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COST ANALYSIS OF DEGRADED MACHINING SYSTEM WITH SPARE, COMMON CAUSE FAILURE AND OPERATING UNDER VARIABLE SERVICE RATE

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Abstract. In this Paper, we have studied the machining system with finite number of operating machines along with warm standby machines under the supervision of R heterogeneous repairmen. The repairmen are servicing with variable service rate. Whenever any machine fails, it is immediately replaced by available standby machine. Machine may also fail due to common cause. Once all the standbys are exhausted, the system has extra burden to share load of failures and it starts to work under degraded mode. It is assumed that, the time-to-failure and time-to-service of the machines follows the exponential distribution. Steady state probability is evaluated using recursive algorithm. We have developed a cost model to obtain the optimum number of repairmen and standbys maintaining the system availability at minimum specified level. Under optimum operating conditions, various system performance measures are evaluated and sensitivity analysis is also performed.

1. Introduction

We deal with a degraded machining system with warm standbys and common cause failure under variable service rate where failed machines (operating or standby) are operated by one or more repairmen on availability at repair facilities. Each repairman is servicing of failed machines with repair rate μ_1 until system gets $i(\geq R)$ failed

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machine, after this repairmen switched on to service rate $\mu(\geq \mu_1)$.

Chien [2] presented a model for preventive replacement of failure. By this model, at the outset, before the ordering spares we obtain the optimal number of minimum repairs which is helpful in minimizing the system cost. For the smooth functioning of any machining system, availability of standby support is very much helpful. It's also prevents the loss of production as well as improve the system reliability.

The machine repair problem with spares has been studied by many researchers since long including Gupta and Rao [4] , Jain et al. ([8],[9]), Sharma [18], Jain and Preeti [6], Shree et al. [19] , Jharotia and Sharma [11], Shekhar et al. ([21],[22]) etc. Once all the standbys machines are exhausted then the system has extra burden to share load of failure and it starts to work under degraded mode. Jain and Upadhyaya [10] studied the degraded machining system with multiple vacations and multiple type spares under threshold N-policy. Kumar and Jain ([15],[16]), Jharotia and Sharma [13] also studied the machine repair problem with degraded failure.

Common cause failure affects the reliability and availability of the repairable system. Several situations like voltage shock fluctuation, humidity, temperature are very common failure in machining system. Works in this area have been done by Hughes [5], Kvam and Miller [17], Dai et al. [3] etc. Jain et al. [7] investigated the machine repair problem using common cause failures and switching failure. Jharotia and Sharma [12] also discussed the common cause failures in machining system.

Failed machines are operated by one or more repairmen on availability at repair facilities. To avoid the backlog of failed machine, repairmen may increase its service rate; such type of service rate is called variable service rate. Sivazlian and Wang [20], Wang and Sivazlian [23], Wang [25] etc. have studied the machine repair problem by considering variable service rates.

Analysis of any machining system under various cost parameters has been studied by several researchers including Wang and Sivazlian [23], Wang [24], Wand and Wu

[26], Ke and Wang [14], Sharma [18], Jharotia and Sharma ([12], [13]). To the best of our knowledge, the analysis of degrading machining system with warm standby and common cause failure under variable service rate has never been investigated. This model is more profitable with respect to above mentioned models. The main purpose of our paper is threefold.

- Firstly, we solved the steady state equations of our model by using recursive approach.
- Secondly, a cost model is developed to determine optimum number of repairmen and spare machines simultaneously to maintaining the system availability at minimum specified level. Some performance measures like expected number of failed machines, standby machines ideal and busy servers in the system have been evaluated through numerical example.
- Thirdly, we performed sensitivity analysis to study the effect of system parameters on the optimum system cost.

2. Model Descriptions

We have considered a machining system with W warm standbys and M operating machines under supervision of R heterogeneous repairmen. Following are some notations and assumptions taken into consideration for formulation of model.

- The operating as well as standby machines fail under the exponential distribution with mean rate of λ and $\alpha(0 < \alpha < \lambda)$ respectively.
- The failure rate of machining system due to common cause failure follows the Poisson distribution with mean rate λ_c.
- Once all standby machines are exhausted and failure occurs, system work under degraded mode with rate λ_d.

- Whenever operated machine fails, it is immediately replaced by available standby, and failed machine is sent to repair station,where it is repaired with rate μ_1 or μ depending on the number of failed machines in the system.
- It is also assumed that, after switching of standby machine into operating state, the failed characteristics of standby machine is similar to an operating machine and the switch over time is negligible.
- Once a failed machine is repaired, it is as good as a new machine and joins into standbys or operating state. It may go into operating state if there are less number of operating machines as required initially i.e. the system is in short mode.
- Each repairman serve of failed machines with rate μ_1 until there are $i(\geq R)$ failed machines in the system, after this repairman switch on to service rate $\mu(\geq \mu_1)$.

3. Steady-state Analysis

In this section, we determine the steady-state probability equations of our model. For which system states describe as follows: $P(i) : i = 0, 1, ..., M + W$, where i denotes the number of failed machines of system. The combination of failure rate and service rate of system are as follows -

$$\lambda_i = \begin{cases} M\lambda + (W - i)\alpha + \lambda_c, & 0 \leq i \leq W \\ (M + W - i)\lambda_d, & W + 1 \leq i \leq M + W - 1 \\ 0, & \text{otherwise} \end{cases}$$

$$\mu_i = \begin{cases} i\mu_1, & \text{if} 1 < i < R \\ R\mu, & \text{if} R \leq i \leq M + W \\ 0, & \text{otherwise.} \end{cases}$$

In steady-state, let

$P(0) \equiv$ Probability that there are no failed machines in the system,

$P(i) \equiv$ Probability that there are i failed machines in the system, where $i = 0, 1...M + W$. The steady state equations of model are as follows:

Case I: For $R \leq W$

(3.1)

$$(M\lambda + W\alpha + \lambda_c)P(0) = \mu_1 P(1)$$

(3.2)

$$[M\lambda + (W - i)\alpha + \lambda_c + i\mu_1]P(i) = [M\lambda + (W - i + 1)\alpha + \lambda_c]P(i - 1) + (i + 1)\mu_1 P(i + 1),\ 1 \leq i < R - 1$$

(3.3)

$$[M\lambda + (W - i)\alpha + \lambda_c + i\mu_1]P(i) = [M\lambda + (W - i + 1)\alpha + \lambda_c]P(i - 1) + R\mu P(i + 1),\ i = R - 1$$

(3.4)

$$[M\lambda + (W - i)\alpha + \lambda_c + R\mu]P(i) = [M\lambda + (W - i + 1)\alpha + \lambda_c]P(i - 1) + R\mu P(i + 1),\ R \leq i < W$$

(3.5)

$$[M\lambda_d + R\mu]P(i) = [M\lambda + (W - i + 1)\alpha + \lambda_c]P(i - 1) + R\mu P(i + 1),\ i = W$$

(3.6)

$$[(M + W - i)\lambda_d + R\mu]P(i) = [(M + W - i + 1)\lambda_d]P(i - 1) + R\mu P(i + 1),\ W < i < M + W$$

(3.7)

$$\lambda_d P(M + W - 1) = R\mu P(M + W)$$

Case II: For $R > W$

(3.8)

$$(M\lambda + W\alpha + \lambda_c)P(0) = \mu_1 P(1)$$

(3.9)

$$[M\lambda + (W-i)\alpha + \lambda_c + i\mu_1]P(i) = [M\lambda + (W-i+1)\alpha + \lambda_c]P(i-1) + (i+1)\mu_1 P(i+1),\ 1 \le i < W$$

(3.10)

$$[M\lambda_d + i\mu_1]P(i) = [M\lambda + (W-i+1)\alpha + \lambda_c]P(i-1) + (i+1)\mu_1 P(i+1),\ i = W$$

(3.11)

$$[(M+W-i)\lambda_d + i\mu_1]P(i) = [(M+W-i+1)\lambda_d]P(i-1) + (i+1)\mu_1 P(i+1), W < i < R-1$$

(3.12)

$$[(M+W-i)\lambda_d + i\mu_1]P(i) = [(M+W-i+1)\lambda_d]P(i-1) + R\mu P(i+1),\ i = R-1$$

(3.13)

$$[(M+W-i)\lambda_d + R\mu]P(i) = [(M+W-i+1)\lambda_d]P(i-1) + R\mu P(i+1),\ R \le i < M+W$$

(3.14)

$$\lambda_d P(M+W-1) = R\mu P(M+W)$$

Using this result of general birth and death model

$$P(i) = \prod_{j=1}^{i} \frac{\lambda_{j-1}}{\mu_j} P_0, \tag{3.15}$$

Equations (1) to (14) solved recursively for $R \leq W$ and $R > W$ to obtain the steady-state solution $P(i), i = 1, 2, \ldots, M + W$ given as,

Case I : For $R \leq W$

(3.16)

$$P(i) = \frac{1}{i!}\prod_{j=0}^{i-1}[M\delta_\lambda + (W-j)\delta_\alpha + \delta_{\lambda_c}]P_0,\ 1 \leq i < R$$

(3.17)

$$P(i) = \frac{(\frac{\mu_1}{\mu})^{i-R+1}}{R!R^{i-R}}\prod_{j=0}^{i-1}[M\delta_\lambda + (W-j)\delta_\alpha + \delta_{\lambda_c}]P_0,\ R \leq i \leq W$$

(3.18)

$$P(i) = \frac{(\frac{\mu_1}{\mu})^{i-R+1}}{R!R^{i-R}}\prod_{k=M+W+1-i}^{k=1}(k\delta_{\lambda_d})\prod_{j=0}^{W}[M\delta_\lambda + (W-j)\delta_\alpha + \delta_{\lambda_c}]P_0,\ W < i \leq M + W$$

Case II: For $R > W$

(3.19)

$$P(i) = \frac{1}{i!}\prod_{j=0}^{i-1}[M\delta_\lambda + (W-j)\delta_\alpha + \delta_{\lambda_c}]P_0,\ 1 \leq i \leq W$$

(3.20)

$$P(i) = \frac{1}{i!}\prod_{k=M+W+1-i}^{k=M+W+2-R}(k\delta_{\lambda_d})\prod_{j=0}^{W-1}[M\delta_\lambda + (W-j)\delta_\alpha + \delta_{\lambda_c}]P_0,\ W < i < R$$

(3.21)

$$P(i) = \frac{(\frac{\mu_1}{\mu})^{i-R+1}}{R!R^{i-R}}\prod_{k=M+W+1-i}^{k=1}(k\delta_{\lambda_d})\prod_{j=0}^{W}[M\delta_\lambda + (W-j)\delta_\alpha + \delta_{\lambda_c}]P_0,\ R \leq i \leq M + W$$

where $\delta_\lambda = \frac{\lambda}{\mu_1}, \delta_\alpha = \frac{\alpha}{\mu_1}, \delta_{\lambda_c} = \frac{\lambda_c}{\mu_1}, \delta_{\lambda_d} = \frac{\lambda_d}{\mu_1}$
Since, the number of states is finite therefore the steady state solutions $P(i)(i = 0, 1, \ldots, M + W)$ is always exist. $P(0)$ is obtained from the normalizing condition,

$$\sum_{i=0}^{M+W} P(i) = 1 \tag{3.22}$$

4. Performance Measures

With the help of steady state probabilities derived in the previous section, we obtain the following performance measures of our model:

- The expected number of failed machines in the system

$$L_f = \sum_{i=0}^{M+W} iP(i) \tag{4.1}$$

- The expected number of failed machines in the system after all standby machines have failed

$$L_{fs} = \sum_{i=W+1}^{M+W} iP(i) \tag{4.2}$$

- The expected number of operating machines in the system

$$E[O] = M - L_{fs} \tag{4.3}$$

- The expected number of standbys machines in the system

$$E[W] = \sum_{i=0}^{W} (W - i)P(i) \tag{4.4}$$

- The expected number of idle servers in the system

$$E[I] = \sum_{i=0}^{R-1} (R - i)P(i) \tag{4.5}$$

- The expected number of busy servers in the system

$$E[B] = R - E[I] \tag{4.6}$$

Using the concept of Benson and Cox [1], the Machine Availability (MA) and the Operative Utilization (OU) are defined as:

- The machine availability of the system

$$M.A. = 1 - \frac{L_f}{M + W} \tag{4.7}$$

- The fraction of busy servers (operative utilization)

$$O.U. = \frac{E[B]}{R} \tag{4.8}$$

- The availability of the system

$$A_v = \sum_{i=0}^{W} P(i) \tag{4.9}$$

5. Cost Model

For the worthiness of machining system, we constructed the expected cost function with two discrete decision variables R and W. Our main purpose is to determine the optimum number of repairmen R say $R*$ and optimum number of standby machines W, say $W*$, so that we can minimize the cost of the system, maintaining the system availability. The Cost parameters for model defined are as follows:

C_f : − Cost per unit time when one of the machines is failed in the repair facility after all standby machines is exhausted.

C_W : − Cost per unit time when one machine is working as a warm standby,

C_I : − Cost per unit time when one of the repairman is in idle state,

C_B : − Cost per unit time when one of the repairman is in busy state,

The optimum value (R^*, W^*) obtained by cost minimization problem which is stated as follows -

$$Min.F(R, W) = C_f L_{fs} + C_W E[W] + C_I E[I] + C_B E[B]$$

$$s.t. A_\nu \geq A_0$$

Where, $A_v \equiv$ System availability, which is defined by Eq.(29) in section performance measures,

$A_0 \equiv$ Minimum specified level of system availability.

6. Performance Analysis

The decision variables (R and W) having discrete property and to be obtained by any optimization approach. Objective function of the model is highly non-linear and complex. It is too difficult to attain analytical results for the optimum solution (R^*, W^*). So, we may use heuristic approach (direct search method) to find the optimum value of repairmen and spares. We use direct substitution of subsequent values of R and W into the above mentioned expected cost function until the optimum value of $F(R, W)$, say $F(R^*, W^*)$ is achieved and all the constraints are satisfied. A Numerical example has been provided in consideration of the following cost parameters: $C_f = C_O = 75, C_W = 50, C_I = 60, C_B = 100$.

Initially, we fixed number of operating machines $M = 5$, upper bound of decision variables R and W is $2M$. We choose $A_0 = 0.9, \alpha = 0.15, \mu_1 = 1, \mu = 3, \lambda = 0.4, \lambda_c = 0.1, \lambda_d = 0.5$. The expected cost of the system $F(R, W)$ and system availability A_v are presented by table 1 for different value of R and W. In this table, we observe that the minimum cost per day 257.80 and system availability $A_v = 0.901$ is obtained at $R^* = 2$ and $W^* = 3$. The maximum expected cost $F(R, W)$, system availability A_v and other performance measures $L_f, E[O], E[W], E[I], E[B], MA$ and $O.U.$ are shown in Table 2 for different values of λ. Table 2 shows that $F(R^*, W^*)$ as well as optimum number of spare (W^*) increases as λ increases.

Effect of system parameters on optimum cost: In figure 1, we fix the value of $\mu_1 = 1, \mu = 3, \lambda_c = 0.1, \lambda_d = 0.5$ and for the different values of $\alpha = 0.1, 0.15, 0.2$, we plot the curves. Figure 1 shows that optimum cost decrease significantly as failure rate λ increase for different values of α but after $\lambda = 0.5$ optimum cost almost constant.

TABLE 1. The expected cost $F(R, W)$ and the system availability A_v under optimum operating condition

R W	1	2	3	4	5
1	152.25	161.65	179.5	202.23	227.02
	0.433	0.567	0.661	0.727	0.773
2	221.09	226.25	**257.80**	300.37	346.58
	0.506	0.772	**0.901**	0.957	0.981
3	286.57	299.34	320.63	363.28	411.09
	0.501	0.711	0.913	0.975	0.992
4	367.58	361.91	388.18	422.72	469.55
	0.335	0.709	0.851	0.967	0.992
5	439.60	440.82	449.2	485.66	528.14
	0.250	0.550	0.851	0.931	0.988

TABLE 2. The expected cost $F(R^*, W^*)$, system availability A_v and other performance measures under optimum operating condition.

λ	0.4	0.5	0.6	0.7
$R*, W*$	(1,5)	(2,3)	(2,4)	(2,5)
$F(R*, W*)$	234.28	257.80	299.12	341.50
A_v	0.908	0.901	0.901	0.907
$E[O]$	4.8878	4.8945	4.8959	4.9031
$E[W]$	2.6773	1.6608	2.4453	3.2740
$E[I]$	0.1998	0.8286	0.7737	0.7366
$E[B]$	0.8001	1.1113	1.2262	1.2623
MA	0.7565	0.8194	0.8156	0.8177
OU	0.8001	0.5856	0.6131	0.6316

In figure 2, we fix the value of $\mu_1 = 1, \mu = 3, \lambda_c = 0.1, \lambda_d = 0.5$ and plot the curves corresponding to different values of $\lambda = 0.3, 0.4, 0.5$ respectively. Figure 2 shows that how the failure rate of spares affect the optimum cost by varying the failure rate λ. It shows that optimum cost decrease as λ increases. In figure 3, we note the effect of

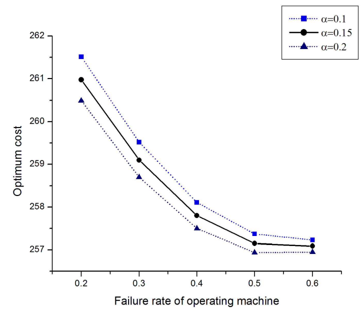

FIGURE 1. Effect of failure rate of operating machines.

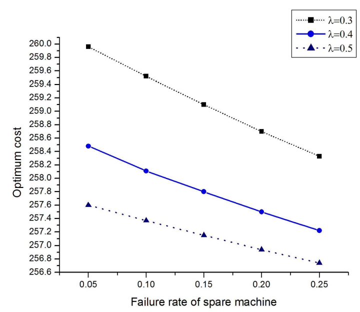

FIGURE 2. Effect of failure rate of spare machines.

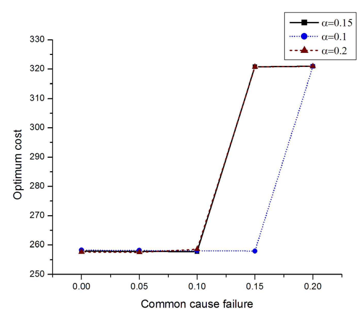

FIGURE 3. Effect of common cause failure.

common cause failure λ_c on the optimum cost by varying the failure rate λ for fixed values of $\mu_1 = 1, \mu = 3, \alpha = 0.15, \lambda_d = 0.5$. Figure 3 shows that optimum value is almost constant as common cause failure increases but variate after $\lambda_c = 0, 10$. The effect of degraded failure mode on optimum cost described in figure 4. In figure 5,

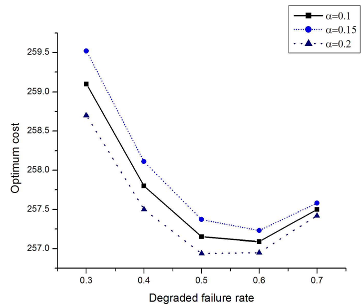

FIGURE 4. Effect of degraded failure mode.

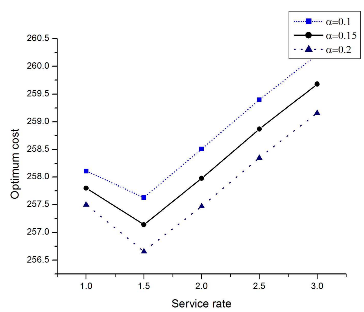

FIGURE 5. Effect of service rate of operating machines.

we have drawn the graph between optimum cost and service rate μ for various values of α. Initially we fix $\mu_1 = 1, \alpha = 0.15, \lambda_c = 0.1, \lambda_d = 0.15$. In figure 5, we see that the optimum cost decrease as service rate μ increase and optimum cost increase after $\mu = 1.5$.

Overall conclusion from above analysis is that system is more cost effective as well as high system availability achieve for decreasing of failure rate of operating units. Common cause failure rate $\mu = 0.10$ is optimum value for the system at this value our model is more cost effective.

TABLE 3. Comparative study with Wang and Sivazlian (1992)

	Present work	**Wang and Sivazlian (1992)**
$R*, W*$	(2,3)	(3,7)
$F(R*, W*)$	257.80	446.15
A_ν	0.901	(0.906

7. Comparative Study

Table-3, presents the comparative study of optimum number of standbys, repairmen and optimum cost of the machining system with Wang and Sivazlian (1992). From this analysis, we see that the present study is more reliable.

8. CONCLUDING REMARKS

In this paper, we have studied the degraded machining system to analyse the performance of the system under the concept of common cause failure. This stochastic model is deals with different embedded engineering systems including electronics and communication system/ electrical and computer systems etc. The optimum number of repairmen and spare machines are determined with the help of a cost model, the system availability is ensured at its minimum level ($A_0 = 0.9$). The model produced by us is more cost effective and reliable in comparison to earlier works done by Wang and Sivazlian (1992) as shown in table 3. Numerical illustration is very much helpful to analyze the effects on performance measures by variation of system parameters. Our model may be helpful in production and manufacturing industries and may help decision makers to design an effective system. Our model can be extended for other types of spares or non-spares.

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