$, then $I \cup J$ is a weakly 3-prime semi-ideal of P. If $(a,b,c)^l \subseteq J$, then $I \cup J$ is a weakly 3-prime semi-ideal of P. Otherwise $(a,b,c)^l \nsubseteq I$ and $(a,b,c)^l \nsubseteq J$. Then there exist $v, w \in (a,b,c)^l$ such that $v \notin I$ and $w \notin J$. Since $I \cup J$ is a u-semi-ideal of P, we have $(w,v)^u \cap (I \cup J) \neq \phi$. Let $k \in (v,w)^u \cap (I \cup J)$. If $k \in I$, then $v \in I$, a contradiction. If $k \in J$, then $w \in J$, a contradiction. Thus $I \cup J$ is a weakly 3-prime semi-ideal of P. □

The following examples shows that the condition $I \cup J$ is a u-semi-ideal is not superficial in Theorem 2.3.

Example 2.3. *Let $P = \{1,2,3,5,6,10,15\}$. Then P is a poset under the relation division. Here $I = \{1,2\}$ and $J = \{1,3\}$ are weakly 3-prime semi-ideals of P. Clearly $I \cup J$ is not u-semi-ideal of P. Also, $I \cap J$ is not weakly 3-prime semi-ideal as $(6,10,15)^l \subseteq I \cap J$ with $(6,10)^l \nsubseteq I \cap J$, $(10,15)^l \nsubseteq I \cap J$ and $(6,15)^l \nsubseteq I \cap J$.*

Example 2.4. *Let $P = \{1, 2, 3, 4, 5, 9, 10, 36, 60, 90\}$. Then P is a poset under the relation division. Here $I = \{1, 2\}$ and $J = \{1, 3\}$ are weakly 3-prime semi-ideals of P. Clearly $I \cup J$ is not a u-semi-ideal of P. Also, $I \cup J$ is not a weakly 3-prime semi-ideal as $(36, 60, 90)^l \subseteq I \cup J$ with $(36, 60)^l \nsubseteq I \cup J$, $(60, 90)^l \nsubseteq I \cup J$ and $(36, 90)^l \nsubseteq I \cup J$.*

The following theorem shows the equivalent condition for a weakly 3-prime semi-ideal.

Theorem 2.4. *Let P_1 and P_2 be two posets and K_1 be a proper semi-ideal of P_1. Then the assertions given below are equivalent:*

(i) $K_1 \times P_2$ is a weakly 3-prime semi-ideal of $P_1 \times P_2$.

(ii) K_1 is a weakly 3-prime semi-ideal of P_1.

Proof. $(i) \Rightarrow (ii)$ Let $(d_1, d_2, d_3)^l \subseteq K_1$ for some $d_1, d_2, d_3 \in P_1$. Then $((d_1, l_1), (d_2, l_2), (d_3, l_3))^l \subseteq K_1 \times P_2$ for any $l_1, l_2, l_3 \in P_2$ which implies either $((d_1, l_1), (d_2, l_2))^l \subseteq K_1 \times P_2$ or $((d_1, l_1), (d_3, l_3))^l \subseteq K_1 \times P_2$ or $((d_2, l_2), (d_3, l_3))^l \subseteq K_1 \times P_2$, and so $(d_1, d_2)^l \subseteq K_1$ or $(d_1, d_3)^l \subseteq K_1$ or $(d_2, d_3)^l \subseteq K_1$.

$(ii) \Rightarrow (i)$ Let $((s_1, u_1), (s_2, u_2), (s_3, u_3))^l \subseteq K_1 \times P_2$ for some $s_1, s_2, s_3 \in P_1$ and $u_1, u_2, u_3 \in P_2$. Then $(s_1, s_2, s_3)^l \subseteq K_1$ which implies either $(s_1, s_2)^l \subseteq K_1$ or $(s_1, s_3)^l \subseteq K_1$ or $(s_2, s_3)^l \subseteq K_1$. So either $((s_1, u_1), (s_2, u_2))^l \subseteq K_1 \times P_2$ or $((s_1, u_1), (s_3, u_3))^l \subseteq K_1 \times P_2$ or $((s_2, u_2), (s_3, u_3))^l \subseteq K_1 \times P_2$. □

Corollary 2.1. *Let P_1 and P_2 be two posets and K be a proper semi-ideal of P_1. If $K \times P_2$ is a weakly 3-prime semi-ideal of $P_1 \times P_2$, then $(K : d_1)$ is a weakly 3-prime semi-ideal of P_1 for each $d_1 \in P_1 \backslash K$.*

Proof. It is evident from Lemma 2.1 and Theorem 2.4. □

Corollary 2.2. *Let P_1 and P_2 be two posets and K be a proper semi-ideal of P_1. If $(K : d_1)$ is a weakly 3-prime semi-ideal of P_1 for each $d_1 \in P_1 \backslash K$, then $K \times P_2$ is a weakly 4-prime semi-ideal of $P_1 \times P_2$.*

Proof. It is evident from Theorem 2.2 and Theorem 2.4. □

The equivalent condition of weakly 3-prime semi-ideal of direct product of posets is given below.

Theorem 2.5. *Let P_1 and P_2 be two posets with greatest elements $'e_1'$ and $'e_2'$ respectively. Let K be a proper semi-ideal of P_1 and L be a semi-ideal of P_2. Then the given assertions are equivalent:*
(i) $K \times L$ is a weakly 3-prime semi-ideal of $P_1 \times P_2$.
(ii) $L = P_2$ and K is a weakly 3-prime semi-ideal of P_1 or L is a prime semi-ideal of P_2 and K is a prime semi-ideal of P_1.

Proof. $(i) \Rightarrow (ii)$ Assume that $K \times L$ is a weakly 3-prime semi-ideal of $P_1 \times P_2$. If $L = P_2$, then by Theorem 2.4, K is a weakly 3-prime semi-ideal of P_1. Suppose that $L \neq P_2$. We show that L is a prime semi-ideal of P_2. Let $a, b \in P_2$ such that $(a, b)^l \subseteq L$ and $i(\neq 0) \in K$. Then $((i, e_2), (e_1, a), (e_1, b))^l \subseteq K \times L$. Since $((e_1, a), (e_1, b))^l \nsubseteq K \times L$, we have either $((i, e_2), (e_1, a))^l \subseteq K \times L$ or $((i, e_2), (e_1, b))^l \subseteq K \times L$ which implies $a \in L$ or $b \in L$ and so L is prime. Let $c, d \in P_1$ such that $(c, d)^l \subseteq K$ and $j(\neq 0) \in L$. Then $((c, e_2), (d, e_2), (e_1, j))^l \subseteq K \times L$. Since $((c, e_2), (d, e_2))^l \nsubseteq K \times L$, we conclude that either $((c, e_2), (e_1, j))^l \subseteq K \times L$ or $((d, e_2), (e_1, j))^l \subseteq K \times L$ which implies $c \in K$ or $d \in K$, and so K is a prime semi-ideal of P_1.
$(ii) \Rightarrow (i)$ If $L = P_2$ and K is a weakly 3-prime semi-ideal of P_1, then $K \times L$ is a weakly 3-prime semi-ideal of $P_1 \times P_2$ by Theorem 2.4. Assume that K and L are prime semi-ideals of P_1 and P_2 respectively. Suppose $((s_1, v_1), (s_2, v_2), (s_3, v_3))^l \subseteq K \times L$ for some $s_1, s_2, s_3 \in P_1$ and $v_1, v_2, v_3 \in P_2$. Then $(s_1, s_2, s_3)^l \subseteq K$ and $(v_1, v_2, v_3)^l \subseteq L$ which imply that at least one of s_1, s_2, s_3 is in K and v_1, v_2, v_3 is in L. If $s_1 \in K$ and $v_1 \in L$, then $((s_1, v_1), (s_2, v_2))^l \subseteq K \times L$. Similarly, we can check the other cases. So $K \times L$ is a weakly 3-prime semi-ideal of $P_1 \times P_2$. □

The example given below shows that one cannot drop the condition greatest elements in Theorem 2.5.

Example 2.5. *Let $P_1 = \{1, 3, 9\}$ and $P_2 = \{1, 2, 3\}$. Then $(P_1, /)$ and $(P_2, /)$ are posets. Here $U = \{1\}$ and $V = \{1\}$ are semi-ideals of P_1 and P_2 respectively and $U \times V = \{(1, 1)\}$ is a weakly 3-prime semi-ideal but V is not a prime semi-ideal of P_2 as well as $V \neq P_2$.*

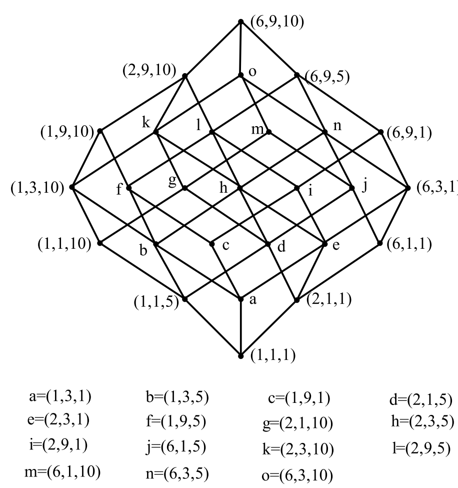
Theorem 2.6. *Let P_1, P_2 and P_3 be posets with greatest elements e_1, e_2 and e_3 respectively. Let K_1, K_2 and K_3 be proper semi-ideals of P_1, P_2 and P_3 respectively. Then the following assertions holds:*
(i) If $I = K_1 \times K_2 \times K_3$ is a weakly 3-prime semi-ideal of $P_1 \times P_2 \times P_3$, then K_1, K_2 and K_3 are prime semi-ideals of P_1, P_2 and P_3 respectively.
(ii) If $K_1 \times K_2 \times K_3$ is a weakly 3-prime semi-ideals of $P_1 \times P_2 \times P_3$, then $K_1 \times K_2 \times P_3$, $K_1 \times P_2 \times K_3$ and $P_1 \times K_2 \times K_3$ are weakly 3-prime semi-ideals of $P_1 \times P_2 \times P_3$.

Proof. (i) Let $(a', b')^l \subseteq K_1$ for some $a', b' \in P_1$.
Then $((a', e_2, e_3),\ (e_1, i, i),\ (b', e_2, e_3))^l \subseteq I$. Since I is a weakly 3-prime semi-ideal and $((a', e_2, e_3),\ (b', e_2, e_3))^l \nsubseteq I$, we have either $((a', e_2, e_3), (e_1, i, i))^l \subseteq I$ or $((e_1, i, i), (b', e_2, e_3))^l \subseteq I$ which implies $a' \in K_1$ and $b' \in K_1$ and so K_1 is a prime semi-ideal of P_1.
Suppose that $(s', t')^l \subseteq K_2$ for some $s', t' \in P_2$. Then $((e_1, s', e_3), (e_1, i, i), (e_1, t', e_3))^l \subseteq I$. Since I is a weakly 3-prime semi-ideal and $((e_1, s', e_3),\ (e_1, t', e_3))^l \nsubseteq I$, we have either $((e_1, s', e_3),\ (e_1, i, i))^l \subseteq I$ or $((e_1, i, i),\ (e_1, t', e_3))^l \subseteq I$ which implies $s' \in K_2$ and $t' \in K_2$ and so K_2 is a prime semi-ideal of P_2. Similarly, we can show that K_3 is prime.
(ii) Now by (i), K_1, K_2 and K_3 are prime semi-ideals of P_1, P_2 and P_3 respectively. Suppose $((u_1, v_1, c_1),\ (u_2, v_2, c_2), (u_3, v_3, c_3))^l \subseteq K_1 \times K_2 \times P_3$ for some $u_1, u_2, u_3 \in P_1$, $v_1, v_2, v_3 \in P_2$, $c_1, c_2, c_3 \in P_3$. Then $(u_1, u_2, u_3)^l \subseteq K_1$ and $(v_1, v_2, v_3)^l \subseteq K_2$ which

implies any one of $u_1, u_2, u_3 \in K_1$ and $v_1, v_2, v_3 \in K_2$. Without loss of generality, let us take $u_1 \in K_1$ and $v_2 \in K_2$. Then $((u_1, v_1, c_1),\ (u_2, v_2, c_2))^l \subseteq K_1 \times K_2 \times P_3$ and hence $K_1 \times K_2 \times P_3$ is a weakly 3-prime semi-ideal of $P_1 \times P_2 \times P_3$. Similarly, we can prove that $K_1 \times P_2 \times K_3$ and $P_1 \times K_2 \times K_3$ are weakly 3-prime semi-ideals of $P_1 \times P_2 \times P_3$. □

The following example shows that the converse of Theorem 2.6 fails.

Example 2.6. *Let* $P_1 = \{1, 2, 6\}$, $P_2 = \{1, 3, 9\}$ *and* $P_3 = \{1, 5, 10\}$ *be posets under the relation division. Then the Hasse diagram of* $P_1 \times P_2 \times P_3$ *is given below.*



Here $K_1 = \{1, 2\}, K_2 = \{1\}$ *and* $K_3 = \{1, 5\}$ *are prime semi-ideals of* P_1, P_2 *and* P_3 *respectively. Also* $K_1 \times K_2 \times K_3$ *is not a weakly 3-prime semi-ideal of* $P_1 \times P_2 \times P_3$ *as* $((2, 9, 10), (6, 1, 10), (6, 9, 5))^l \subseteq K_1 \times K_2 \times K_3$, *but* $((2, 9, 10), (6, 1, 10))^l \nsubseteq K_1 \times K_2 \times K_3$ *and* $((2, 9, 10), (6, 9, 5))^l \nsubseteq K_1 \times K_2 \times K_3$ *and* $((6, 1, 10), (6, 9, 5))^l \nsubseteq K_1 \times K_2 \times K_3$.

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