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LOCAL PROPERTIES OF THE TOTAL GRAPH $T(\Gamma(\mathbb{Z}_n))$

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Abstract. Let R be a commutative ring with unity. The total graph of R, $T(\Gamma(R))$, is the simple graph with vertex set R and two distinct vertices x and y are adjacent if $x+y \in Z(R)$, where $Z(R)$ is the set of all zero divisors of R. This paper presents a study of some local properties of the graph $T(\Gamma(\mathbb{Z}_n))$. We answer the question " when is $T(\Gamma(\mathbb{Z}_n))$ locally connected?". We also prove that the neighborhoods of any two distinct vertices in $T(\Gamma(\mathbb{Z}_n))$ induce isomorphic graphs if and only if n is even.

1. Introduction

In this paper, R will be used to denote a commutative ring with a non-zero unity. Let $Z(R)$ be the set of all zero-divisors of R. The total graph of R, $T(\Gamma(R))$, is the simple graph with vertex set R, where two distinct vertices x and y are adjacent if $x+y \in Z(R)$. This concept was introduced by Anderson and Badawi [1] in 2008. For a survey on the total graph of a commutative ring, the reader may refer to [7].

The graphs $Z(\Gamma(R))$ and $\mathrm{Reg}(\Gamma(R))$ are defined to be the subgraphs of $T(\Gamma(R))$ that are induced by the set of zero divisors, $Z(R)$, and the set of regular elements, $\mathrm{Reg}(R)$, of R respectively. The following theorem describes the graph $T(\Gamma(R))$ when $Z(R)$ is an ideal of R.

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Theorem 1.1. ([1, Theorem 2.1,Theorem 2.2, Theorem 2.4]) *Let R be a ring such that $Z(R)$ is an ideal of R. Let $|Z(R)| = \lambda$, $|R/Z(R)| = \mu$.*

(i) *If $2 \in Z(R)$, then* $\text{Reg}(\Gamma(R))$ *is the union of $\mu - 1$ disjoint $K_\lambda{}'s$.*

(ii) *If $2 \notin Z(R)$, then* $\text{Reg}(\Gamma(R))$ *is the union of $(\mu - 1)/2$ disjoint $K_{\lambda,\lambda}{}'s$.*

(iii) *$Z(\Gamma(R))$ is the complete graph, K_λ.*

(iv) $\text{Reg}(\Gamma(R))$ *is connected if and only if $R/Z(R) \cong \mathbb{Z}_2$ or $R/Z(R) \cong \mathbb{Z}_3$.*

The total graph for the ring of integers modulo n, $\mathbb{Z}_n$ is investigated in both [2] and [3]. In [2], the authors answer the question " when is $T(\Gamma(\mathbb{Z}_n))$ regular, Eulerian, or self-centered?" they also computed the independence number and the clique number of $T(\Gamma(\mathbb{Z}_n))$. In [3], it is proved that the domination number of $T(\Gamma(\mathbb{Z}_n)) = p$, where p is the smallest prime divisor of n. In the same paper, a characterization of all γ-sets in $T(\Gamma(\mathbb{Z}_n))$ is given. Furthermore, the total and perfect domination numbers of $T(\Gamma(\mathbb{Z}_n))$ are determined. Let G be a graph with vertex set $V(G)$ and edge set $E(G)$. For any vertex $v \in V(G)$, the open neighborhood of v, denoted by $N(v)$, is defined by $N(v) = \{u \in V(G) \mid uv \in E(G)\}$. If the open neighborhood of each vertex in G satisfies a certain property P, then G is called locally P. Local properties of graphs were the subject of study for many authors. For survey articles on this topic, see [8] and [9]. The study of local properties of graphs was motivated by this question: "For what graphs H is there a graph G such that neighborhood of each vertex in G induces a graph which is isomorphic to H ?". This question was raised by Zykov [11]. In his book, Some unsolved problems in the graph theory, Vizing listed this question as one of the most important problems in combinatorics. A graph G is said to be locally connected if for all $v \in V(G)$, $N(v)$ induces a connected subgraph of G. A graph is called locally homogeneous if the graph induced by the neighborhoods of any two vertices are isomorphic. Let H be a given graph. A graph G is called locally H if for each vertex $v \in V(G)$, the subgraph induced by the set of neighbors of v,

$N(v)$, is isomorphic to H. Locally H graphs are also called locally homogeneous [10]. Graphs which have certain symmetrical properties are of special interest. Amongst those graphs, are graphs associated with algebraic structures. Vince [10] provided a method for constructing locally homogeneous graphs from groups. It is natural then to investigate local homogeneity in the total graphs associated with rings. In this paper, first, we will consider local connectedness in $T(\Gamma(\mathbb{Z}_n))$. Then, we study local homogeneity in each of $T(\Gamma(\mathbb{Z}_n))$, $Z(\Gamma(R))$, and $\mathrm{Reg}(\Gamma(R))$.

2. Main Results

The complete graph and the complete bipartite graph are two well-known graphs that play a crucial role in the study of the structure of total graphs of commutative rings. Obviously, the complete graph K_n is locally connected, the complete bipartite graph is not locally connected. Thus, if G is a union of complete graphs, then G is locally connected. If a graph G has a bipartite component, other than $K_{1,1}$, then it is not locally connected. We address the problem "when is $T(\Gamma(\mathbb{Z}_n))$ locally connected?". The proof shows the interplay between graph theory, number theory, and ring theory. It also sheds light on the structure of $T(\Gamma(\mathbb{Z}_n))$; this graph will be viewed as the overlay of layers of graphs. Each layer is a spanning subgraph of $T(\Gamma(\mathbb{Z}_n))$. The components of each layer are either complete graphs or complete bipartite graphs.

Before proceeding, we recall the following two classic theorems from number theory that could be found in [6].

Theorem 2.1. ([6, Lemma 7.2, p 327]) *Let p be a prime and a any integer such that p does not divide a, Then the least residues of the integers $a, 2a, 3a, \ldots, (p-1)a$ modulo p are permutation of the integers $1, 2, 3, \ldots, p-1$.*

Theorem 2.2. *(The Chinese Remainder Theorem)* ([6, Theorm 6.1, p 297]) *The linear system of congruences $x \equiv a_i \pmod{m_i}$, where the moduli are pairwise relatively prime and $1 \le i \le k$, has a unique solution modulo $m_1 m_2 \dots m_k$.*

To understand the structure of $T(\Gamma(\mathbb{Z}_n))$, $n = p_1^{r_1} p_2^{r_2} \dots p_m^{r_m}$, where $p_i's$ are all distinct primes, and $r_i's$ are positive integers. Note that $Z(\mathbb{Z}_n) = \bigcup_{i=1}^{m} < p_i >$ where for each $i \in \{1, 2, \dots, m\}$, $< p_i >$ is the ideal generated by p_i . Let H_i be the spanning subgraph of $T(\Gamma(\mathbb{Z}_n))$ where two distinct vertices x and y are adjacent in H_i if $x + y \in < p_i >$. Thus $E(T(\Gamma(\mathbb{Z}_n))) = \bigcup_{i=1}^{m} E(H_i)$. Let $x \in \mathbb{Z}_n$, suppose that for each $i \in \{1, 2, \dots, m\}$, $x \equiv a_i \pmod{p_i}$, where $0 \le a_i \le p_i - 1$, and that $a_i + b_i \equiv 0 \pmod{p_i}$, then $N(x) = \bigcup_{i=1}^{m} (b_i + < p_i >)$. Let y be in $\mathbb{Z}_n$, assume that $y \equiv c_i \pmod{p_i}$, and $c_i + d_i \equiv 0 \pmod{p_i}$, where $i \in \{1, 2, \dots, m\}$ and $0 \le c_i \le p_i - 1$. We claim that there exists $z \in \mathbb{Z}_n$ such that $z \in N(x) \cap N(y)$. If such z exists, then z must satisfy

$z \equiv b_j \pmod{p_j}$ for some j and,

$z \equiv d_k \pmod{p_k}$ for some k, where $1 \le j, k \le m$.

If $j = k$, then, this system reduces to one congruence $z \equiv b_j \pmod{p_j}$ for some j thus, the solution set is the coset $b_j + < p_j >$. Now, assume that $j \ne k$. By the Chinese remainder theorem, this system of linear congruences has a unique solution modulo $p_j p_k$ since p_j and p_k are primes and hence, relatively prime. This means that the solution set is precisely an equivalence class modulo $p_j p_k$ and has exactly $n/p_j p_k$ elements. Thus a coset of $< p_1 >$ and a coset of $< p_2 >$ have exactly $p_1^{r_1 - 1} p_2^{r_2 - 1} p_3^{r_3} \dots p_m^{r_m}$ elements in common. This leads to the following lemma.

Lemma 2.1. *Let R be the ring of integers modulo n, let $n = p_1^{r_1} p_2^{r_2} \dots p_m^{r_m}$, where $p_i's$ are all distinct primes, and $r_i's$ are positive integers. Then the intersection of a coset of the ideal $< p_i >$ and a coset of the ideal $< p_j >$ has exactly $n/(p_i p_j)$ elements.*

In this paper, for $x \in \mathbb{Z}_n$, the subgraph of $T(\Gamma(\mathbb{Z}_n))$, induced by $N(x)$, will be denoted by $G_T[N(x)]$.

Theorem 2.3. (i) *If $n = p^r$, where p is a prime, and r is a positive integer, then $T(\Gamma(\mathbb{Z}_n))$ is locally connected if and only if either $p = 2$ or p is an odd prime and $r = 1$,*

(ii) *if $n = p_1p_2$, where p_1 and p_2 are two distinct primes, then $T(\Gamma(\mathbb{Z}_n))$ is not locally connected,*

(iii) *if $n = p_1^{r_1}p_2^{r_2}$ where p_1, p_2 are distinct primes and r_1, r_2 are positive integers such that either $r_1 \geq 2$ or $r_2 \geq 2$, then $T(\Gamma(\mathbb{Z}_n))$ is locally connected,*

(iv) *if $n = p_1^{r_1}p_2^{r_2}\dots p_m^{r_m}, m \geq 3$ where p_1, p_2,..., p_m are distinct primes and r_1, r_2,..., r_m are positive integers, then $T(\Gamma(\mathbb{Z}_n))$ is locally connected.*

Proof. (i) If $n = p^r$, where p is a prime, then $Z(\mathbb{Z}_n)$ is an ideal. So, the result follows by Theorem 1.1.

(ii) If $n = 2p$ where p is an odd prime, then $N(1) = 1+ < 2 > \cup\{p - 1\}$. Note that $p - 1$ is an isolated vertex in $N(1)$, so $N(1)$ is not connected. Hence, $T(\Gamma(\mathbb{Z}_{2p)})$ is not locally connected. Now, we may assume that $n = p_1p_2$, where p_1 and p_2 are distinct odd primes. To show that $T(\Gamma(\mathbb{Z}_n))$ is not locally connected, it suffices to find a vertex x and two distinct nonadjacent vertices, y and z in the neighborhood of x with no path joining y to z in $G_T[N(x)]$. Choose $x = p_1$, assume $x \equiv a_2 \pmod{p_2}$ and $a_2 + b_2 \equiv 0 \pmod{p_2}$. Take $y \in b_2+ < p_2 >$, and assume $y \in c+ < p_1 >$, where $y \neq x$. Clearly, $y \in N(x)$. Let z be any vertex in the neighborhood of x which is different from y. Suppose that $w_1 - w_2 - \cdots - w_k$ is a path in $G_R[N(x)]$ joining y to z. Since $w_1 \in N(x) \cap N(y)$. Then $w_1 \in \{(< p_1 > \backslash p_1) \cup (b_2+ < p_2 >)\} \cap \{(p_1 - c+ < p_1 >) \cup (a_2+ < p_2 >)\} = \{(< p_1 > \backslash p_1) \cap (a_2+ < p_2 >)\} \cup \{(p_1 - c+ < p_1 >) \cap (b_2+ < p_2 >)\}$. By Lemma 2.3, we obtain $(< p_1 > \backslash p_1) \cap (a_2+ < p_2 >) = \varphi$, therefore, $w_1 \in \{(p_1 - c+ < p_1 >) \cap \ (b_2+ < p_2 >)\}$.

Similarly, since $w_2 \in N(w_1) \cap N(x)$, then $w_2 \in c+ < p_1 > \cap\, b_2+ < p_2 >$. By Lemma 2.3, $c+ < p_1 > \cap\, b_2+ < p_2 >$ contains exactly one element, thus, $w_2 = y$, which is a contradiction. So, no such path exists. Consequently $T(\Gamma(\mathbb{Z}_n))$ is not locally connected.

(iii) $n = p_1^{r_1} p_2^{r_2}$ where p_1, p_2 are distinct primes and r_1, r_2 are positive integers and either $r_1 \geq 2$ or $r_2 \geq 2$. By Lemma 2.3, the intersection of a coset of the ideal $< p_1 >$ and a coset of the ideal $< p_2 >$ contains at least two distinct vertices. Let $x \in \mathbb{Z}_n$, assume that $x \in a_1+ < p_1 > \cap\, a_2+ < p_2 >$ and that $a_i + b_i \equiv 0 \pmod{p_i}$ for $i = 1, 2$. Let $y, z \in N(x)$. If both $y, z \in b_1+ < p_1 >$ then, by Lemma 2.3, there is $w \in a_1+ < p_1 > \cap\, b_2+ < p_2 >$, such that $w \neq x, y$, thus we get the path $y - w - z$. Now, assume that $y \in b_1+ < p_1 >$ and $z \in b_2+ < p_1 >$, pick $u \in a_1+ < p_1 > \cap\, b_2+ < p_2 >$, and $w \in b_1+ < p_1 > \cap\, a_2+ < p_2 >$, this gives the path $y - u - w - z$. Thus, if $n = p_1^{r_1} p_2^{r_2}$ where either r_1 or r_2 is greater than 1, then $T(\Gamma(\mathbb{Z}_n))$ is locally connected.

(iv) $n = p_1^{r_1} p_2^{r_2} \ldots p_m^{r_m}, m \geq 3$. Let $x \in \mathbb{Z}_n$ and $y, z \in N(x)$. Assume that $x \equiv a_i \pmod{p_i}$, and $a_i + b_i \equiv 0 \pmod{p_i}$, $i = 1, 2, \ldots, m$. Then $N(x) = \bigcup_{i=1}^{m} (b_i+ < p_i >)$. If both y and z are congruent to some b_j modulo p_j, then there is a vertex $u \neq x, y$ or z, such that $u \in a_j+ < p_j > \cap\, b_k+ < p_k >$, $k \neq j$. Thus $y - u - z$ is a path joining y to z in $G_T[N(x)]$. Now, without loss of generality, we may assume that $y \equiv b_1 \pmod{p_1}$, while $y \equiv r_k \pmod{p_k}$, where $k \neq 1$, and $z \equiv b_2 \pmod{p_2}$, $z \equiv s_t \pmod{p_t}$, where $t \neq 2$. By the Chinese remainder theorem, there is a vertex w such that $w \equiv b_1 \pmod{p_1}$, $w \equiv (p_2 - r_2) \pmod{p_2}$, and $w \equiv (p_3 - s_3) \pmod{p_3}$. Thus $w \in N(x) \cap N(y) \cap N(z)$. □

Next, we move to study local homogeneity in the graph $T(\Gamma(\mathbb{Z}_n))$. Since a locally homogeneous graph is indeed regular, we need the following theorem

Theorem 2.4. ([2, Theorem 2.5]) *$T(\Gamma(\mathbb{Z}_n))$ is regular if and only if n is even.*

If $n = 2^r$ where r is a positive integer, then $T(\Gamma(\mathbb{Z}_n))$ is the union of two copies of the complete graph K_m, where $m = 2^{r-1}$, in this case $T(\Gamma(\mathbb{Z}_n))$ is locally K_{m-1}. The next thorem shows that for even n, the neighborhoods of any two vertices in $T(\Gamma(\mathbb{Z}_n))$ induces isomorphic graphs.

Theorem 2.5. *Let x and y be two distinct vertices in $T(\Gamma(\mathbb{Z}_n))$. Then, $G_T[N(x)]$ is isomorphic to $G_T[N(y)]$ if and only if n is even.*

Proof. If $n = 2$, then $T(\Gamma(\mathbb{Z}_n))$ consists of two isolated vertices. Thus, we may assume $n > 2$. In view of Theorem 2.4, it suffices to show that if n is even, then for each $x \neq y \in \mathbb{Z}_n$, $G_T[N(x)]$ is isomorphic to the $G_T[N(y)]$.

Define the function $\varphi : N(x) \longrightarrow N(y)$ by

$$\varphi(z) = \begin{cases} z + y - x, & \text{if } z \text{ and } x \text{ have the same parity;} \\ z - y + x, & \text{if } z \text{ and } x \text{ have different parities.} \end{cases}$$

Where φ is defined for each $z \in N(x)$. Let z_1 and $z_2 \in N(x)$. If z_1 and z_2 have the same parity, then it can easily be seen that $\varphi(z_1) = \varphi(z_2)$ implies that $z_1 = z_2$. On the other hand, if z_1 and z_2 have different parities, say z_1 has the same parity as x and z_2 has a different parity from that of x, then they could not have the same image, otherwise, $z_1 + y - x = z_2 - y + x$ which implies that $z_1 + 2y - 2x = z_2$ which is a contradiction. So, φ is one-to-one. But $|N(x)| = |N(y)|$ and $T(\Gamma(\mathbb{Z}_n))$ is a finite graph, therefore, φ is onto. It remains to prove that φ is an isomorphism from $N(x)$ to $N(y)$. Let z_1 and z_2 be two adjacent vertices in $G_T[N(x)]$. If z_1 and z_2 have the same parity as that of x, then $\varphi(z_1)+\varphi(z_2) = (z_1+y-x)+(z_2+y-x) = z_1+z_2+2y-2x$ which belongs to $Z(\mathbb{Z}_n)$. If both z_1 and z_2 have the same parity which is different from the parity of x, then $\varphi(z_1) + \varphi(z_2) = (z_1 - y + x) + (z_2 - y + x) = z_1 + z_2 - 2y + 2x$ which again belongs to $Z(\mathbb{Z}_n)$. If z_1 and z_2 have different parities, say z_1 has the same parity as x and z_2 has a different parity from that of x, then $\varphi(z_1) + \varphi(z_2) = (z_1 + y - x) + (z_2 - y + x) = z_1 + z_2 \in Z(\mathbb{Z}_n)$. □

Theorem 2.6. ([4, Theorem 3.6, Theorem 3.2]) *Let R be a finite commutative ring with unity. Then:*

(i) *The graph $Z(\Gamma(R))$ is regular if and only if R is a local ring.*

(ii) *The graph $\mathrm{Reg}(\Gamma(R))$ is regular.*

Theorem 2.7 part (i) together, with Theorem 1.1, give the following theorem

Theorem 2.7. (i) *If $n = p^m$, where p is a prime, m is a positive integer, then $Z(\Gamma(R))$ is locally $K_{p^{m-1}-1}$,*

(ii) *otherwise, there is no graph H such that $Z(\Gamma(\mathbb{Z}_n))$ is locally H.*

Next, we study $\mathrm{Reg}(\Gamma(\mathbb{Z}_n))$. Before proceeding, we elaborate on the structure of $\mathrm{Reg}(\Gamma(\mathbb{Z}_n))$. Let $n = p_1^{r_1} p_2^{r_2} \ldots p_k^{r_k}$, where $p_i's$ are all distinct odd primes, and $r_i's$ are positive integers. By Theorem 2.1, for each $i = 1, 2, \ldots, k$, a coset of $< p_i >$, has a fixed number of regular elements say n_i. The graph $\mathrm{Reg}(\Gamma(\mathbb{Z}_n))$ assumes a layered structure; to see this, for each $i = 1, 2, \ldots, k$, let G_i be the spanning subgraph of $\mathrm{Reg}(\Gamma(\mathbb{Z}_n))$ where two distinct vertices x and y of $\mathrm{Reg}(\Gamma(\mathbb{Z}_n))$ are adjacent in G_i if $x + y \in < p_i >$. Obviously, $\mathrm{Reg}(\Gamma(\mathbb{Z}_n))$ is the (not necessarily edge disjoint) overlay of the k subgraphs $G_1, G_2, \ldots, G_k$. For $i = 1, 2, \ldots, k$, the graph G_i is a union of complete bipartite graphs K_{n_i,n_i}. Thus we obtain

Lemma 2.2. *For each $i = 1, 2, \ldots, k$, let G_i and n_i be as defined above, then each $G_i = K_{n_i,n_i}$ and hence, is locally homogeneous.*

For the sake of simplicity, in the following, we will refer to the subgraph of $\mathrm{Reg}(\Gamma(\mathbb{Z}_n))$ induced by the neighborhood of a vertex x by $G_R[N(x)]$.

Theorem 2.8. *Let x and y be two distinct vertices in* $\text{Reg}(\Gamma(\mathbb{Z}_n))$. *Then,* $G_R[N(x)]$ *is isomorphic to* $G_R[N(y)]$.

Proof. If n is even, then all regular elements in $\mathbb{Z}_n$ are odd and hence, $\text{Reg}(\Gamma(\mathbb{Z}_n))$ is a complete graph which is locally homogeneous. So, we may assume that n is odd. If $n = p^m$ where p is an odd prime and m is a positive integer, then by part (ii) of Theorem 1.1, $\text{Reg}(\Gamma(R))$ is the union of $K_{\lambda,\lambda}{}'s$, where $\lambda = |Z(\mathbb{Z}_n)|$. Let $n = p_1^{r_1} p_2^{r_2} \dots p_k^{r_k}$, where $p_i's$ are all distinct odd primes, $r_i's$ are positive integers, and $k \geq 2$. For $i = 1, 2, \dots, k$, let G_i be the spanning subgraphs of $\text{Reg}(\Gamma(\mathbb{Z}_n))$ where two distinct vertices x and y of $\text{Reg}(\Gamma(\mathbb{Z}_n))$ are adjacent in G_i if $x + y \in < p_i >$. The graph $\text{Reg}(\Gamma(\mathbb{Z}_n))$ is the (not necessarily edge disjoint) overlay of the graphs $G_1, G_2, \dots, G_k$. We call each G_i the ith layer. Let x, y be any two distinct vertices in $\text{Reg}(\mathbb{Z}_n)$. Let $N_i(x)$, be the neighborhoods of x in G_i, $i = 1, 2, \dots, k$. The sets $N_i(y)$ are defined analogously. Then $N(x) = N_1(x) \cup N_2(x) \cup \dots \cup N_k(x)$, and $N(y) = N_1(y) \cup N_2(y) \cup \dots \cup N_k(y)$. By Lemma 2.9, each of G_i , is locally homogeneous, then $N_i(x)$ and $N_i(y)$ induce isomorphic graphs for each $i = 1, 2, \dots, k$. Note that $G_R[N(x)]$ is the overlay of $G_R[N_1(x)]$, $G_R[N_2(x)] \dots G_R[N_k(x)]$. Thus $N(x)$ and $N(y)$ induce isomorphic graphs. Thus, $\text{Reg}(\Gamma(\mathbb{Z}_n))$ is also locally homogeneous. □

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