Jordan Journal of Mathematics and Statistics (JJMS), 14(4), 2021, pp 777 - 785
DOI: https://doi.org/10.47013/14.4.11

SUPER FACE MAGIC LABELING OF SUBDIVIDED PRISM GRAPH

SHAHID ZAMAN[(1)], JAWAD ALAM*[(2)] AND ASMA JABEEN[(3)]

Abstract. This paper is about the problems of labeling vertices, edges and faces of subdivided prism graph. We show that the generalized r-subdivided prism $G_{r,t}$ without the central (and external) face has a super face-magic total ($SFMT$)-labeling of type-(1,1,1).

1. Introduction

In this paper, all graphs are finite, simple, undirected and planar. The planar graph $G = (V, E, F)$ has vertex set $V(G)$, edge set $E(G)$ and face set $F(G)$. We follow either Wallis [1] or West [2] for most terminology of graph theory that have been used in this paper.

The notion of graph labeling extends widely to many scientific field like astronomy, coding theory, cryptography, x-ray crystallography, communication networks design and circuit design.

Labeling of a graph is a mapping that assigns a collection of graph elements to a set of numbers, which are generally positive integer numbers. The labeling is known as vertex labeling, edge labeling or face labeling, respectively if the domain is the vertex set, edge set or the face set. A bijection $\eta : V \cup E \cup F \rightarrow \{1, 2, 3...|V(G)| + |E(G)| +$

2000 *Mathematics Subject Classification.* 05C38, 05C78.

Key words and phrases. r-subdivided prism graph, weight, $SFMT$-labeled graph.

Received: July. 13, 2020 Accepted: Feb. 21, 2021 .

$|F(G)|\}$ is known as type-(1,1,1) labeling. The weight of every face in a graph labeling reflects the sum of the face labeling, vertex labeling and edge labeling where these edges and vertices are around that face. Type-(1,1,1) is known as face-antimagic (or face-magic resp.) in a labeled graph, if all faces of a graph have different (or same resp.) weight.

The notion of magic labeling for planar graph is described by Lih [3]. In this paper, Lih investigated magic labeling with type-(1,1,0) for prism, wheel and friendship graph. In [4], [5] and [6], the magic labeling for honeycomb, families of wheel and grid graph respectively, are given. And in [7], Ahmad proved the admittance of magic labeling of type-(1,1,1) in subdivided ladders. For the planar graph families with 3-sided, 5-sided and 6-sided faces, the magic labeling of type-(1,1,1) was given by Kathiresan and Gokulakrishnan in [8]. Same as for a planar graph with one infinite external face [8].

In [9], [10] and [11], we can find the d-antimagic labeling for grids, generalized Petersen graphs and hexagonal planar maps. The d-antimagic labeling for prisms are described in [12] and [13] by Lin *et al.* and Sugeng *et al.* respectively. When the possible smallest labels appear on the vertices, then that anti-magic labeling is referred as super anti-magic labeling. In [14] and [15], the super anti-magic labeling of type-(1,1,1) for disjoint union of prisms and antiprism was given. The labeling is known as $SFMT$-labeling if the possible smallest labels appear on the vertices.

In this paper we prove that the subdivided prism $G_{r,t}$ admits the $SFMT$-labeling.

2. Main Results

Suppose $G_{r,t}$ denotes the subdivided prism where $r \geq 1$ is the subdivision of the graph and $t \geq 3$ is the number of faces of the graph. The subdivided prism $G_{r,t}$ has $3rt$ vertices of degree 2, $2t$ vertices of degree 3, $3t(r+1)$ edges and t internal $(4r+4)$-sided faces. We do not consider the external and central $(rt+t)$-sided face.

Let $G_{r,t}$ defines the sets of vertices, edges, and faces as follows

$$
\begin{aligned}
V(G_{r,t}) = &\{x_j; 1 \leq j \leq t\} \cup \{y_j; 1 \leq j \leq t\} \cup \{v_{jk}; 1 \leq j \leq t, 1 \leq k \leq r\} \\
&\cup \{u_{jk}; 1 \leq j \leq t, 1 \leq k \leq r\} \cup \{w_{jk}; 1 \leq j \leq t, 1 \leq k \leq r\} \\
E(G_{r,t}) = &\{x_j u_{jk}; 1 \leq j \leq t, k = 1\} \cup \{y_j w_{jk}; 1 \leq j \leq t, k = 1\} \cup \{x_j v_{jk}; 1 \leq j \leq t, k = 1\} \\
&\cup \{x_{j+1} u_{jk}; 1 \leq j \leq t, k = r\} \cup \{y_j v_{jk}; 1 \leq j \leq t, k = r\} \\
&\cup \{y_{j+1} w_{jk}; 1 \leq j \leq t, k = r\} \cup \{u_{jk} u_{jk+1}; 1 \leq j \leq t, 1 \leq k \leq r-1\} \\
&\cup \{v_{jk} v_{jk+1}; 1 \leq j \leq t, 1 \leq k \leq r-1\} \cup \{w_{jk} w_{jk+1}; 1 \leq j \leq t, 1 \leq k \leq r-1\} \\
F(G_{r,t}) = &\{f_j; 1 \leq j \leq t\}
\end{aligned}
$$

Theorem 2.1. *The generalized r-subdivided prism $G_{r,t}$, $r \geq 1$ and $t = 3$, without the central (and external) face is SFMT-labeling when r is positive integer.*

Proof. Define a labeling $\eta : V(G_{r,t}) \cup E(G_{r,t}) \cup F(G_{r,t}) \to \{1, 2, 3..., 6t(r+1)\}$ for $t = 3$ and $r \geq 1$ in the following way

$$\eta(x_j) = \left\{ 2j - 1, \quad 1 \leq j \leq 3 \right.$$

$$\eta(y_j) = \left\{ 10 - j, \quad 1 \leq j \leq 3 \right.$$

$$
\eta(w_{jk}) = \begin{cases}
9 - j + 4k, & 1 \leq j \leq 3, \quad k = 1 \\
10k + 2j, & 1 \leq j \leq 2, \quad k = 2 \\
10k + j, & j = 3, \quad k = 2 \\
9k + 2j + 2, & 1 \leq j \leq 2, \quad 3 \leq k \leq r \\
9k + j + 2, & j = 3, \quad 3 \leq k \leq r
\end{cases}
$$

$$\eta(u_{jk}) = \begin{cases} 6-2j, & 1 \le j \le 2, \quad k=1 \\ 6, & j=3, \quad k=1 \\ 8k-j+5, & 1 \le j \le 3, \quad k=2 \\ 8k+j+2, & j=3, \quad k=2 \\ 9k-j+3, & 1 \le j \le 2, \quad 3 \le k \le r \\ 9k+j, & j=3, \quad 3 \le k \le r \end{cases}$$

$$\eta(v_{jk}) = \begin{cases} 6+j+3k, & 1 \le j \le 3, \quad 1 \le k \le 2 \\ 9k+j-3, & 1 \le j \le 3, \quad k=3 \end{cases}$$

$$\eta(x_j v_{jk}) = \begin{cases} 9r+2j+20, & 1 \le j \le 2 \\ 9r+j+20, & j=3 \end{cases}$$

$$\eta(v_{jk} v_{jk+1}) = \begin{cases} 27+9r-2j+2k, & 1 \le j \le 2, \quad k=1 \\ 27+9r-j+2k, & j=3, \quad k=1 \\ 9r+9k-2j+20, & 1 \le j \le 2, \quad 2 \le k \le r \\ 9r+9k-j+20, & j=3, \quad 2 \le k \le r \end{cases}$$

$$\eta(x_j u_{jk}) = \Big\{ \; 15+9r+j, \quad 1 \le j \le 3$$

$$\eta(u_{jk} u_{jk+1}) = \begin{cases} 9r+5k-j+17, & 1 \le j \le 3, \quad k=1 \\ 9r+9k-j+13, & 1 \le j \le 3, \quad 2 \le k \le r \end{cases}$$

$$\eta(y_j w_{jk}) = \Big\{ \; 9r-j+13, \quad 1 \le j \le 3$$

$$\eta(w_{jk} w_{jk+1}) = \begin{cases} 9r+3k-j+13, & 1 \le j \le 3, \quad k=1 \\ 9r+9k-j+16, & 1 \le j \le 3, \quad 2 \le k \le r \end{cases}$$

$$\eta(f_j) = \Big\{ \; 6+9r+j, \quad 1 \le j \le 3$$

The $4(r+1)$-sided face weight, in which each $j = \{1,2,3\}$ is computed as follows

$$W(f_{4(r+1),j}) = \eta(x_j) + \eta(y_j) + \eta(u_{jk}) + \eta(v_{jk}) + \eta(w_{jk}) + \eta(w_{jk}y_j) + \eta(v_{jk}v_{jk+1})$$
$$+ \eta(u_{jk}u_{jk+1}) + \eta(w_{jk}w_{jk+1}) + \eta(w_{jk}y_j) + \eta(x_j u_{jk}) + \eta(x_j v_{jk}) + \eta(f_j)$$

An example of $SFMT$-labeling of $G_{r,t}$ is shown in Figure 1. Note that the weight of every face in a labeling graph reflects the sum of the face labeling and the vertices and edges around that face labeling.

Based on the definition of the weight of each face, we get

In Fig: 1.1, $W(f_j) = 314$ for $j = \{1,2,3\}$.

In Fig: 1.2, $W(f_j) = 675$ for $j = \{1,2,3\}$.

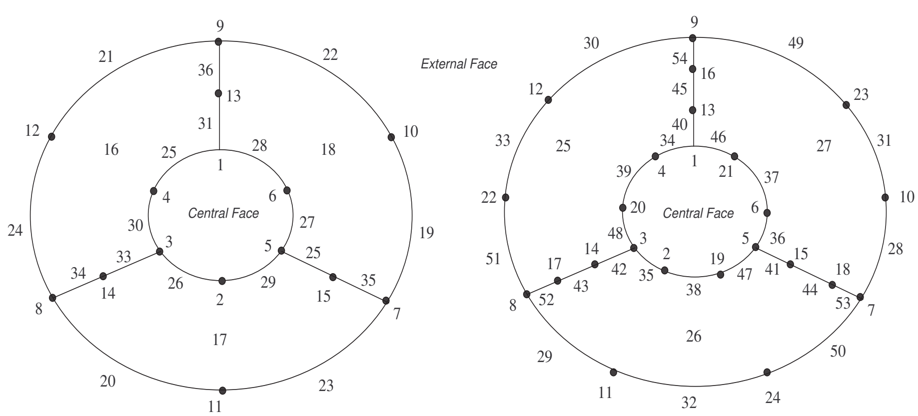

Figure 1.1 :The graph $G_{1,3}$

Figure 1.2 : The graph $G_{2,3}$

Figure 1. $SFMT$-Labeling for $G_{1,3}$ *and* $G_{2,3}$.

Theorem 2.2. *The generalized r-subdivided prism $G_{r,t}$, r=1 and $t \geq 4$, without the central (and external) face is SFMT-labeling when n is positive integer.*

Proof. Define a labeling $\eta : V(G_{r,t}) \cup E(G_{r,t}) \cup F(G_{r,t}) \to \{1,2,3...,6t(r+1)\}$ for

$t \geq 4$ and $r = 1$ in the following way

$$\eta(x_j) = \begin{cases} 2j-1, & 1 \leq j \leq t \end{cases}$$

$$\eta(y_j) = \begin{cases} 3t-j+1, & 1 \leq j \leq t \end{cases}$$

$$\eta(u_{jk}) = \begin{cases} 2t-2j, & 1 \leq j \leq t-1 \\ 2t, & j = t \end{cases}$$

$$\eta(w_{jk}) = \begin{cases} 4t, & j = 1 \\ 3t+j-1, & 2 \leq j \leq t \end{cases}$$

$$\eta(v_{jk}) = \begin{cases} 4t+j, & 1 \leq j \leq t \end{cases}$$

$$\eta(x_j v_{jk}) = \begin{cases} 10t+j, & j = 1 \\ 11t-j+2, & 2 \leq j \leq t \end{cases}$$

$$\eta(v_{jk} y_j) = \begin{cases} 12t, & j = 1 \\ 11t+j-1, & 2 \leq j \leq t \end{cases}$$

$$\eta(y_j w_{jk}) = \begin{cases} 7t-j+1, & 1 \leq j \leq t \end{cases}$$

$$\eta(y_{j+1} w_{jk}) = \begin{cases} 7t+j, & j = 1 \\ 8t-j+2, & 2 \leq j \leq t \end{cases}$$

$$\eta(x_j u_{jk}) = \begin{cases} 9t-j+1, & 1 \leq j \leq t \end{cases}$$

$$\eta(x_{j+1} u_{jk}) = \begin{cases} 10t-j+1, & 1 \leq j \leq t \end{cases}$$

$$\eta(f_j) = \begin{cases} 5t+j, & 1 \leq j \leq t \end{cases}$$

The 8-sided face weight, in which each $j = \{1, 2, 3, ...t\}$ is computed as follows

$$W(f_{8,j}) = \eta(x_j) + \eta(y_j) + \eta(u_{jk}) + \eta(v_{jk}) + \eta(w_{jk}) + \eta(v_{jk}y_j) + \eta(x_jv_{jk})$$
$$+ \eta(x_{j+1}u_{jk}) + \eta(x_ju_{jk}) + \eta(y_{j+1}w_{jk}) + \eta(y_jw_{jk}) + \eta(f_j)$$

An example of $SFMT$-labeling of $G_{r,t}$ is shown in Figure 2. Note that the weight of every face in a labeling graph reflects the sum of the face labeling and the vertices and edges around that face labeling. Based on the definition of the weight of each face, we get In Fig: 2.1, $W(f_j) = 416$ for $j = \{1, 2, 3, 4\}$.
In Fig: 2.2, $W(f_j) = 518$ for $j = \{1, 2, 3, 4, 5\}$.

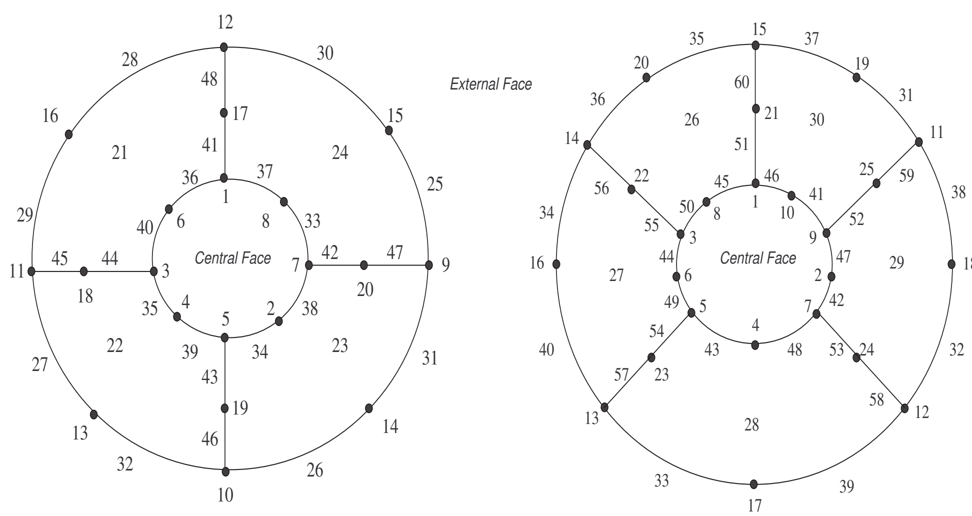

Figure 2.1: The graph $G_{1,4}$ Figure 2.2: The graph $G_{1,5}$

Figure 2. $SFMT$-Labeling for $G_{1,4}$ *and* $G_{1,5}$.

3. Concluding remarks

In this paper, we examined $SFMT$-labeling of graphs $(G_{r,t})$ derived from a subdivided prism. We found that such graphs accept $SFMT$-labeling for $t \geq 4$, $r = 1$ and $t = 3$, $r \geq 1$.

Acknowledgement

The authors would like to express their sincere gratitude to the referees and the editor for their careful reading of this article and for all their insightful comments, which leads a number of improvements to this paper.

References

[1] W. D. Wallis, *Magic Graphs*, Birkhäuser, Boston - Basel - Berlin, 2001

[2] D. B. West, *An Introduction to Graph Theory*, Prentice-Hall, 1996

[3] K. Lih, On magic and consecutive labelings of plane graphs, *Utilitas Math* **24** (1983), 165-197

[4] M. Bača, On magic labelings of honeycomb, *Discrete Math* **105** (1992), 305-311

[5] K. Ali, M. Hussain, A. Ahmad, M. Miller, Magic labelings of type (a,b,c) of families of wheels, *Math. Computer Sci.* **7** (2013), 315-319

[6] M. Bača, On magic labelings of grid graphs, *Ars Combinatoria* **33** (1992), 295-299

[7] S. Ahmad, On the evaluation of a subdivision of the ladder graph, *Punjab Univ. J. Math.* **47**(1) (2015), 15-19

[8] K. M. Kathiresan, S. Gokulakrishnan, On magic labelings of type (1,1,1) for the special classes of plane graphs, *Utilitas Math.* **63** (2003), 25-32

[9] M. Bača, Y. Lin, M. Miller, Antimagic labelings of grids, *Utilitas Math.* **72** (2007), 65-75

[10] M. Bača, S. Jendroľ, M. Miller, J. Ryan, Antimagic labelings of generalized Petersen graphs that are plane, *Ars Combin.* **73** (2004), 115-128

[11] M. Bača, E. T. Baskoro, S. Jendroľ, M. Miller, Antimagic labelings of hexagonal planar maps, *Utilitas Math.* **66** (2004), 231-238

[12] Y. Lin, Slamin, M. Bača, M. Miller, On d-antimagic labelings of prisms, *Ars Combin.* **72** (2004), 65-76

[13] K. A. Sugeng, M. Miller, Y. Lin, M. Bača, Face antimagic labelings of prisms, *Utilitas Math.* **71** (2006), 269-286

[14] M. Bača, F. Bashir, A. Semaničovečovčvace antimagic labelings of antiprisms, *Utilitas Math.* **84** (2011), 209-224

[15] G. Ali, M. Bača, F. Bashir, A. Semaničovečovčvn face antimagic labelings of disjoint union of prisms, *Utilitas Math.* **85** (2011), 97-112

(1) Faculty of Mathematics and Statistics, Central China Normal University, Wuhan 430079, P.R. China

Email address: zaman.ravian@@gmail.com

(2) (*Corresponding author) Department of Mathematics, Govt. Degree College Kahuta Haveli , Azad Kashmir 12508, Pakistan

Email address: mjalam@@live.com

(3) Department of Mathematics, University of Management and Technology, Sialkot 51310, Pakistan

Email address: asmagcwu@@gmail.com