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NEW FAMILIES OF 4-TOTAL PRIME CORDIAL GRAPH

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Abstract. Let G be a (p,q) graph. Let $f : V(G) \rightarrow \{1,2,\ldots,k\}$ be a map where $k \in \mathbb{N}$ is a variable and $k > 1$. For each edge $u,v \in V$, assign the label $\gcd\{f(u),f(v)\}$. f is called k-total prime cordial labeling of G if $|t_f(i) - t_f(j)| \leq 1$, $i,j \in \{1,2,\cdots,k\}$ where $t_f(x)$ denotes the total number of vertices and the edges labeled with x. A graph with a k-total prime cordial labeling is called k-total prime cordial graph. In this paper we investigate the 4-total prime cordial labeling of some graphs like triangular ladder and armed crown, subdivision of jelly fish and subdivision of triangular ladder.

1. Introduction

Graphs considered here are finite, simple and undirected. The notion of cordial labeling of graphs was introduced by Cahit [1] in 1987. Sundaram, Ponraj and Somasundaram [11] have introduced the notion of prime cordial labeling. Ponraj et al. [5], have been introduced the concept of k-total prime cordial labeling motivated by the prime cordial labeling and investigate the k-total prime cordial labeling of certain graphs. Also in [5, 6, 7, 8, 9, 10], we investigate the 4-total prime cordial labeling behaviour of path, cycle, star, bistar, some complete graphs, comb, double comb, triangular snake, double triangular snake, ladder, friendship graph, flower graph, gear graph, Jelly fish, book, irregular triangular snake, prism, helm, dumbbell

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graph, sunflower graph, corona of irregular triangular snake, corona of some graphs and subdivision of some graphs. In this paper we investigate the 4-total prime cordiality of graphs like triangular ladder and armed crown, subdivision of jelly fish and subdivision of triangular ladder.

2. k-TOTAL PRIME CORDIAL LABELING

Definition 2.1. Let G be a (p, q) graph. Let $f : V(G) \to \{1, 2, \ldots, k\}$ be a function where $k \in \mathbb{N}$ is a variable and $k > 1$. For each edge uv, assign the label $\gcd(f(u), f(v))$. f is called k-total prime cordial labeling of G if $|t_f(i) - t_f(j)| \leq 1$, $i, j \in \{1, 2, \cdots, k\}$ where $t_f(x)$ denotes the total number of vertices and the edges labeled with x. A graph with a k-total prime cordial labeling is called *k-total prime cordial graph.*

3. PRELIMINARIES

Definition 3.1. If $e = uv$ is an edge of G, then e is said to be *subdivided* when it is replaced by the edges uw and wv. The graph obtained by subdividing each edge of a graph G is called the subdivision graph of G and is denoted by $S(G)$.

Definition 3.2. The *armed crown* AC_n is obtained from the cycle C_n with $V(AC_n) = V(C_n) \cup \{v_i, w_i : 1 \leq i \leq n\}$ and $E(AC_n) = E(C_n) \cup \{u_i w_i, w_i v_i : 1 \leq i \leq n\}$.

Definition 3.3. The *triangular ladder* TL_n is obtained from the path $u_1 u_2 \ldots u_n$ ans $v_1 v_2 \ldots v_n$ with $V(TL_n) = \{u_i, v_i : 1 \leq i \leq n\}$ and $E(TL_n) = \{u_i u_{i+1}, v_i v_{i+1} : 1 \leq i \leq n-1\} \cup \{u_i v_i : 1 \leq i \leq n\}$

Definition 3.4. *Jelly fish* graph $J(m, n)$ is obtained from a cycle $C_4 : uxvwu$ by joining x and w with an edge and appending m pendent edges to u and n pendent edges to v.

Definition 3.5. Let f be a map from V(G) to $\{0, 1, \ldots, k-1\}$, where k is an integer, $2 \leq k \leq |V(G)|$. For each edge uv, assign the label $f(u)f(v)(modk)$. f is called k-total product cordial labeling of G if $|f(i) - f(j)| \leq 1$, $i, j \in \{0, 1, \cdots, k-1\}$ where $f(x)$ denotes the total number of vertices and the edges labeled with $x = 0, 1, 2, ..., k-1$. A graph with a k-total product cordial labeling is called *k-total product cordial graph.*

Theorem 3.1. [5]$K_2 + mK_1$ is not 4-total prime cordial iff $m > 1$.

Remark 3.1. [4] *2- total prime cordial graph is 2-total product cordial graph.*

4. Main Results

Theorem 4.1. The triangular ladder graph TL_n is 4-total prime cordial iff $n \notin \{2, 3\}$.

Proof. Let $V(TL_n) = \{u_i, v_i : 1 \leq i \leq n\}$ and $E(TL_n) = \{u_iu_{i+1}, v_iv_{i+1}, u_iv_{i+1} : 1 \leq i \leq n-1\} \cup \{u_iv_i : 1 \leq i \leq n\}$. Clearly $|V(TL_n)| + |E(TL_n)| = 6n - 3$. We consider the following cases according to the nature of n.

Case 1. $n \equiv 0 \pmod 4$.

Let $n = 4r$, $r > 1$ and $r \in \mathbb{N}$. Assign the label 4 to the vertices $u_1, u_2, \ldots, u_r$ and assign the label 2 to the vertices $u_{r+1}, u_{r+2}, \ldots, u_{2r}$. Next we assign the label 3 to the vertices $u_{2r+1}, u_{2r+2}, \ldots, u_{3r+1}$ then we assign the label 1 to the vertices $u_{3r+2}, u_{3r+3}, \ldots, u_{4r-1}$. Finally we assign the label 4 to the vertex u_{4r}. Next we consider the vertices v_i $(1 \leq i \leq n)$. Assign the label 4 to the vertices $v_1, v_2, \ldots, v_r$ and assign the label 2 to the vertices $v_{r+1}, v_{r+2}, \ldots, v_{2r}$. Next we assign the label 3 to the vertices $v_{2r+1}, v_{2r+2}, \ldots, v_{3r}$. Next we assign the label 4 to the vertex v_{3r+1}. Finally we assign the label 1 to the vertices $v_{3r+2}, v_{3r+3}, \ldots, v_{4r}$. Clearly $t_f(1) = t_f(3) = t_f(4) = 6r - 1$ and $t_f(2) = 6r$.

Case 2. $n \equiv 1 \pmod 4$.

Let $n = 4r+1$, $r > 1$ and $r \in \mathbb{N}$. In this case, assign the same label as in case 1 to the vertices u_i, v_i $(1 \leq i \leq n-3)$. Next we assign the labels 3, 1, 2, 4, 1, 1 respectively to the vertices u_{4r-2}, u_{4r-1}, u_{4r}, v_{4r-2}, v_{4r-1} and v_{4r}. Here $t_f(1) = t_f(2) = t_f(3) = 6r+1$ and $t_f(4) = 6r$.

Case 3. $n \equiv 2 \pmod 4$.

Let $n = 4r+2$, $r > 1$ and $r \in \mathbb{N}$. Assign the same label as in case 2 to the vertices u_i, v_i $(1 \leq i \leq n-2)$. Finally we assign the labels 2, 4, 3, 4 respectively to the vertices u_{4r-1}, u_{4r}, v_{4r-1} and v_{4r}. It is easy to verify that $t_f(1) = t_f(2) = t_f(3) = 6r+2$ and $t_f(4) = 6r+3$.

Case 4. $n \equiv 3 \pmod 4$.

Let $n = 4r+3$, $r > 1$ and $r \in \mathbb{N}$. In this case, assign the same label as in case 3 to the vertices u_i, v_i $(1 \leq i \leq n-4)$. Then we assign the labels 3, 2, 1, 2, 4, 4, 1, 3 to the vertices u_{4r-3}, u_{4r-2}, u_{4r-1}, u_{4r}, v_{4r-3}, v_{4r-2}, v_{4r-1} and v_{4r} respectively. Here $t_f(1) = t_f(3) = t_f(4) = 6r+4$ and $t_f(2) = 6r+3$.

Case 5. $n = 2$.

Theorem 3.1 gives $n = 2$ is not a 4-total prime cordial.

Case 6. $n = 3$.

Any one of the following types occur:

Type 1: $t_f(1) = t_f(2) = t_f(3) = 4$ and $t_f(4) = 3$.

Type 2: $t_f(2) = t_f(3) = t_f(4) = 4$ and $t_f(1) = 3$.

Type 3: $t_f(1) = t_f(3) = t_f(4) = 4$ and $t_f(2) = 3$.

Type 4: $t_f(1) = t_f(2) = t_f(4) = 4$ and $t_f(3) = 3$.

Type 1 and Type 2: $t_f(3) = 4$. $f(u_1) = 3$, $f(u_3) = 3$ and $f(v_2) = 3$. All the others are symmetry. To get the label 2, it is easy to verify that $f(u_2) = f(v_1) = f(v_3) = 2$. This implies $t_f(4) = 0$, a contradiction.

Type 3: In this case, $t_f(3) = 4$. By symmetry, assume $f(u_1) = 3$, $f(u_3) = 3$ and $f(v_2) = 3$. To get the label 4, clearly $f(u_2) = f(v_1) = f(v_3) = 4$. This implies $t_f(2) = 0$, a contradiction.

Type 4: In this case, $t_f(4) = 4$. Therefore by symmetry, assume $f(u_1) = f(u_3) = f(v_2) = 4$. To get the label 3, clearly $f(u_2) = f(u_3) = 3$. Therefore, $t_f(2) = 3$, a contradiction.

Case 7. $n = 4, 5, 6, 7$.

n	4	5	6	7
u_1	4	4	4	4
u_2	4	4	4	4
u_3	2	2	2	2
u_4	3	4	3	2
u_5		3	2	3
u_6			4	3
u_7				4
v_1	4	4	4	4
v_2	2	2	2	4
v_3	3	3	4	2
v_4	3	3	3	1
v_5		3	3	3
v_6			1	3

v_7				2

Table 1: A 4-total prime cordial labeling of $n = 4, 5, 6, 7$

□

Illustration 4.1.

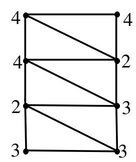

FIGURE 1. A 4-total prime cordial labeling of TL_4

Theorem 4.2. The subdivision of triangular ladder TL_n, $S(TL_n)$ is 4-total prime cordial for all n.

Proof. Let $V(TL_n) = \{u_i, v_i : 1 \leq i \leq n\}$ and $E(TL_n) = \{u_iu_{i+1}, v_iv_{i+1}, u_iv_{i+1} : 1 \leq i \leq n-1\} \cup \{u_iv_i : 1 \leq i \leq n\}$. Let y_i, x_i, w_i and z_i be the vertices which subdivide the edges u_iu_{i+1}, u_iv_i, v_iv_{i+1} and u_iv_{i+1} respectively. Clearly $|V(S(TL_n))| + |E(S(TL_n))| = 14n - 9$. We consider the following cases according to the nature of n.

Case 1. $n \equiv 0 \pmod 4$.

Let $n = 4r$, $r > 1$ and $r \in \mathbb{N}$. Assign the label 4 to the vertices $u_1, u_2, \ldots, u_r$ and $v_1, v_2, \ldots, v_r$. Assign the label 2 to the vertices $u_{r+1}, u_{r+2}, \ldots, u_{2r}$ and $v_{r+1}, v_{r+2}, \ldots, v_{2r}$. Next we assign the label 3 to the vertices u_{2r+1}, $u_{2r+2}, \ldots$, u_{3r} and $v_{2r+1}, v_{2r+2}, \ldots, v_{3r}$ then we assign the label 1 to the vertices $u_{3r+1}, u_{3r+2}, \ldots, u_{4r-1}$ and $v_{3r+1}, v_{3r+2}, \ldots, v_{4r-1}$. Finally, we assign the labels 4 and 3 to the vertices u_{4r} and v_{4r} respectively. Next we consider the vertices x_i $(1 \leq i \leq n)$. Assign the label 4 to the vertices $x_1, x_2, \ldots, x_r$

and assign the label 2 to the vertices $x_{r+1}, x_{r+2}, \ldots, x_{2r}$. Now we assign the label 3 to the vertices $x_{2r+1}, x_{2r+2}, \ldots, x_{3r}$. Finally we assign the label 1 to the vertices $x_{3r+1}, x_{3r+2}, \ldots, x_{4r}$. Now we consider the vertices y_i, w_i $(1 \leq i \leq n-1)$. Assign the label 4 to the vertices $y_1, y_2, \ldots, y_r$ and $w_1, w_2, \ldots, w_r$. Assign the label 2 to the vertices $y_{r+1}, y_{r+2}, \ldots, y_{2r-1}$ and $w_{r+1}, w_{r+2}, \ldots, w_{2r-1}$. Next we assign the label 3 to the vertices $y_{2r}, y_{2r+1}, \ldots, y_{3r-1}$ and $w_{2r}, w_{2r+1}, \ldots, w_{3r-1}$ then we assign the label 1 to the vertices $y_{3r}, y_{3r+1}, \ldots, y_{4r-2}$ and $w_{3r}, w_{3r+1}, \ldots, w_{4r-2}$, w_{4r-1}. Finally, we assign the labels 2 vertex y_{4r-1}. Next we consider the vertices z_i $(1 \leq i \leq n-1)$. Assign the label 4 to the vertices $z_1, z_2, \ldots, z_r$ and assign the label 2 to the vertices $z_{r+1}, z_{r+2}, \ldots, z_{2r}$. Now we assign the label 3 to the vertices $z_{2r+1}, z_{2r+2}, \ldots, z_{3r}$. Finally we assign the label 1 to the vertices $z_{3r+1}, z_{3r+2}, \ldots, z_{4r-1}$.

Case 2. $n \equiv 1 \pmod 4$.

Let $n = 4r+1$, $r > 1$ and $r \in \mathbb{N}$. In this case, assign the same label as in case 1 to the vertices u_i $(1 \leq i \leq n-1)$, v_i $(1 \leq i \leq n-1)$, x_i $(1 \leq i \leq n)$, y_i $(1 \leq i \leq n-2)$, w_i $(1 \leq i \leq n-2)$ and z_i $(1 \leq i \leq n-3)$. Finally we assign the labels 2, 2, 4, 3, 4, 3 respectively to the vertices u_{4r}, v_{4r}, y_{4r-1}, w_{4r-1}, z_{4r-2} and z_{4r-1}.

Case 3. $n \equiv 2 \pmod 4$.

Let $n = 4r+2$, $r > 1$ and $r \in \mathbb{N}$. Assign the same label as in case 2 to the vertices u_i $(1 \leq i \leq n-1)$, v_i $(1 \leq i \leq n-1)$, x_i $(1 \leq i \leq n-1)$, y_i $(1 \leq i \leq n-2)$, w_i $(1 \leq i \leq n-2)$ and z_i $(1 \leq i \leq n-4)$. Finally we assign the labels 4, 3, 2, 4, 3, 1, 4, 3 to the vertices u_{4r}, v_{4r}, x_{4r}, y_{4r-1}, w_{4r-1}, z_{4r-3}, z_{4r-2} and z_{4r-1} respectively.

Case 4. $n \equiv 3 \pmod 4$.

Let $n = 4r+3$, $r > 1$ and $r \in \mathbb{N}$. In this case, assign the same label as in case 3 to the vertices u_i $(1 \leq i \leq n-1)$, v_i $(1 \leq i \leq n-1)$, x_i $(1 \leq i \leq n-1)$, y_i $(1 \leq i \leq n-2)$, w_i $(1 \leq i \leq n-2)$ and z_i $(1 \leq i \leq n-5)$. Finally we assign the labels 3, 4, 2, 3, 4, 2, 4, 3, 1 respectively to the vertices u_{4r}, v_{4r}, x_{4r}, y_{4r-1}, w_{4r-1}, z_{4r-4}, z_{4r-3}, z_{4r-2} and z_{4r-1} respectively.

Case 5. $n = 2, 3, 4, 5, 6, 7$.

n	2	3	4	5	6	7
u_1	4	4	4	4	4	4
u_2	3	4	4	4	4	4
u_3		1	3	2	2	2
u_4			3	3	3	2
u_5				3	3	3
u_6					1	3
u_7						1
v_1	4	2	4	4	4	4
v_2	3	3	2	4	4	4
v_3		3	1	2	2	2
v_4			3	3	3	2
v_5				1	3	3
v_6					1	3
v_7						1
x_1	4	4	4	4	4	4
x_2	3	2	2	4	4	4
x_3		4	1	2	2	2
x_4			3	3	3	1
x_5				1	3	3
x_6					1	2
x_7						1
y_1	2	4	4	4	4	4
y_2		3	2	2	2	4
y_3			3	3	2	2

y_4				3	3	3
y_5					1	3
y_6						1
w_1	1	2	2	4	4	4
w_2		3	2	2	2	4
w_3			3	3	2	2
w_4				2	3	3
w_5					1	3
w_6						1
z_1	2	2	4	2	4	4
z_2		4	3	1	2	2
z_3			1	3	1	2
z_4				1	3	3
z_5					1	3
z_6						1

Table 2: A 4-total prime cordial labeling $n = 2, 3, 4, 5, 6, 7$

Illustration 4.2.

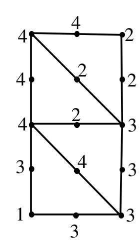

FIGURE 2. A 4-total prime cordial labeling of $S(TL_3)$

□

Theorem 4.3. The armed crown graph AC_n is 4-total prime cordial for all $n \geq 3$.

Proof. Let C_n be the cycle $u_1u_2\dots u_nu_1$. Then armed crown is obtained from the cycle C_n with $V(AC_n) = V(C_n) \cup \{v_i, w_i : 1 \leq i \leq n\}$ and $E(AC_n) = E(C_n) \cup \{u_iw_i, w_iv_i : 1 \leq i \leq n\}$. Clearly $|V(AC_n)| + |E(AC_n)| = 6n$. We consider the following cases according to the nature of n.

Case 1. $n \equiv 0 \pmod 4$.

Let $n = 4r$, $r > 1$ and $r \in \mathbb{N}$. Assign the label 4 to the vertices $u_1, u_2, \dots, u_r$ and assign the label 2 to the vertices $u_{r+1}, u_{r+2}, \dots, u_{2r}$. Next we assign the label 3 to the vertices $u_{2r+1}, u_{2r+2}, \dots, u_{3r}$ then we assign the label 1 to the vertices $u_{3r+1}, u_{3r+2}, \dots, u_{4r}$. Now we move to the vertices w_i $(1 \leq i \leq n)$. Assign the label 4 to the vertices $w_1, w_2, \dots, w_r$ and assign the label 2 to the vertices $w_{r+1}, w_{r+2}, \dots, w_{2r}$. Next we assign the label 3 to the vertices $w_{2r+1}, w_{2r+2}, \dots, w_{3r}$ then we assign the label 1 to the vertices $w_{3r+1}, w_{3r+2}, \dots, w_{4r-1}$. Finally we assign the label 3 to the vertex w_{4r}. Next we move to the vertices v_i $(1 \leq i \leq n)$. Assign the label 4 to the vertices $v_1, v_2, \dots, v_r$ and assign the label 2 to the vertices $v_{r+1}, v_{r+2}, \dots, v_{2r}$. Next we assign the label 3 to the vertices $v_{2r+1}, v_{2r+2}, \dots, v_{3r}$ then we assign the label 1 to the vertices $v_{3r+1}, v_{3r+2}, \dots, v_{4r-1}$. Finally we assign the label 4 to the vertex v_{4r}. Clearly $t_f(1) = t_f(2) = t_f(3) = t_f(4) = 6r$.

Case 2. $n \equiv 1 \pmod 4$.

Let $n = 4r + 1$, $r > 1$ and $r \in \mathbb{N}$. Assign the same label as in case 1 to the vertices u_i $(1 \leq i \leq n-1)$, w_i $(1 \leq i \leq n-1)$ and v_i $(1 \leq i \leq n-1)$. Finally we assign the labels 3, 4, 2 respectively to the vertices u_{4r}, w_{4r} and v_{4r}. It is easy to verify that $t_f(1) = t_f(2) = 6r + 2$ and $t_f(3) = t_f(4) = 6r + 1$.

Case 3. $n \equiv 2 \pmod 4$.

Let $n = 4r + 2$, $r > 1$ and $r \in \mathbb{N}$. In this case, assign the same label as in case 1 to the vertices u_i $(1 \leq i \leq n-2)$, w_i $(1 \leq i \leq n-2)$ and v_i $(1 \leq i \leq n-2)$. Finally we assign the labels 2, 3, 4, 3, 4, 2 respectively to the vertices u_{4r-1}, u_{4r}, w_{4r-1}, w_{4r}, v_{4r-1} and v_{4r}. Here $t_f(1) = t_f(2) = t_f(3) = t_f(4) = 6r + 3$.

Case 4. $n \equiv 3 \pmod 4$.

Let $n = 4r + 3$, $r > 1$ and $r \in \mathbb{N}$. Assign the same label as in case 2 to the vertices u_i $(1 \leq i \leq n-2)$, w_i $(1 \leq i \leq n-2)$ and v_i $(1 \leq i \leq n-2)$. Finally we assign the labels 2, 4, 3, 1, 3, 4 respectively to the vertices u_{4r-1}, u_{4r}, w_{4r-1}, w_{4r}, v_{4r-1} and v_{4r}. Here $t_f(1) = t_f(2) = 6r + 5$ and $t_f(3) = t_f(4) = 6r + 4$.

Case 5. $n = 3, 4, 5, 6, 7$.

n	3	4	5	6	7
u_1	4	4	4	4	4
u_2	2	2	4	4	4
u_3	3	3	2	2	2
u_4		1	3	3	2
u_5			3	3	3
u_6				1	3
u_7					1
w_1	4	4	4	4	4
w_2	2	2	2	4	4
w_3	3	3	2	2	2
w_4		4	3	3	2
w_5			1	3	3
w_6				1	3
w_7					1
v_1	4	4	4	4	4
v_2	1	2	2	2	4
v_3	3	3	1	2	2

v_4		3	3	3	1
v_5			1	2	3
v_6				1	3
v_7					1

Table 3: A 4-total prime cordial labeling $n = 3, 4, 5, 6, 7$

□

Illustration 4.3.

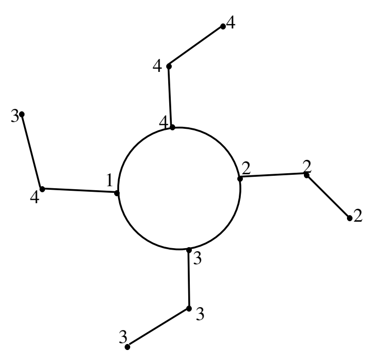

FIGURE 3. A 4-total prime cordial labeling of AC_4

Theorem 4.4. The subdivision of jelly fish $J(n, n)$, $S(J(n, n))$ is 4-total prime cordial for all values of n.

Proof. Let u, w_2, v, w_5 be the vertices such that u, v are adjacent to w_5 and w_2, w_5 is adjacent to w_2. Let u_i be the pendent vertices adjacent to u and y_i be the pendent vertices adjacent to v. Let x_i, y_i, w_1, w_3, w_4, w_6 and w_7 be the vertex which subdivide the edge uu_i, vv_i, uw_2, w_2v, vw_5, w_5u and w_2w_5 respectively. Clearly $|V(S(J(n, n)))|+|E(S(J(n, n)))| = 8n+19$. We consider the following cases according to the nature of n.

Case 1. $n \equiv 0 \pmod 4$.

Let $n = 4r$, $r > 1$ and $r \in \mathbb{N}$. Assign the labels 4, 2, 2, 1, 3, 3, 3, 1, 1 respectively to the vertices u, w_1, w_2, w_3, v, w_4, w_5, w_6 and w_7. Assign the label 4 to the vertices $u_1, u_2, \ldots, u_{2r}$ and assign the label 2 to the vertices $u_{2r+1}, u_{2r+2}, \ldots, u_{4r}$. Now we consider the vertices x_i $(1 \le i \le n)$. Assign the label 4 to the vertices $x_1, x_2, \ldots, x_{2r}$ and assign the label 2 to the vertices $x_{2r+1}, x_{2r+2}, \ldots, x_{4r}$. Next we move to the vertices y_i $(1 \le i \le n)$. Assign the label 3 to the vertices $y_1, y_2, \ldots, y_{2r}$. Next we assign the label 4 to the vertex y_{2r+1}. Finally we assign the label 1 to the vertices $y_{2r+2}, y_{2r+3}, \ldots, y_{4r}$. Now we consider the vertices v_i $(1 \le i \le n)$. Assign the label 3 to the vertices $v_1, v_2, \ldots, v_{2r}$. Next we assign the label 4 to the vertices v_{2r+1} and v_{2r+2}. Finally we assign the label 1 to the vertices $v_{2r+3}, v_{2r+4}, \ldots, v_{4r}$. Clearly $t_f(1) = t_f(3) = t_f(4) = 8r + 5$ and $t_f(2) = 8r + 4$.

Case 2. $n \equiv 1 \pmod 4$.

Let $n = 4r + 1$, $r > 1$ and $r \in \mathbb{N}$. In this case, assign the same label as in case 1 to the vertices u, w_1, w_2, w_3, v, w_4, w_5, w_6, w_7, u_i $(1 \le i \le n-1)$, x_i $(1 \le i \le n-1)$, y_i $(1 \le i \le n-1)$ and v_i $(1 \le i \le n-1)$. Finally we assign the labels 3, 4, 2, 2 respectively to the vertices u_{4r}, x_{4r}, y_{4r} and v_{4r}. Here $t_f(1) = t_f(2) = t_f(4) = 8r + 7$ and $t_f(3) = 8r + 6$.

Case 3. $n \equiv 2 \pmod 4$.

Let $n = 4r + 2$, $r > 1$ and $r \in \mathbb{N}$. Assign the same label as in case 2 to the vertices u, w_1, w_2, w_3, v, w_4, w_5, w_6, w_7, u_i $(1 \le i \le n-1)$, x_i $(1 \le i \le n-1)$, y_i $(1 \le i \le n-1)$ and v_i $(1 \le i \le n-1)$. Finally we assign the labels 3, 3, 2, 4 respectively to the vertices u_{4r}, x_{4r}, y_{4r} and v_{4r}. Obviously $t_f(1) = t_f(2) = t_f(3) = 8r + 9$ and $t_f(4) = 8r + 8$.

Case 4. $n \equiv 3 \pmod 4$.

Let $n = 4r + 3$, $r > 1$ and $r \in \mathbb{N}$. Assign the same label as in case 3 to the vertices u, w_1, w_2, w_3, v, w_4, w_5, w_6, w_7, u_i $(1 \le i \le n-1)$, x_i $(1 \le i \le n-1)$,

y_i $(1 \leq i \leq n-1)$ and v_i $(1 \leq i \leq n-1)$. Finally we assign the labels 2, 4, 3, 4 respectively to the vertices u_{4r}, x_{4r}, y_{4r} and v_{4r}. It is easy to verify that $t_f(1) = 8r+10$ and $t_f(2) = t_f(3) = t_f(4) = 8r+11$.

Case 5. $n = 1, 2, 3, 4, 5, 6, 7$.

n	1	2	3	4	5	6	7
u	4	4	4	4	4	4	4
w_1	4	2	2	2	2	2	2
w_2	2	2	2	2	2	2	2
w_3	1	1	1	1	1	1	1
v	3	3	3	3	3	3	3
w_4	3	1	1	1	1	1	1
w_5	2	2	2	2	2	2	2
w_6	2	2	2	2	2	2	2
w_7	1	1	1	1	1	1	1
u_1	4	4	4	4	4	4	4
u_2		4	4	4	4	4	4
u_3			2	4	4	4	4
u_4				2	2	4	4
u_5					2	2	2
u_6						2	2
u_7							2
x_1	4	4	4	4	4	4	4
x_2		4	4	4	4	4	4
x_3			4	4	4	4	4
x_4				2	2	4	4
x_5					2	2	4

x_6						2	2
x_7							2
y_1	3	3	3	3	3	3	3
y_2		3	3	3	3	3	3
y_3			3	3	3	3	3
y_4				1	3	3	3
y_5					1	1	3
y_6						1	1
y_7							1
v_1	3	3	3	3	3	3	3
v_2		3	3	3	3	3	3
v_3			1	3	3	3	3
v_4				1	1	3	3
v_5					1	1	1
v_6						1	1
v_7							1

Table 4: 4-total prime cordial labeling $n = 1, 2, 3, 4, 5, 6, 7$

$\square$

Illustration 4.4.

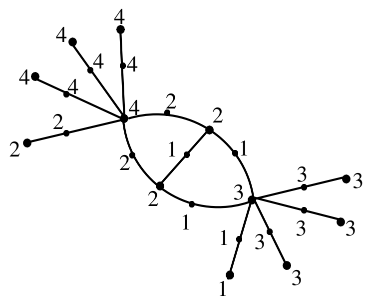

FIGURE 4. A 4-total prime cordial labeling of $S(J(4,4))$

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