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ON NEARLY COMPACT SPACES VIA PRE-OPEN SETS

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Abstract. Recently, the first author of this paper in a collaborative research work redefined the concept of nearly compact spaces by pre-open sets and obtained some new properties on nearly compact spaces when they are studied from this new perspective. In this paper, we continue to study the idea of near compactness via pre-open sets and obtain some new characterizations on nearly compact spaces.

1. Introduction

Let X be a nonempty set and $\mathscr{T}$ be a topology on X. According to usual convention, we write X to denote the topological space $(X, \mathscr{T})$. Unless otherwise mentioned, for any subset A of X, the closure (resp. interior) of A is denoted by $Cl(A)$ (resp. Int(A)).

In the literature of topological spaces, we find several generalizations of open sets of topological spaces. One among such generalizations of open sets is locally dense sets in topological spaces introduced by Corson and Michael [3]: a subset A of a topological space X is called locally dense if there exists an open set U such that $A \subset U \subset Cl(A)$. However, locally dense sets of topological spaces widely studied after the name pre-open sets coined by Mashhour et al. [8]. The complement of a pre-open set of a topological space X is called pre-closed in X. It is easy to follow that a subset A of X is pre-open if $A \subset Int(Cl(A))$ and is pre-closed if $Cl(Int(A)) \subset A$.

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Note that a subset A of X is regular open [11, p. 29] (resp. regular closed) if $A = Int(Cl(A)$ (resp. $A = Cl(Int(A))$). The complement of a regular open set is regular closed and conversely.

We recall that the intersection of all pre-closed sets containing a subset A of X is called the pre-closure [4, 9] of A and it is denoted by $pCl(A)$. Similarly, the union of all pre-open sets contain in a subset A of X is called the pre-interior [9] of A and it is denoted by $pInt(A)$.

We also write $PO(X)$ (resp. $PC(X)$) to denote the collection of all pre-open (resp. pre-closed) sets of X. Throughout the paper, $\mathbb{R}$ stands for the set of real numbers and k, l, m, n etc. stand to denote the natural numbers.

2. Definitions and Results

Firstly, we recall some known notions and results to make the article self-sufficient as far as possible.

Theorem 2.1 (Mashhour et al. [9]). *The intersection of two pre-open sets in a topological space X is pre-open if the closure in X is preserved under finite intersection property.*

The closure operator in a topological space is preserved under finite intersection property if for any two subsets A, B of X with $A \cap B \neq \emptyset$, we have $Cl(A) \cap Cl(B) = Cl(A \cap B)$.

Definition 2.1 (Kar and Bhattacharyya [6]). A topological space X is called pre-T_2 if for each pair of distinct points x and y in X, there exist pre-open sets U and V such that $x \in U, y \in V$ and $U \cap V = \emptyset$.

Definition 2.2 (Jafari [7]). A subset A of a topological space X is called pre-regular p-open (resp. pre-regular p-closed) if $A = pInt(pCl(A))$ (resp. $A = pCl(pInt(A))$).

It follows that the complement of a pre-regular p-open set in X is pre-regular p-closed and conversely.

Theorem 2.2 (Jafari [7]). *If $A \in PO(X)$, then $pInt(pCl(A))$ is pre-regular p-open in X and if $A \in PC(X)$, $pCl(pInt(A))$ is pre-regular p-closed in X.*

Theorem 2.3 (Andrijević [1]). *For any subset A of a topological space X, $pInt(A) = A \cap Int(Cl(A))$ and $pCl(A) = A \cup Cl(Int(A))$.*

Definition 2.3 (Singal and Mathur [10]). A topological space X is called nearly compact if for each open cover $\mathscr{U}$ of X, there exists a finite subcollection $\mathscr{V} \subset \mathscr{U}$ such that $\bigcup\{Int(Cl(V)) \mid V \in \mathscr{V}\} = X$.

However, the notion of near compactness is redefined via pre-open sets by Bagchi et al. [2] follows below. Firstly, recall that a collection of subsets of X is pre-open collection if the collection consists of pre-open sets of X only. A pre-open collection $\mathscr{A}$ of X is a pre-open cover of X if $\bigcup_{A\in\mathscr{A}} A = X$. Then the notions of 'pre-closed collections', 'pre-regular p-open covers', 'pre-regular p-closed collections' etc. are apparent.

Definition 2.4 (Bagchi et al. [2]). Let $\mathscr{S}$ be a pre-open collection of X. For each $A \in \mathscr{S}$, there exists an open set U such that $A \subset U \subset Cl(A)$. We define $\mathscr{U} = \{U \mid A \in \mathscr{S}, A \subset U \subset Cl(A)\}$. Then $\mathscr{U}$ is said to be an 'open super-collection' of $\mathscr{S}$.

We see that there always exists an open super-collection corresponding to a pre-open collection of a topological space X. Also note that $\mathscr{U}$ is a cover of X if $\mathscr{S}$ is a cover of X. If it is, $\mathscr{U}$ is called an open super-cover of the pre-open cover $\mathscr{S}$. Again, if there exists a finite subcollection $\mathscr{V}$ of $\mathscr{U}$ such that $\{G \mid G \in \mathscr{V}\}$ covers X then $\mathscr{V}$ is called a finite open super-cover of $\mathscr{S}$.

Definition 2.5 (Bagchi et al. [2])**.** A topological space X is said to be po-compact if each pre-open cover of X has a finite open super-cover.

We note that the notions of po-compactness and near compactness are equivalent (Theorem 2.1, [2]). So in this paper also as of [2], we study po-compact spaces after the name nearly compact spaces i.e., we study nearly compact spaces via pre-open sets of topological spaces.

Theorem 2.4 (Bagchi et al. [2])**.** *A topological space X is nearly compact if and only if each pre-open cover $\mathscr{A}$ of X has a finite subcollection $\mathscr{B}$ such that $\{Int(Cl(B)) \mid B \in \mathscr{B}\}$ covers X.*

We introduce the following idea on pre-open sets.

Definition 2.6. A subset A of X is said to be lightly pre-open if $A \subset G \subset pCl(A)$ for all open sets G of X satisfying $A \subset G \subset Cl(A)$.

We see that lightly pre-open sets are pre-open but converse need not be true.

Example 2.1 (Jun et al. [5])**.** *Let $X = \{a, b, c, d, e\}$ and $\mathscr{T} = \{\emptyset, X, \{a\}, \{c, d\}, \{a, c, d\}\}$. The pre-open sets of the topological space $(X, \mathscr{T})$ are $\mathscr{T} \cup \{\{c\}, \{d\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, e\}, \{a, d, e\}, \{a, b, c, d\}, \{a, b, c, e\}, \{a, b, d, e\}, \{a, c, d, e\}\}$. We see that $\{c\}$ is a pre-open set in X and $Cl(\{c\}) = \{b, c, d, e\}$, $pCl(\{c\}) = \{c\}$. Also $\{c, d\}$ is the only open set in X such that $\{c\} \subset \{c, d\} \subset Cl(\{c\})$. So there does not exist an open set G in X such that $\{c\} \subset G \subset pCl(\{c\})$. Hence $\{c\}$ is not lightly pre-open in X.*

Example 2.2. *Let $X = \{a, b, c\}$ and $\mathscr{T} = \{\emptyset, X, \{a\}\}$. In the topological space $(X, \mathscr{T})$, $PO(X) = \{\emptyset, X, \{a\}, \{a, b\}, \{c, a\}\}$ and all pre-open sets are lightly pre-open. We also see that $\mathscr{T} \neq PO(X)$.*

In view of Example 2.2, we conclude that even if all pre-open sets in a topological space $(X, \mathscr{T})$ are lightly pre-open, $\mathscr{T} = PO(X)$ may not hold.

Theorem 2.5. *A topological space X is nearly compact if each pre-regular p-open cover of X has a finite subcover.*

Proof. Let $\mathscr{U} = \{U_\alpha \mid \alpha \in A\}$ be a pre-open cover of X. By Theorem 2.2, we see that $\mathscr{V} = \{pInt(pCl(U_\alpha)) \mid \alpha \in A\}$ is a pre-regular p-open cover of X. It yields a finite subcover $\{pInt(pCl(U_{\alpha_k})) \mid \alpha_k \in A, k \in \{1, 2, \ldots, m\}\}$ of $\mathscr{V}$. By Theorem 2.3, we get $pInt(pCl(U_{\alpha_k})) = pCl(U_{\alpha_k}) \cap Int(Cl(pCl(U_{\alpha_k}))) \subset Int(Cl(pCl(U_{\alpha_k})))$ which in turn implies $U_{\alpha_k} \subset pInt(pCl(U_{\alpha_k})) \subset Int(Cl(U_{\alpha_k})) \subset Cl(U_{\alpha_k})$ for each $k \in \{1, 2, \ldots, m\}$. So $\{Int(Cl(U_{\alpha_k})) \mid \alpha_k \in A, k \in \{1, 2, \ldots, m\}\}$ is a finite open super-cover of $\mathscr{U}$. □

Example 2.3. *Let b be a fixed real number. We define $\mathscr{T} = \{\emptyset, \mathbb{R}, (-\infty, b], (b, \infty)\}$. The topological space $(\mathbb{R}, \mathscr{T})$ is compact and hence nearly compact. For any $x \in \mathbb{R}$, both $\{x\}$ and $\mathbb{R} - \{x\}$ are pre-open. As $pInt(pCl(\{x\})) = \{x\}$, $\{x\}$ is pre-regular p-open in $(\mathbb{R}, \mathscr{T})$. The pre-regular p-open cover $\{\{x\} \mid x \in \mathbb{R}\}$ of $(\mathbb{R}, \mathscr{T})$ has no finite subcover. So we see that the converse of Theorem 2.5 need not be true.*

However, we have the following characterizations of nearly compact spaces.

Theorem 2.6. *If each pre-open set of a topological space X is lightly pre-open, then the following statements are equivalent:*

(i) X is nearly compact.

(ii) Each pre-regular p-open cover of X has a finite subcover.

(iii) Each collection of pre-regular p-closed sets of X with finite intersection property have the nonempty intersection.

(iv) Let $\mathscr{F}$ be any collection of pre-closed sets of X with the following property: for any finite subcollection $\mathscr{E}$ of $\mathscr{F}$, $\bigcap_{E\in\mathscr{E}} pCl(pInt(E)) \neq \emptyset$. Then $\bigcap_{F\in\mathscr{F}} F \neq \emptyset$.

Proof. (i)⇒(ii): Let $\mathscr{U} = \{U_\alpha \mid \alpha \in A\}$ be a pre-regular p-open cover of X. Since a pre-regular p-open set in X is also a pre-open set in X, $\mathscr{U}$ is a pre-open cover of X. So we obtain a finite open super-cover $\{G_{\alpha_k} \mid k \in \{1, 2, \ldots, n\}\}$ of $\mathscr{U}$. For each $k \in \{1, 2, \ldots, n\}$, there exists a pre-open set U_{α_k} such that $U_{\alpha_k} \subset G_{\alpha_k} \subset pCl(U_{\alpha_k}) \subset Cl(U_{\alpha_k})$. Since the open sets of X are also pre-open, we see that $U_{\alpha_k} \subset G_{\alpha_k} \subset pInt(pCl(U_{\alpha_k}))$. By Theorem 2.2, $pInt(pCl(U_{\alpha_k}))$ is pre-regular p-open for each $k \in \{1, 2, \ldots, n\}$. So it follows that $\{pInt(pCl(U_{\alpha_k})) \mid k \in \{1, 2, \ldots, n\}\}$ is a finite pre-regular p-open subcover of $\mathscr{U}$.

(ii)⇒(iii): We suppose that $\mathscr{F} = \{F_\alpha \mid \alpha \in \Delta\}$ is a collection of pre-regular p-closed sets of X such that the intersection of any finitely many members of $\mathscr{F}$ is nonempty. If possible, let $\bigcap_{\alpha\in\Delta} F_\alpha = \emptyset$. Then $\mathscr{G} = \{X - F_\alpha \mid \alpha \in \Delta\}$ is a pre-regular p-open cover of X. So by (ii), there exists a finite subcover $\{X - F_{\alpha_k} \mid \alpha_k \in \Delta, k \in \{1, 2, \ldots, m\}\}$ of $\mathscr{G}$. Hence we have $\bigcap_{k=1}^{m} F_{\alpha_k} = \emptyset$, a contradiction to our assumption.

(iii)⇒(iv): We see that for any $F \in \mathscr{F}$, $pCl(pInt(F))$ is pre-regular p-closed with $pCl(pInt(F)) \neq \emptyset$. So $\{pCl(pInt(F)) \mid F \in \mathscr{F}\}$ is a collection of pre-regular p-closed sets having the property of (iii). Our conclusion by *(iii)* is $\bigcap_{F\in\mathscr{F}} pCl(pInt(F)) \neq \emptyset$. Since each $F \in \mathscr{F}$ is pre-closed, we have $pCl(pInt(F)) \subset F$. It means that $\bigcap_{F\in\mathscr{F}} F \neq \emptyset$.

(iv)⇒(i): Let $\mathscr{G} = \{G_\alpha \mid \alpha \in \Delta\}$ be any pre-open cover of X. We have to show that there exists a finite open super-cover of $\mathscr{G}$. In particular, it suffices to show that there exists a finite subcollection $\mathscr{H}$ of $\mathscr{G}$ such that $\bigcup_{H\in\mathscr{H}} Int(Cl(H)) = X$ according to Theorem 2.4. If possible, let for any finite subcollection $\mathscr{H}$ of $\mathscr{G}$, $\bigcup_{H\in\mathscr{H}} Int(Cl(H)) \neq X$. Then

$$\bigcap_{H\in\mathscr{H}} Cl(Int(X - H)) \neq \emptyset \tag{2.1}$$

for any finite subcollection $\{X - H \mid H \in \mathscr{H}\}$ of the family of pre-closed sets $\{X - G_\alpha \mid \alpha \in \Delta\}$. For any $\alpha \in \Delta$,

$$\begin{aligned} pCl(pInt(X - G_\alpha)) &= pInt(X - G_\alpha) \cup Cl(Int(pInt((X - G_\alpha))) \\ &\qquad\qquad \text{(by Theorem 2.3)} \\ &\supset Cl(Int(pInt((X - G_\alpha))) \\ &\supset Cl(Int(X - G_\alpha)) \text{ since } pInt((X - G_\alpha)) \supset Int((X - G_\alpha)). \end{aligned}$$

So from (2.1), we get $\bigcap_{H \in \mathscr{H}} pCl(pInt(X - H)) \neq \emptyset$. It means that $\{X - G_\alpha \mid \alpha \in \Delta\}$ is a collection of pre-closed sets satisfying the hypothesis of (iv). So by (iv), we get $\bigcap_{\alpha \in \Delta}(X - G_\alpha) \neq \emptyset$ which in turn implies that $\bigcup_{\alpha \in \Delta} G_\alpha \neq X$. It is a contradiction to our assumption that $\mathscr{G}$ is a cover of X. □

The all pre-open sets in the topological space $(\mathbb{R}, \mathscr{T})$ of Example 2.3 are not lightly pre-open. That is why, the each pre-regular p-open cover of the topological space of Example 2.3 does not have a finite subcover. It means that the condition of Theorem 2.6 is essential.

Theorem 2.7. *If A is any pre-regular p-closed set of X and each cover $\mathscr{C}$ of A by pre-open sets of X has a finite subcollection $\mathscr{E}$ such that $A \subset \bigcup_{E \in \mathscr{E}} pInt(pCl(E))$, then X is nearly compact.*

Proof. Let $\mathscr{G} = \{G_\gamma \mid \gamma \in \Gamma\}$ be a pre-open cover of X. As $\mathscr{G}$ is a cover of X, there exists $A \in \mathscr{G}$ such that $A \neq \emptyset$. By Theorem 2.2, $pInt(pCl(A))$ is pre-regular p-open in X and so $X - pInt(pCl(A))$ is pre-regular p-closed in X. By the assumption, we get a finite subcollection $\{G_{\gamma_k} \mid \gamma_k \in \Gamma, k \in \{1, 2, \ldots, n\}\}$ such that $X - pInt(pCl(A)) \subset \bigcup_{k=1}^{n} pInt(pCl(G_{\gamma_k}))$ which in turn implies $X = \bigcup_{k=1}^{n}(pInt(pCl(G_{\alpha_k})))\cup pInt(pCl(A))$. As of Theorem 2.5, $pInt(pCl(A)) \subset Int(Cl(A))$. So $\{G_{\gamma_k} \mid \gamma_k \in \Gamma, k \in \{1, 2, \ldots, n\}\} \cup \{A\}$ is a finite subcollection of $\mathscr{G}$ such that

$X = \bigcup_{k=1}^{n}(Int(Cl(G_{\alpha_k}))) \cup Int(Cl(A))$. Hence X is nearly compact by Theorem 2.4. □

Theorem 2.8. *Suppose X is a nearly compact space such that each pre-open sets of X is lightly pre-open. Then each cover $\mathscr{B}$ by pre-open sets of X of a pre-regular p-closed set A of X has a finite subcollection $\mathscr{C}$ such that $A \subset \bigcup_{C\in\mathscr{C}} pInt(pCl(C))$.*

Proof. For each $B \in \mathscr{B}$, $pInt(pCl(B))$ is pre-regular p-open in X. So it follows that $\mathscr{D} = \{pInt(pCl(B)) \mid B \in \mathscr{B}\} \cup \{X - A\}$ is a pre-regular p-open cover of X. Since X is nearly compact, there exists a finite subcover $\mathscr{E}$ of $\mathscr{D}$ by Theorem 2.6. We put $\mathscr{F} = \mathscr{E} - \{X - A\}$. Then $\mathscr{F}$ is a subcollectin of $\{pInt(pCl(B)) \mid B \in \mathscr{B}\}$ such that $A \subset \bigcup_{F\in\mathscr{F}} F$. Since the collection $\{pInt(pCl(B)) \mid B \in \mathscr{B}\}$ is constructed by the members of $\mathscr{B}$ and $\mathscr{F}$ is finite, we have a finite subcollection $\mathscr{C}$ of $\mathscr{B}$ such that $\mathscr{F} = \{pInt(pCl(C)) \mid C \in \mathscr{C}\}$. So $\mathscr{C}$ is a finite subcollection of $\mathscr{B}$ such that $A \subset \bigcup_{C\in\mathscr{C}} pInt(pCl(C))$. □

Theorem 2.9. *Each cover $\mathscr{C}$ by pre-open sets of X of a regular closed set A of a nearly compact space X has a finite subcollection $\mathscr{D}$ such that $\{Int(Cl(D)) \mid D \in \mathscr{D}\}$ is a cover of A.*

Proof. We see that $\mathscr{C} \cup \{X - A\}$ is a pre-open cover of X. By Theorem 2.4, there exists a finite subcollection $\mathscr{E}$ of $\mathscr{C} \cup \{X - A\}$ such that $\bigcup_{C\in\mathscr{E}} Int(Cl(C)) = X$. Since A is regular closed, $X - A = Int(Cl(X - A))$. So $\mathscr{E} - \{X - A\}$ is our required $\mathscr{D}$. □

Definition 2.7. A subset A of a topological space X is said to be pN-closed relative to X if for any cover of A by pre-open sets of X has a finite open super-cover.

Theorem 2.10. *A pN-closed set relative to a pre-T_2 topological space X is pre-closed if closure in X is preserved under finite intersection and pre-open sets in X are lightly pre-open.*

Proof. Let A be pN-closed in X. We choose $y \notin A$. Since X is pre-T_2, for each $x \in A$, there exist pre-open sets U_x, V_x of X such that $x \in U_x, y \in V_x$ and $U_x \cap V_x = \emptyset$. Then $\mathscr{U} = \{U_x \mid x \in A\}$ is a cover of A by pre-open sets of X. As A is pN-closed relative to X, there exists a finite open super-cover $\mathscr{G}_n = \{G_{x_1}, G_{x_2}, \ldots, G_{x_n}\}$ of $\mathscr{U}$ such that $A \subset \bigcup_{k=1}^{n} G_{x_k}$. For each $k \in \{1, 2, \ldots, n\}$, we also have a $U_{x_k} \in \mathscr{U}$ such that $U_{x_k} \subset G_{x_k} \subset Cl(U_{x_k})$. We also see that $U_{x_k} \cap V_{x_k} = \emptyset$. We put $H = \bigcap_{k=1}^{n} V_{x_k}$ which is pre-open by Theorem 2.1. We now show that $H \cap A = \emptyset$. If possible, let $z \in H \cap A$. Then $z \in V_{x_k}$ for each $k \in \{1, 2, \ldots, n\}$ and $z \in G_{x_k}$ for some $k \in \{1, 2, \ldots, n\}$. Let $z \in G_{x_k}$ for $k = l$. So $V_{x_l} \cap G_{x_l} \neq \emptyset$. From $U_{x_l} \subset G_{x_l} \subset Cl(U_{x_l})$, we get $U_{x_l} \subset G_{x_l} \subset pCl(U_{x_l}) \subset Cl(U_{x_l})$ and from $U_{x_l} \cap V_{x_l} = \emptyset$, we have $pCl(U_{x_l}) \cap V_{x_l} = \emptyset$. Since $G_{x_l} \subset pCl(U_{x_l})$, we conclude that $G_{x_l} \cap V_{x_l} = \emptyset$ which is a contradiction to the fact $V_{x_l} \cap G_{x_l} \neq \emptyset$. Thus we find $y \in H \subset X - A$. It means that $X - A$ is a pre-open set and so A is a pre-closed set in X. □

Example 2.4. *Let $X = \{a, b, c\}$ and $\mathscr{T} = \{\emptyset, X, \{a, b\}\}$. In the topological space $(X, \mathscr{T})$, $PO(X) = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{c, a\}, \{b, c\}\}$. The topological space $(X, \mathscr{T})$ is pre-T_2, closure of subsets of X are preserved under finite intersection but all pre-open sets are not lightly pre-open. We also see that $\{a, b\}$ is pN-closed in X but $\{a, b\}$ is not pre-closed.*

We now consider Example 2.2. All the pre-open sets of the space are lightly pre-open, the closure of subsets of X is preserved under finite intersection. We see that $\{c, a\}$ is pN-closed relative to the topological space X. But $\{c, a\}$ is not pre-closed. It happens as the topological space X is not *pre*-T_2.

Due to above Examples, we conclude that the conditions in Theorem 2.10 can not be dropped.

Now we obtain a preservation theorem on nearly compact spaces. For it, we suppose that $(X, \mathscr{T})$ and $(Y, \mathscr{P})$ are two topological spaces, and $f : X \to Y$ is a singled valued function.

Definition 2.8 (Mashhour et al. [9])**.** A mapping $f : X \to Y$ is called a precontinuous (resp. M-precontinuous) function if the inverse image of each open (resp. pre-open) set in Y is pre-open in X.

Theorem 2.11 (Mashhour et al. [9])**.** *A precontinuous and open function is an M-precontinuous function.*

In the following theorem, we write $Cl_X(A)$ to denote the closure of $A \subset X$.

Theorem 2.12. *Let there exist an open, continuous and precontinuous function $f : X \to Y$ such that $Y = f(X)$. Then Y is nearly compact if X is a nearly compact space.*

Proof. Let $\mathscr{A} = \{A_\alpha \mid \alpha \in \Delta\}$ be a pre-open cover of X. For each $A \in \mathscr{A}$, $f^{-1}(A)$ is pre-open in X as f is M-precontinuous by Theorem 2.11. Then it follows that $\{f^{-1}(A) \mid A \in \mathscr{A}\}$ is a pre-open cover of X. By the near compactness of X, we obtain a finite open-super cover $\{G_1, G_2, \ldots, G_n\}$ of $\{f^{-1}(A) \mid A \in \mathscr{A}\}$. For each $k \in \{1, 2, \ldots, n\}$, there is an $\alpha_k \in \Delta$ such that $f^{-1}(A_{\alpha_k}) \subset G_k \subset Cl_X(f^{-1}(A_{\alpha_k})) \subset f^{-1}(Cl_Y(A_{\alpha_k}))$ by the continuity of f. It implies that $A_{\alpha_k} \subset f(G_k) \subset Cl_Y(A_{\alpha_k})$. Since f is open, $\{f(G_k) \mid k \in \{1, 2, \ldots, n\}$ is an open super-collection of the pre-open cover $\mathscr{A}$. Finally, $Y = f(X) = f(\bigcup_{k=1}^n G_k) = \bigcup_{k=1}^n f(G_k)$. So we get a finite open super-cover $\{f(G_k) \mid k \in \{1, 2, \ldots, n\}$ of the pre-open cover $\mathscr{A}$. □

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