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OPTIMAL PARAMETER SELECTION FOR A MACHINE REPAIR SYSTEM WITH SERVERS VACATION AND CONTROLLING F-POLICY

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Abstract. In this article, we proposed an algorithm for determining optimal parameters of the queueing system that allows server vacation under F-policy. The queueing system parameters such as the threshold of the F-policy and the system service rate are designed by minimize the cost associated with the queuing system. We first propose an efficient algorithm to determine steady-state probabilities of the system, and second, a nonlinear integer programming problem is formulated to determine optimal system parameters. The nonlinear integer programming problem is then relaxed to a nonlinear optimization and is solved using Quasi-Newton's method. Subsequently, various system performance measures are studied for different system parameters.

1. Introduction

Now-a-days queueing modeling approaches for machine repair systems with various threshold policies have found applications in technological and industrial domains such as communication, production and manufacturing/machining systems. The queueing model comprises of an arrival process that introduces entities in the queue and the service that is provided to these entities. A controlled arrival policy restricts the queue that decreases the waiting time of customers and avoids over burden on the service. Therefore, in real life scenario, controlled arrival policies are very cost effective. A queueing system with controlled service (N-policy with vacation [1]) that enables the server to go on vacation

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once the system customers drops below N and resumes its services as the customers in the system exceeds N. On the contrary, an arrival controlled queueing system is first proposed by [2] that allows the arrival of customers unless the system reaches its full capacity and don't resumes arrival before the customer strength in the system falls below a threshold level. In addition, [2] establishes an interrelationship between the queueing systems with F-policy and N-policy. Various researchers have contributed to queueing systems with F-policy such as [3] proposes an optimal policy of an $M/G/1/K$ queueing system that considers a F-policy with exponential start-up time, and [4] employs recursive techniques for obtaining steady state probabilities of a $G/M/1/K$ queueing system with F-policy having an exponential start-up time.

Recently, an optimal management that includes system performance measures and sensitivity analysis of a finite capacity $M/M/1$ queueing system with F Policy is explored in [5] where unsatisfied customers may demand another service besides the essential service. To generalized further, [6] employs matrix analytic method to determine the steady state probabilities of a F policy $M/M/1/K$ queueing system with working vacation. The optimal system capacity K and the queue length F for such queueing systems are determined by employing direct search techniques and optimization approaches like Quasi-Newton method. Another F policy for a finite capacity for a $M/H_2/1$ queueing system is investigated in [7] that determines the optimal capacity K by optimizing system performance measures. In the sequel, various queueing models have been explored such as the arrival control is discussed in [8].

A rich literature exists on machine repair problems with server vacation [9]. Specifically, $M/M/1$ machine repair problem with vacation is explored in [10] , and [11] examines the reliability characteristics of queueing system with m operating units and s spare with one repairman is removable and other repairman works according to N-policy. Moreover, [12] analysed the N-policy machine repair system with spare and reneging. [13] investigates the machine repair problem with partial server vacation policy in which first server always available for giving the service to failed machine while second server works according N-policy. A optimal value of N is obtained for a defined performance objective and its sensitivity analysis is also studied. A brief overview of research literature on vacation queueing systems my be found in [14].

[15] considers the queueing system that comprises of multi servers having distinct vacation policies with a finite source. [16] studies a deteriorating system that contains servers with multiple vacation and employs supplementary variable technique to find reliability indexes. [17] considers a batch arrival queueing system having unreliable server with multiple vacation, and [18] studies a machine repair system with spares having two repairmen with partial server vacation policy such that the first repairman is invariably available for servicing the failed units and the second repairman is allowed to go on vacation if failed units fall below N. [19] presented a multiple vacation machine repair problem in which profit optimization is done by employing particle swarm optimization technique to find an optimal number of standby and operating machines.

[20] first studies queueing system with working vacation where the servers, instead of stopping its service, reducing its service rate, and it is further generalized in [21] to a $G1/M/1$ queuing system with multiple working vacation as well as to a $M/G/1$ model in [22]. Subsequently, [23] introduces a new policy which is vacation interruption with working vacation where a server can terminate the vacation once certain no. of customer accumulated in the system during vacation period and uses matrix geometric method to calculated the system steady state. [24] uses a direct search method and Newton's method for the performance optimization in queuing systems with working vacation. Recently, [25] employ matrix method to find steady state probabilities of the F policy queueing system with working vacation. A detailed survey on the queueing system with various arrival and service processes considering different vacation models is available in [26]. A more realistic approach in which [27] investigates a fault tolerant system having multiple components with a single unreliable server and determine steady state probability using

successive over relaxation method. More recently, the matrix analytic method for finding performance measures of the queueing system with single working vacation is explored in [28]. Some of the recent work on queueing systems with vacation may be found in [29, 30, 31].

A vast amount of literature has been devoted to mathematical study of queueing systems with various vacation policies. However, there is no substantial study available on vacation queueing models with arrival control policy in which the server follows the single vacation policy. In this article, we have developed a queueing system with randomized arrival control policy with single vacation policy of the server. We propose an efficient algorithm to determine steady-state probabilities of the system, and a nonlinear integer programming problem is proposed to determine optimal system parameters for a given performance objective. The nonlinear integer programming problem is then relaxed to a nonlinear optimization and is solved using Quasi-Newton's method. In addition, various system performance measures are studied for different system parameters.

2. Model Description

Let us consider M/M/1/K queueing system with F-policy in which the system has M operating machines and S warm standby machines. Let us assume that the arrival of failed units follow the Poisson process with parameter λ and the service of failed units having FCFS discipline follow the exponential distribution with parameter μ. The system has a capacity of K number of units and the server provides its services according to F-policy i.e., when the number of customers in the system reaches its capacity K, the arrival of failed units is suspended until a certain number of failed unit have been served so that the queue length drops to a predetermined threshold $F\,(0 \leq F \leq K-1)$. Once the queue length drops below F, the server allows the entry of failed units to the system with a startup time which is exponentially distributed with parameter β. It is further considered that when an operating machine fails, it is immediately replaced by a spare machine (if any available) with negligible switch over time, and the replaced machine failure rate is identical to those of an operating machine. Furthermore, we introduce a server vacation scheme in which when there is no failed machine in the system, the server goes on a vacation having an exponential vacation rate with parameter η, and whenever any failed unit arrives in the system, server returns from vacation with rate θ and starts the service.

In order to determine the steady state probabilities of the governing model with exponentially distributed life time and repair time of the operating/standby units and the server respectively, we construct the Chapman-Kolmogrov equations by using the birth and death process. Let $X(t)$ denote the number of failed machines in the system at time t and

$$J(t) := \begin{cases} 0, & \text{server is busy and failed machines are not allowed} \\ 1, & \text{server is busy and failed machines are allowed} \\ 2, & \text{server is on vacation and failed machines are allowed} \end{cases}$$

represents the state of the system at time t. Then,

$$\{(X(t), J(t)) \mid t \geq 0\}$$

represents a continuous-time Markov chain (CTMC) with the state space

$$S := \{(n, j) \cup (K, 0) : n = 0, 1, 2, \ldots, K-1 \text{ and } j = 0, 1, 2\}.$$

Moreover,

$$P_{n,j}(t) := P[X(t) = n, J(t) = j] \text{ for } n = 0, 1, 2, \ldots, K-1 \text{ and } j = 0, 1, 2,$$

denotes the time-dependent state probability when the CTMC is in state $\{n, j\}$ at time t, and the state dependent failure rates are given by

$$\lambda_n := \begin{cases} M\lambda + (S-n)\alpha, & \text{if } 0 \leq n < S \\ (M+S-n)\lambda_d, & \text{if } S \leq n < K \\ 0, & \text{otherwise} \end{cases}$$

where λ_d is the degraded mode failure rate and α is the failure rate of the standby machines. After establishing all necessary notions, we are now in a position to define steady state equations for the queueing system described in Figure 1.

2.1. **Steady state equations.** The steady state model of the queueing system, see Figure 1, is derived by balancing the in-flow and the out-flow rates of birth and death processes, and is given by

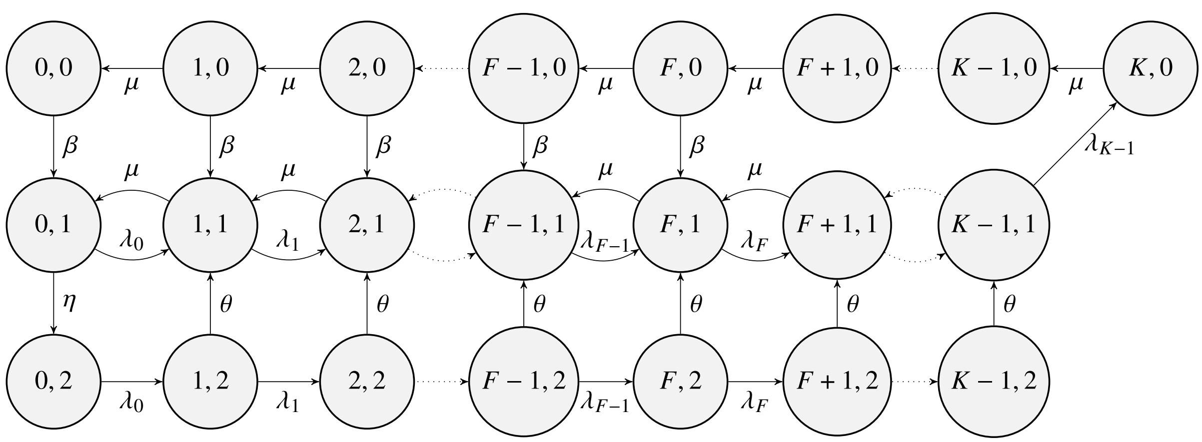

FIGURE 1. Queueing system

(2.1a)

$$J=0\quad\begin{cases}\mu P_{1,0}=\beta P_{0,0}\\ \mu P_{n+1,0}=(\mu+\beta)P_{n,0}; & 1\leq n\leq F\\ P_{n+1,0}=P_{n,0}; & F+1\leq n\leq K-1\\ \lambda_{K-1}P_{K-1,1}=\mu P_{K,0}\end{cases}$$

(2.1b)

$$J=1\quad\begin{cases}\mu P_{1,1}+\beta P_{0,0}=(\lambda_0+\eta)P_{0,1}\\ \lambda_{n-1}P_{n-1,1}+\mu P_{n+1,1}+\beta P_{n,0}+\theta P_{n,2}=(\lambda_n+\mu)P_{n,1}; & 1\leq n\leq F\\ \lambda_{n-1}P_{n-1,1}+\mu P_{n+1,1}+\theta P_{n,2}=(\lambda_n+\mu)P_{n,1}; & F+1\leq n\leq K-2\\ \lambda_{K-2}P_{K-2,1}+\theta P_{K-1,2}=(\lambda_{K-1}+\mu)P_{K-1,1}\end{cases}$$

(2.1c)

$$J=2\quad\begin{cases}\eta P_{0,1}=\lambda_0 P_{0,2}\\ \lambda_n P_{n,2}=(\theta+\lambda_{n+1})P_{n+1,2} & 1\leq n\leq K-3\\ \lambda_{K-2}P_{K-2,2}=\theta P_{K-1,2}\end{cases}$$

normalizing condition:

(2.1d)

$$\sum_{n=0}^{K}P_{n,0}+\sum_{j=1}^{2}\sum_{n=0}^{K-1}P_{n,j}=1.$$

It is worth noting that the algebraic system (2.1) is linear in probabilities $P_{n,0}$ for $n=0,\dots K$, and $P_{n,j,}$ for $n=0,\dots K-1$ and $j=1,2$. In general, algorithms for solving linear systems have cubic computational complexity with the size of the system. However, due to special structure and sparsity of the linear system (2.1), it has linear computational complexity.

We now propose a technique to solve the linear system (2.1) with linear computational complexity:

Algorithm 1: Newton's method for solution of the state space system

Result: $P_{n,0}$ for $n = 0, \ldots, K$, and $P_{n,j}$ for $n = 0, \ldots K-1$ and $j = 1, 2$.

Initialize: $P_{0,0} = \alpha > 0, P_{0,1} = \beta > 0, k_0 = K$, and $k_j = K - 1$ for $j = 1, 2$.

for $j = 0$ *to* 2 **do**

  for $n = 0$ *to* k_j **do**

   Evaluate $P_{n,j}$ in terms of $P_{0,0}$ and $P_{0,1}$ using system (2.1).

  end

end

Evaluate:

$$F\left(P_{0,0}, P_{0,1}\right) := \begin{pmatrix} \lambda_{K-1} P_{K-1,1} - \mu P_{K,0} \\ \sum_{n=0}^{K} P_{n,0} + \sum_{j=1}^{2} \sum_{n=0}^{K-1} P_{n,j} - 1 \end{pmatrix}$$

Find $P_{0,0}$ and $P_{0,1}$ using Newton's update:

$$\begin{pmatrix} P_{0,0} \\ P_{0,1} \end{pmatrix} = \begin{pmatrix} P_{0,0} \\ P_{0,1} \end{pmatrix} + DF^{-1}\left(P_{0,0}, P_{0,1}\right) F\left(P_{0,0}, P_{0,1}\right).$$

for $j = 0$ *to* 2 **do**

  for $n = 0$ *to* k_j **do**

   Evaluate $P_{n,j}$ in terms of $P_{0,0}$ and $P_{0,1}$ using system (2.1).

  end

end

Note that the algorithm 1 need only $O(K)$ evaluations where K is the number of non-zero elements in the Linear system. Therefore, the algorithm 1 solves the linear system (2.1) with optimal computational complexity.

3. System Performance

For a given set of parameters, we solve the linear system (2.1) and subsequently evaluate the performance of the queueing system.

3.1. **Performance Measures.**

Let us introduce some important system performance measures by using steady state probabilities $P_{i,j}$ for M/M/1/K queuing system with F-policy.

- Expected number of failed machines in the system

$$E(n) = \sum_{j=0}^{2} \sum_{n=0}^{K-1} nP_{n,j} + KP_{K,0}.$$

- Expected number of operating machines in the system

$$E(o) = M - \sum_{j=0}^{2} \sum_{n=S}^{K-1} (n-S)P_{n,j} - (K-S)P_{K,0}.$$

- Expected number of standby machines in the system

$$E(s) = \sum_{j=0}^{2} \sum_{n=0}^{S} (S-n)P_{n,j}.$$

- Probability that the server is on vacation

$$P(v) = \sum_{n=0}^{K-1} P_{n,2}.$$

- Probability that the server is busy

$$P(b) = \sum_{j=0}^{1} \sum_{n=0}^{K-1} P_{n,j} + P_{K,0}.$$

- Probability that the server is idle
- Probability that the server allows the failed machines to join the system

$$P(f) = \sum_{n=0}^{K} P_{n,1} + \sum_{n=0}^{K} P_{n,2}.$$

- Probability that the system is blocked

$$P(d) = \sum_{n=0}^{K} P_{n,0}.$$

3.2. **Cost optimization.** Let us define the operating cost of the system that is the sum of all performance measures multiplied by a constant factor. The cost associated with each performance measure is given as:

- $C_h \equiv$ Holding cost per unit time for each failed machine
- $C_s \equiv$ cost per unit time for spare machine
- $C_o \equiv$ cost per unit time for operating machine
- $C_b \equiv$ Cost per unit time for the busy server
- $C_d \equiv$ Fixed cost per unit time when the system is blocked
- $C_f \equiv$ Cost per unit time for allowing customers to enter the system
- $C_m \equiv$ Cost per unit time for providing the service with rate μ
- $C_k \equiv$ Fixed cost associated to the capacity of the system
- $C_e \equiv$ Fixed cost per unit time due to vacation rate θ
- $C_{bt} \equiv$ Cost per unit time for setup of the server

Therefore, the operating cost of the system in steady state is given by

$$\begin{aligned} \mathscr{C} =& C_h E(n) + C_s * E(s) + C_o * E(o) + C_b P(b) + C_v * P(v) + C_d P(d) \\ &+ C_i * P(i) + C_f P(f) + C_m \mu + C_k K + C_e \eta + C_{bt}\beta \end{aligned} \tag{3.1}$$

One of the key ingredients in design of a queueing system is to find a right set of parameters such that the system operating cost (3.1) is minimal. Suppose we wish to design a queueing system in which the queue length threshold F and the service time μ are to be chosen such that the operating cost (3.1) of that queueing system is minimal. The constrained static optimization for minimizing the system operating cost is given by

$$\begin{aligned} &\underset{\{F,\mu\}}{\text{minimize}}\ \mathscr{C}(F,\mu) \\ &\text{subject to} \begin{cases} 0 \le \mu \le 6, \\ 1 \le F \le S, \\ F \in \mathbb{N}. \end{cases} \end{aligned} \tag{3.2}$$

It is evident that the optimization problem (3.2) is a mixed integer nonlinear program as $F \in \mathbb{N}$, and therefore, standard optimization techniques can't be employed directly. However the cost function (3.1) in the feasible region allows its convex extension to the reals along F. Such convex relaxation of the integer programming problem (3.2) leads to a nonlinear programming problem which is relatively easy to solve as compared to the integer program. To fix notation, let us define

$$\lfloor F \rfloor \coloneqq \sup_{y} \{y \in \mathbb{N} \mid y \le F\}, \quad \text{and} \quad \lceil F \rceil \coloneqq \inf_{y} \{y \in \mathbb{N} \mid y \ge F\}.$$

Let us define the convex extension and establish the fact that an optimal solution of the relaxed problem corresponds to a solution of the integer programming problem (3.2). The convex extension of the cost is given by:

$$\mathbb{R}_{\ge 0} \times \mathbb{R} \ni (F,\mu) \mapsto \widetilde{\mathscr{C}}(F,\mu) \coloneqq \mathscr{C}(\lfloor F \rfloor,\mu) + (F - \lfloor F \rfloor)\left(\mathscr{C}(\lceil F \rceil,\mu) - \mathscr{C}(\lfloor F \rfloor,\mu)\right) \in \mathbb{R}_{\ge 0}. \tag{3.3}$$

Therefore the nonlinear programming corresponding to the integer program (3.2) for the cost function (3.3) is defined as

$$\begin{aligned} &\underset{\{F,\mu\}}{\text{minimize}}\ \widetilde{\mathscr{C}}(F,\mu) \\ &\text{subject to} \begin{cases} 0 \le \mu \le 6, \\ 1 \le F \le S. \end{cases} \end{aligned} \tag{3.4}$$

Lemma 3.1. Suppose the cost function (3.3) is convex in the feasible region

$$\widetilde{\Omega} := \{(F,\mu) \in \mathbb{R}_{\geq 0} \times \mathbb{R} \mid 1 \leq F \leq S \text{ and } 0 \leq \mu \leq 6\},$$

and (F^*, μ^*) be an optimal solution of the nonlinear programming problem (3.4). Then the set

$$\Omega^* := \{(\lceil F^* \rceil, \mu^*),\ (\lfloor F^* \rfloor, \mu^*)\},$$

characterizes optimal solutions of the integer programming problem (3.2).

Proof. Let

$$\Omega := \{(F,\mu) \in \mathbb{N} \times \mathbb{R} \mid 1 \leq F \leq S \text{ and } 0 \leq \mu \leq 6\}.$$

be the feasible region of the integer programming problem (3.2). Note that if $F^* \in \mathbb{N} \cap [1,6]$ then the claim trivially holds true. Suppose $(F^*, \mu^*) \in \widetilde{\Omega} \setminus \Omega$ is an optimal point of the optimization problem (3.4), i.e.,

$$\widetilde{\mathscr{C}}(F^*, \mu^*) \leq \widetilde{\mathscr{C}}(F,\mu) \text{ for all } (F,\mu) \in \Omega.$$

It is easy to conclude, in particular, for $\lfloor F^* \rfloor < F^* < \lceil F^* \rceil$ that

$$\widetilde{\mathscr{C}}(F^*, \mu^*) \leq \widetilde{\mathscr{C}}(\lceil F^* \rceil, \mu^*) \quad \text{and} \quad \widetilde{\mathscr{C}}(F^*, \mu^*) \leq \widetilde{\mathscr{C}}(\lfloor F^* \rfloor, \mu^*). \tag{3.5}$$

Therefore, by definition of $\widetilde{C}$, i.e.,

$$\begin{aligned} \widetilde{\mathscr{C}}(F,\mu) &:= \mathscr{C}(\lfloor F \rfloor, \mu) + (F - \lfloor F \rfloor)\,(\mathscr{C}(\lceil F \rceil, \mu) - \mathscr{C}(\lfloor F \rfloor, \mu)) \\ &= \mathscr{C}(\lceil F \rceil, \mu) + (F - \lceil F \rceil)\,(\mathscr{C}(\lceil F \rceil, \mu) - \mathscr{C}(\lfloor F \rfloor, \mu)) \end{aligned} \tag{3.6}$$

and (3.5), one gets

$$\left(\widetilde{\mathscr{C}}(\lceil F^* \rceil, \mu^*) - \widetilde{\mathscr{C}}(\lfloor F^* \rfloor, \mu^*)\right) \geq 0 \text{ and } \left(\widetilde{\mathscr{C}}(\lceil F^* \rceil, \mu^*) - \widetilde{\mathscr{C}}(\lfloor F^* \rfloor, \mu^*)\right) \leq 0. \tag{3.7}$$

Consequently, using (3.7) and (3.6), it is easy to conclude that

$$\widetilde{\mathscr{C}}(F^*, \mu^*) = \widetilde{\mathscr{C}}(\lceil F^* \rceil, \mu^*) = \widetilde{\mathscr{C}}(\lfloor F^* \rfloor, \mu^*).$$

Hence, we proved that the set Ω^* characterizes optimal points of the nonlinear programming (3.4). Further, we know that $\Omega \subset \widetilde{\Omega}$ and

$$\mathscr{C}(F,\mu) = \widetilde{\mathscr{C}}(F,\mu) \ \text{ for all } (F,\mu) \in \Omega,$$

and hence, for each $(\tilde{F},\tilde{\mu}) \in \Omega^*$,

$$\mathscr{C}(\tilde{F},\tilde{\mu}) \leq \mathscr{C}(F,\mu) \ \text{ for all } \ (F,\mu) \in \Omega.$$

This proves the assertion. □

The optimization (3.4) is solved using nonlinear programming techniques such as interior point methods, sequential quadratic program (SQP). We have conducted our numerical experiments in MATLAB using its optimization toolbox.

Remark 1. It is worth noting that the proposed algorithm for solving the linear system to determine steady-state probabilities has the optimal computational complexity and therefore, the optimization (3.4) is computationally less intensive as compared to the existing literature.

4. Numerical Experiments

The following system parameters have been considered for the numerical experiments:

$$M = 10,\ S = 5,\ \lambda_d = 4.5,\ \lambda = 2,\ \theta = 4,$$

$$\alpha = 5,\ \beta = 1,\ \eta = 5,\ K = 15,\ F = 2.$$

The cost factors associated with performance measures are

$$C_h = 50,\ C_s = 10,\ C_o = 30,\ C_b = 500,\ C_m = 1,\ C_e = 0,$$

$$C_v = 500,\ C_d = 100,\ C_i = 200,\ C_f = 10,\ C_{bt} = 0,\ C_k = 0.$$

Under these set of system constants, the cost function (3.1) is convex in the feasible region

$$\widetilde{\Omega} := \{(F, \mu) \in \mathbb{R}_{\geq 0} \times \mathbb{R} \mid 1 \leq F \leq S \;\text{ and }\; 0 \leq \mu \leq 6\},$$

as is shown in Figure 2. The optimal set of parameters $(F^*, \mu^*) = (2, 4.01)$ are obtained by solving the nonlinear program (3.4) in MATLAB. Moreover, we illustrate variations in

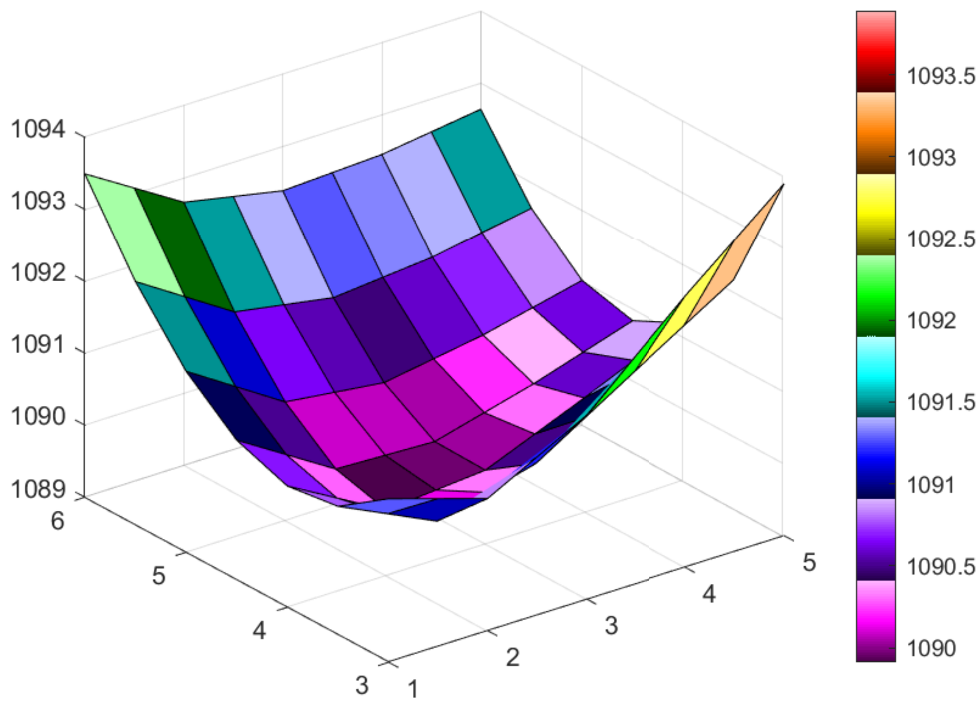

FIGURE 2. The queueing system operating cost.

the expected number of failed machines in the system $E(n)$ with different set of system parameters. Note that the $E(n)$ is convex in nature along μ, and it increases with increase in K and F but decreases with increase in parameters M and λ_d, see Figure 3. In an identical manner, $E(n)$ is concave in nature along λ, and it increases with increase in K and F but decreases with increase in parameters M and λ_d, see Figure 4.

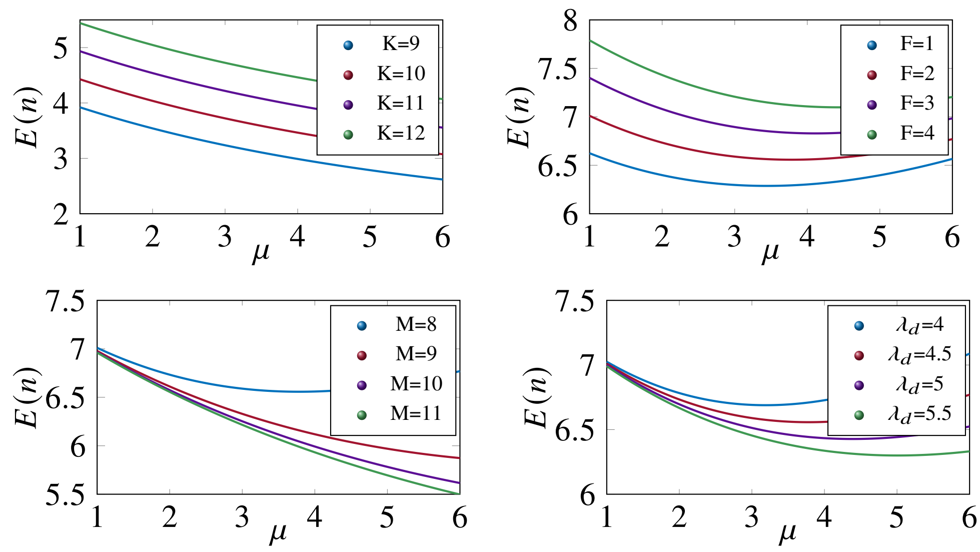

FIGURE 3. Change in $E(n)$ along μ.

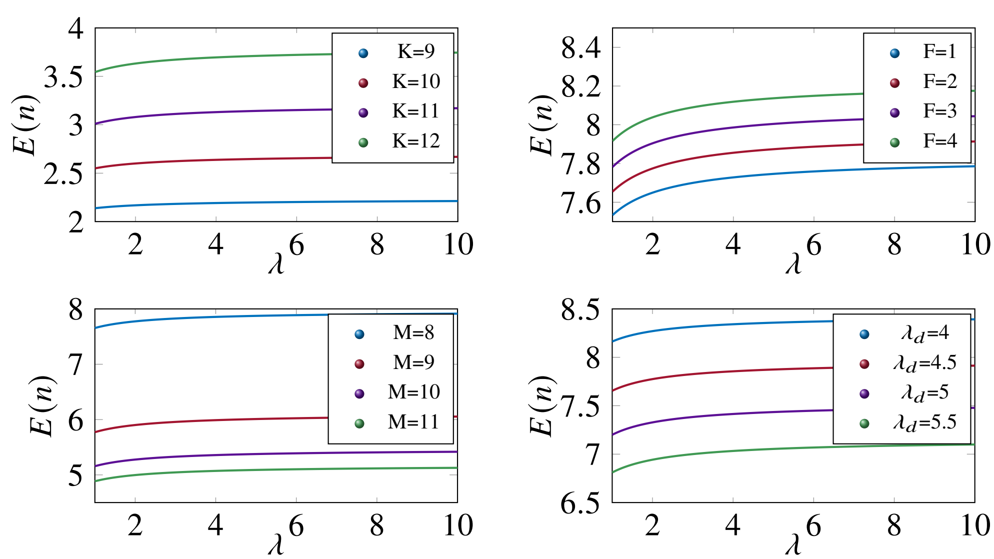

FIGURE 4. Change in $E(n)$ along λ.

5. CONCLUSION AND FUTURE WORK

In this article, we proposed a novel algorithm to determine an optimal policy for the queueing system that allows server vacation under $F-$ policy. The proposed algorithm is computationally less intensive as compared to existing algorithms. In Future, we would like to employ to proposed algorithm to various queuing systems and study various performance measures of such queuing systems.

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