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A NEW VIEW ON FUZZY CODES AND ITS APPLICATION

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Abstract. In this paper, the notion of fuzzy complement, fuzzy intersection and fuzzy union on fuzzy codes are studied with their respective axioms and also the arithmetic operations on fuzzy codes are given. The role of these operators on the dual of fuzzy codes are studied and finally the concept of super increasing sequence of fuzzy codes is introduced along with its application.

1. Introduction

The way toward sending and getting information and data has constantly assumed a noteworthy part in correspondence. In any case, these information when transmitted run over numerous unsettling influence because of which there is a plausibility that a man may not get the message appropriately. Codes are utilized for blunder revision and furthermore in organize frameworks. The notion of coding theory was developed by Shanon in 1948. The linear codes over a finite ring were studied by Blake. The theory of error correcting codes was initially studied by Pless and later this was developed by Mc Williams and Sloane.

The concept of fuzzy sets was introduced by Zadeh in 1967 [5]. A fuzzy set can be defined mathematically by assigning to each possible individual in the universe of discourse a value representing its grade of membership in the fuzzy set[2]. These

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membership grades are represented by real number values ranging in the closed interval between 0 and 1. A new perspective of coding theory based on fuzzy sets was studied by Ayten Ozkan and Mehmet Ozkan[1].

This paper is organized as follows: In Section 2, the operators of Fuzzy t-norm, Fuzzy t-conorm and Fuzzy Complement on Fuzzy Codes and their respective axioms are studied. In Section 3, the concept of Strong Fuzzy Code is introduced and suitable example is given and finally an application of Strong Fuzzy Code is provided.

2. Preliminaries

Definition 2.1. [4] Let F be a set of elements on which two binary operations, called addition $'+'$ and multiplication $'.'$, are defined. The set F together with the two binary operations + and . is a *field* if the following conditions are satisfied:

(i) $(F, +)$ is a commutative group. The identity element with respect to addition is called the *zero* element or the additive identity of F and is denoted by 0.

(ii) $(F-\{0\}, .)$ is a commutative group. The identity element with respect to multiplication is called the *unit* element or the multiplicative identity of F and is denoted by 1.

(iii) Multiplication is *distributive* over addition; that is, for any three elements a,b and c in F,

$$a.(b+c)=a.b + a.c.$$

.

A field with finite order is a *finite field*.

Definition 2.2. [1] A q-ary code is a set of sequences of symbols where each symbol is chosen from a set $F_q=\{\lambda_1,\lambda_2,...,\lambda_q\}$ of q distinct elements.

Definition 2.3. [1] A *Binary Code* is a sequences of 0_s and 1_s which are called codewords.

Definition 2.4. [1] Let (F_q^n) denote the set of all ordered n-tuples a=$a_1, a_2, ...a_n$ where each $a_i \in F_q$. The elements of F_q^n are called *vectors or words*

Definition 2.5. [3] Let F be a finite field and n be a positive integer. Let C be a subspace of the vector space V= F^n. Then C is called a *linear code* over F.

Definition 2.6. [3] Let C be a linear [n,k]-code. Let G be a k×n matrix whose rows form basis of C. Then G is called *generator matrix* of the code C.

Definition 2.7. [1] The weight w(x) of a vector x in F_2^n is defined to be the number of non-zero entries of x.

Definition 2.8. [1] Suppose x is a codeword of C. If $w_1, w_2, ..., w_k$ are defined to be the positions of 1_s in x, then $w_1 + ... + w_k$ are called relative weight of codeword x Since 11...1 is a codeword of C then its relative weight is

$$1 + 2 + ... + n = \frac{n(n+1)}{2}$$

Thus this weight is called a *Maximum Relative Weight* of code C .

Definition 2.9. [1] The *Relative Weight* of a codeword x in $(F_2)^n$ is denoted by

$$J(x) = \frac{\text{w(x)}}{\text{Maximum Relative Weight}}$$

Definition 2.10. [3] The *Exclusive Or* is a basic computer operation denoted by XOR or $\oplus$, which takes two individual bits $\beta \in \{0,1\}$ and $\beta' \in \{0,1\}$ and yields

$$\beta \oplus \beta' = \begin{cases} 0 & \text{if } \beta \text{ and } \beta' \text{ are same} \\ 1 & \text{if } \beta \text{ and } \beta' \text{ are different} \end{cases}$$

Definition 2.11. [1] Let C be a Code. The function $J : C \to [0,1]$ is said to be a *Fuzzy Code* if it satisfies the following conditions:

(1) $J(x+y) \geq$ min {J(x),J(y)}
(2) $J(-x) = J(x)$
(3) $J(xy) \leq$ max {J(x), J(y)}, for all x,y $\in$ C

3. Operators on Fuzzy Codes

In this section, the operators Fuzzy Complement, Fuzzy Intersection and Fuzzy Union of fuzzy codes are defined and suitable examples are provided along with their respective axioms. Also, the arithmetic operations on fuzzy codes are given.

Let $\mathfrak{C}$ be a Code and $\{C_{r_1}, C_{r_2},...,C_{r_m}\}$ be the codewords with varying length in $\mathfrak{C}$. The membership value of $\mathfrak{C}$ is denoted by $\mu_{\mathfrak{C}}$(x), and it is defined in terms of the relative weight of each codeword as follows:

If $\mathfrak{C} = \{C_{r_1}, C_{r_2},...,C_{r_m}\}$, then $\mu_{\mathfrak{C}}(x) = \{J(C_{r_1})$, J$(C_{r_2})$, ..., J$(C_{r_m})\}$, where J$(C_{r_i})$ denotes the relative weight of the codeword C_{r_i} for i = 1,2,...,m.

Definition 3.1. Let $\mathfrak{C}$ be a Code and $\{C_{r_1}, C_{r_2},...,C_{r_m}\}$ be the codewords with varying length in $\mathfrak{C}$. Let $J(C_{r_i})$ be the respective fuzzy codes associated with the codewords C_{r_i} in $\mathfrak{C}$,where i = 1,2,...,m. Then, the fuzzy complement of $J(x)$ denoted by $c(J(x))$ is obtained by subtracting the relative weight of each member of $\mathcal{C}$ from 1.

Example 3.1. Let $\mathfrak{C} = \{0000, 1101, 0100, 1100, 1111\}$. Then the relative weights of the codewords can be written as follows:

In $\mathfrak{C}, J(0000) = 0, J(1101) = 0.7, J(0100) = 0.2, J(1100) = 0.3$ and $J(1111)$=1.
Then c($J(0000)$)=1, c($J(1101)$)= 0.3, c($J(0100)$)=0.8, c($J(1100)$)=0.7, c($J(1111)$)=0.

The fuzzy complement of a fuzzy code satisfies the following axioms for all J(x), J(y) $\in$ [0,1]:

Axiom c1. $c(J(0_c)) = 1$ and $c(J(1_c))$ =0 (*boundary conditions*);

Axiom c2. $J(x) \leq J(y)$, implies $c(J(x)) \geq c(J(y))$ (*monotonicity*);

These two axioms are the axiomatic skeleton for fuzzy complements of fuzzy codes. The other axioms that are generally listed are the following

Axiom c3. c is a continuous function;

Axiom c4. c is involutive, that is $c(c($J(x)$))$= J(x) for each x $\in \mathfrak{C}$.

Definition 3.2. [**Fuzzy Intersection (t-norms)**] Let $\mathfrak{C}$ be a Code and $\{C_{r_1}$, C_{r_2},...,$C_{r_m}\}$ be the codewords with varying length in $\mathfrak{C}$. Let $J(C_{r_i})$ be the respective fuzzy codes associated with the codewords C_{r_i} in $\mathfrak{C}$,where i = 1,2,...,m. Then the fuzzy intersection between any two fuzzy codes $J(C_{r_j})$ and $J(C_{r_k})$ of the codewords C_{r_j} and C_{r_k} is given by, $\mu_{c_{r_j} \cap\, c_{r_k}} = J(C_{r_j}) \cap J(C_{r_k}) = \min\,\{J(C_{r_j}), J(C_{r_k})\ \}$.

Example 3.2. Let $\mathfrak{C} = \{101,\ 01011,\ 1011000,\ 1001,\ 010111\}$. If $C_1 = 01011$, $C_2 = 1011000$ and $C_3 = 010111$ then, J$(C_1) = 0.73$, J$(C_2) = 0.29$ and J$(C_3) = 0.81$. Hence, min $\{$J$(C_1), J(C_2), J(C_3)\} =$ min [0.73, 0.29, 0.81] $= 0.29$ which corresponds to the codeword 1011000.

A fuzzy intersection t-norm is a binary operation that satisfies the following axioms for all $J(x), J(y), J(z) \in [0,1]$:

Axiom i1. i(J(x), 1) = J(x)(*boundary condition*);

Axiom i2. J(y) $\leq$ J(z)implies i(J(x), J(y)) $\leq$ i(J(x), J(z))(*monotonicity*);

Axiom i3. i(J(x), J(y))= i(J(y), J(x))(*commutativity*);

Axiom i4. i(J(x), i(J(y), J(z)))= i(i(J(x), J(y)), J(z))(*associativity*);

These set of axioms are called the *axiomatic skeleton for fuzzy codes* as an intersection t-norm.

Definition 3.3. [**Fuzzy Union (t-conorms)**] Let $\mathfrak{C}$ be a Code and $\{C_{r_1}, C_{r_2}$,...,$C_{r_m}\}$ be the codewords with varying length in $\mathfrak{C}$. Let $J(C_{r_i})$ be the respective fuzzy codes associated with the codewords C_{r_i} in $\mathfrak{C}$,where i = 1,2,...,m. Then the fuzzy union between any two fuzzy codes $J(C_{r_j})$ and $J(C_{r_k})$ of the codewords C_{r_j} and C_{r_k} is given by, $\mu_{c_{r_j} \cup\, c_{r_k}} = J(C_{r_j}) \cup J(C_{r_k}) = \max\,\{J(C_{r_j}), J(C_{r_k})\ \}$.

Example 3.3. *Let* $\mathfrak{C} = \{110,\ 0111001,\ 01010,\ 111000,\ 1101001,\ 01001110\}$. If $C_1 = 0111001$, $C_2 = 01010$ and $C_3 = 01001110$ then, J$(C_1) = 0.57$, J$(C_2) = 0.4$ and J$(C_3) = 0.56$.

Hence, max $\{\mathrm{J}(C_1), J(C_2), J(C_3)\}$ = max[0.57, 0.4, 0.56] = 0.57 which corresponds to the codeword 0111001.

A fuzzy union t-conorm is a binary operation that satisfies the following axioms for all $J(x), J(y), J(z) \in$ [0,1]:

Axiom u1. u(J(x), 0)=J(x)(*boundary condition*);

Axiom u2. J(y) $\leq$ J(z)implies u(J(x), J(y))$\leq$ u(J(x), J(z))(*monotonicity*);

Axiom u3. u(J(x), J(y))= u(J(y), J(x))(*commutativity*);

Axiom u4.u(J(x), u(J(y), J(z)))= u(u(J(x), J(y)), J(z))(*associativity*);

These set of axioms are called the *axiomatic skeleton for fuzzy codes* as an union t-conorm.

Definition 3.4. Let $\mathfrak{C}_1$, $\mathfrak{C}_2$ be any two codes and $\{C_{r_1}, C_{r_2},...,C_{r_m}\}$, $\{C_{s_1}, C_{s_2},...,C_{s_n}\}$ be the codewords of varying length in $\mathfrak{C}_1$ and $\mathfrak{C}_2$ respectively. Let $J(C_{r_i})$ be the respective fuzzy codes associated with the codewords C_{r_i} in $\mathfrak{C}_1$, where i = 1,2,...,m and Let $J(C_{s_j})$ be the respective fuzzy codes associated with the codewords C_{s_j} in $\mathfrak{C}_2$, where j = 1,2,...,n. Then the arithmetic operations on the fuzzy codes are as follows:

- The addition of $J(C_{r_i})$ and $J(C_{s_j})$ is

  $J(C_{r_i}) + J(C_{s_j}) = [J(C_{r_1}) + J(C_{s_1}), J(C_{r_2}) + J(C_{s_2}),..., J(C_{r_m}) + J(C_{s_n})]$
- The subtraction of $J(C_{r_i})$ and $J(C_{s_j})$ is

  $J(C_{r_i}) - J(C_{s_j}) = [J(C_{r_1}) - J(C_{s_n}), J(C_{r_2})$ - $J(C_{s_{n-1}}),..., J(C_{r_m}) - J(C_{s_1})]$
- The multiplication of $J(C_{r_i})$ and $J(C_{s_j})$ is

  $J(C_{r_i}) * J(C_{s_j}) = [J(C_{r_1}) * J(C_{s_1}), J(C_{r_2}) * J(C_{s_2}),..., J(C_{r_m}) * J(C_{s_n})]$

4. Dual of a Fuzzy Code

In this section the concept of dual of a fuzzy code and the duality of fuzzy union and fuzzy intersection with respect to the fuzzy complement is studied.

Definition 4.1. Let $\mathfrak{C}$ be a Code and $\{C_{r_1},\ C_{r_2},...,C_{r_m}\}$ be the codewords with varying length in $\mathfrak{C}$. Let $J(C_{r_i})$ be the respective fuzzy codes associated with the codewords C_{r_i} in $\mathfrak{C}$,where i = 1,2,...,m. Then the *dual* of C_i, denoted by $C_i^{\perp}$ is defined by

$$\mathfrak{C}_{\mathrm{i}}^{\perp} = \{\mathrm{y} \in \mathfrak{C} : \mathrm{x} \otimes \mathrm{y} = 0_c \text{ for all } \mathrm{x} \in \mathfrak{C}_{\mathrm{i}}\}$$

Example 4.1. Let,

$$\mathfrak{C}_1 = \{0000,\ 0100,\ 1111\} \text{ and } \mathfrak{C}_2 = \{0000,\ 1001,\ 1111\}. \text{ Then,}$$
$$\mathfrak{C}_1^{\perp} = \{0000,\ 1011,\ 1111\} \text{ and } \mathfrak{C}_2^{\perp} = \{0000,\ 0110,\ 1111\}$$

Definition 4.2. Let $\mathfrak{CC}$ be the collection of codes. Then $\mathfrak{CC}$ is said to be *Complete*, if for each pair of codes $\mathfrak{C}_1$, $\mathfrak{C}_2$ of $\mathfrak{CC}$, $\mathfrak{C}_1$ should be the dual of $\mathfrak{C}_2$ and $\mathfrak{C}_2$ should be the dual of $\mathfrak{C}_1$.

Example 4.2. Let $\mathfrak{CC}$ be a code and if

$$\mathfrak{C}_1 = \{0000,\ 0101,\ 1111\},\ \mathfrak{C}_2 = \{0000,\ 1010,\ 1111\},\ \mathfrak{C}_3 = \{0000,\ 1110,\ 1111\},$$
$$\mathfrak{C}_4 = \{0000,\ 0001,\ 1111\},\ \mathfrak{C}_5 = \{0000,\ 1000,\ 1111\} \text{ and } \mathfrak{C}_6 = \{0000,\ 0111,\ 1111\}.$$

Clearly, $\mathfrak{C}_1$ is the dual of $\mathfrak{C}_2$, $\mathfrak{C}_3$ is the dual of $\mathfrak{C}_4$ and $\mathfrak{C}_5$ is the dual of $\mathfrak{C}_6$. Thus, $\mathfrak{CC}$ is Complete.

Remark 4.1 If $\mathfrak{C}_1$ is the dual of $\mathfrak{C}_2$ then $\mathfrak{C}_1 \otimes \mathfrak{C}_2 = \mathfrak{C}\ (0_c)$.

The operations of Fuzzy Union and Fuzzy Intersection of fuzzy codes are dual with respect to the Fuzzy Complement of fuzzy codes i.e., they satisfy the De Morgan laws $\overline{A \cup B} = \overline{A} \cap \overline{B}$ and $\overline{A \cap B} = \overline{A} \cup \overline{B}$. The fuzzy t-norm i and fuzzy t-conorm u are dual with respect to a fuzzy complement c iff

$$\mathrm{c}(\mathrm{i}(J(x),\ J(y))) = \mathrm{u}(\mathrm{c}(J(x)),\ \mathrm{c}(J(y)) \text{ and } \mathrm{c}(\mathrm{u}(J(x),\ J(y))) = \mathrm{i}(\mathrm{c}(J(x)),\ \mathrm{c}(J(y))$$

Theorem 4.1. Let $\mathfrak{C}$ be a code. For x, y, z $\in \mathfrak{C}$, let J(x), J(y), J(z) be the fuzzy codes of the codewords x,y and z in $\mathfrak{C}$. Then. given a t-conorm u and an involutive fuzzy complement c, the binary operation i on [0,1] defined by

$$\mathrm{i}(J(x),\ J(y)) = \mathrm{c}(\mathrm{u}(\mathrm{c}(J(x)),\ \mathrm{c}(J(y)))\ (4.1)$$

for all a,b $\in$ [0,1] is a t-norm such that$\langle i, u, c\rangle$ is a dual triple.

Proof. To prove that i is a t-norm, it is sufficient to show that i satisfies axioms i1-i4.
(i1) For any $J(x) \in$ [0,1],

$$\begin{aligned} i(J(x),1) &= c(u(c(J(x)),c(1))) && \text{(by (4.1))} \\ &= c(u(c(J(x)),0)) && \text{(by Axiom c1)} \\ &= c(c(J(x)) && \text{(by Axiom i1)} \\ &= J(x) && \text{(by Axiom c4)} \end{aligned}$$

Hence, i satisfies Axiom i1.
(i2) For any $J(x), J(y), J(z) \in$ [0,1] if J(y) $\leq J$(z) ,then c(J(y)) $\geq$ c (J(z)). Moreover,

$$(\mathrm{u}(\mathrm{c}(J(x)),\ \mathrm{c}(J(y))) \geq (\mathrm{u}(\mathrm{c}(J(x)),\ \mathrm{c}(J(z)))) \quad (\text{ by Axiom i2})$$

Thus, from the given equation

$$\begin{aligned} i(J(x),J(y)) &= c(u(c(J(x)),c(J(y))) \leq (u(c(J(x)), \\ &= c(J(z))) \\ &= i(J(x),J(z)). \end{aligned}$$

Thus, i satisfies Axiom i2.
(i3) For any $J(x), J(y) \in$ [0,1], we have

$$\mathrm{i}(J(x),\ J(y)) = \mathrm{c}(\mathrm{u}(\mathrm{c}(J(x)),\ \mathrm{c}(J(y))) = \mathrm{c}\ (\mathrm{u}(\mathrm{c}(J(y)),\ \mathrm{c}(J(x))) = \mathrm{i}(J(y),\ J(x)).$$

Thus, i satisfies Axiom i3.

(i4) For any $J(x), J(y), J(z) \in [0,1]$,

$$
\begin{aligned}
i(J(x), i(J(y), J(z))) &= c(u(c(J(x))), c(i(J(y), J(z)))) && (\text{ by } (4.1)) \\
&= c(u(c(J(x))), c(c((u(c(J(y)), c(J(z))))) && (\text{ by } (4.1)) \\
&= c(u(c(J(x))), u(c(J(y)), c(J(z)))) && (\text{ by Axiom c4}) \\
&= c(u(u(c(J(x)), c(J(y)), c(J(z)))) && (\text{ by Axiom i4}) \\
&= c(u(c(c(u(c(J(x)), (J(y)))))), c(J(z))) && (\text{ by Axiom c4}) \\
&= i(i(J(x), J(y)), J(z)) && (\text{ by } (4.1))
\end{aligned}
$$

Thus, i satisfies Axiom i4, consequently i is a t-norm.

By applying the given equation and Axiom i4, we can now show that i satisfies the De Morgan laws:

$$\mathrm{c}(\mathrm{i}(J(x),\ J(y)) = \mathrm{c}(\mathrm{c}(\mathrm{u}((\mathrm{c}(J(x),\ \mathrm{c}(J(y))))) = \mathrm{u}(\mathrm{c}(J(x),\ \mathrm{c}(J(y)$$

$$\mathrm{i}(\mathrm{c}(J(x)),\ \mathrm{c}(J(y))))=\mathrm{c}(\mathrm{u}(\mathrm{c}(\mathrm{c}(J(x)),\ (\mathrm{c}(\mathrm{c}(J(y))))=\mathrm{c}(\mathrm{u}(J(x),\ J(y))$$

Hence, i and u are dual with respect to c. □

Theorem 4.2. Let $\mathfrak{C}$ be a code. For x, y, z $\in \mathfrak{C}$, let J(x), J(y), J(z) be the fuzzy codes of the codewords x,y and z in $\mathfrak{C}$. Then. given a t-norm i and an involutive fuzzy complement c, the binary operation u on [0,1] defined by

$$\mathrm{u}(J(x),\ J(y)) = \ \mathrm{c}(\mathrm{i}(\mathrm{c}(J(x)),\ \mathrm{c}(J(y)))$$

for all a,b $\in [0,1]$ is a t-conorm such that $\langle i, u, c\rangle$ is a dual triple.

Proof. The proof is similar to the proof of Theorem 4.1. □

5. Super Increasing Sequence of Fuzzy Codes

In this section, the notion of super increasing sequence of fuzzy codes and other definitions based on the concept of super increasing sequence of fuzzy codes are given.

Definition 5.1. Let $\mathfrak{C}$ be a Code and $\{C_{r_1}, C_{r_2},...,C_{r_m}\}$ be the codewords of varying length in $\mathfrak{C}$. Let, $J(C_i)$ be the respective fuzzy codes associated with the codewords C_i in $\mathfrak{C}$ where i = 1,2,...m. A sequence $\mu_{\mathfrak{C}}(x) = \{J(C_{r_1})$, J$(C_{r_2})$, ..., J$(C_{r_m})\}$ is a super increasing sequence of fuzzy codes, if

$$\mathrm{J}(C_{r_{i+1}}) > \mathrm{J}(C_{r_i}) \text{ for all } 1 \leq \mathrm{i} \leq \mathrm{m} - 1.$$

is satisfied.

Example 5.1. Let $\mathfrak{C} = \{1001100, 101110, 0011, 10111\}$. Then the relative weights of the codewords can be written as follows:

$$\mathrm{J}(1001100)=0.36,\ \mathrm{J}(101110) = 0.62,\ \mathrm{J}(0011)=0.7,\ \mathrm{J}(10111)= 0.87.$$

Clearly, $0.36 < 0.62 < 0.7 < 0.87$. Thus, $\mu_{\mathfrak{C}}(x) = \{$J(1001100), J(101110), J(0011), J(10111)$\}$ is a super increasing sequence of fuzzy codes.

Proposition 5.1. Let $\mathfrak{C}$ be a Code and $\{C_{r_1}, C_{r_2},...,C_{r_m}\}$ be the codewords in $\mathfrak{C}$. Let, $J(C_i)$ be the respective fuzzy codes associated with the codewords C_i in $\mathfrak{C}$ where i = 1,2,...m. Then

$$\mathrm{J}(C_k) > \frac{J(C_{k-1})+...+J(C_2)+J(C_1)}{2} \text{ for all } 2 \leq \mathrm{k} \leq \mathrm{m}$$

Proof. We give the proof by induction. If k = 2, then by Definition 5.1, J(C_2) > J(C_1). Thus, J(C_2) > J(C_1 > $\frac{1}{2}$ J(C_1. Hence the proposition is true for k = 2. Suppose that the proposition is true for all $1 \leq$ m $\leq$ k-1, then we have,

$$\begin{aligned} \mathrm{J}(C_{k+1}) > \mathrm{J}(C_k) &= \tfrac{1}{2}\ \mathrm{J}(C_k) + \tfrac{1}{2}\ \mathrm{J}(C_k) \\ &> \tfrac{1}{2}\ \mathrm{J}(C_k) + \tfrac{1}{2}\ [\mathrm{J}(\mathrm{C}_{k-1})+...+\mathrm{J}(\mathrm{C}_2)+\mathrm{J}(\mathrm{C}_1)] \\ &= \tfrac{1}{2}\ [\mathrm{J}(C_k)+\ \mathrm{J}(\mathrm{C}_{k-1})+...+\mathrm{J}(\mathrm{C}_2)+\mathrm{J}(\mathrm{C}_1)] \end{aligned}$$

which completes the proof. □

Definition 5.2. Let $\mathfrak{C}$ be a Code with four codewords and $J(C_i)$ be the respective fuzzy codes associated with the codewords C_i in $\mathfrak{C}$ where i = 1,2,3,4. Let $\mu_{\mathfrak{C}}(x)$ =

{J(C_{r_1}), J(C_{r_2}), J(C_{r_3}), J(C_{r_4})} be a super increasing sequence of four fuzzy codes. The measure of this sequence $\mu_{\mathfrak{C}}(x)$ denoted by $\mathfrak{M}$ is defined as follows:

$$\mathfrak{M} = \frac{1}{2}[\mathrm{J}(C_{r_1})+\mathrm{J}(C_{r_2})+\frac{1}{2}(\mathrm{J}(C_{r_4} - \mathrm{J}(C_{r_3}))]$$

Definition 5.3. Let $\mathfrak{C}$ be a Code with n codewords and $J(C_i)$ be the respective fuzzy codes associated with the codewords C_i in $\mathfrak{C}$ where i = 1,2,...,n. Let $\mu_{\mathfrak{C}}(x)$ = {J(C_{r_1}), J(C_{r_2}),..., J(C_{r_n}))} be a super increasing sequence of fuzzy codes. The magnitude of this sequence denoted by M$_{C_i}$ is defined as follows:

$$M_{C_i} = \begin{cases} \sum_{i=1}^{n} J(C_i) & \text{whenever } 0 < \sum_{i=1}^{n} J(C_i) < 1 \\ \frac{\sum_{i=1}^{n} J(C_i)}{n} & \text{whenever } \sum_{i=1}^{n} J(C_i) > 1 \\ 0 & \text{otherwise.} \end{cases}$$

6. Mathematical Formulation of the problem

In this section, a method of finding the strongest collection of codewords among a group of codes is found out by applying the idea of super increasing fuzzy codes for each collection.

Consider a collection of fuzzy codewords of various lengths. These codewords are grouped in such a way, so that each group has codewords of all length. The sender has to send his encrypted message through any one of the group which is much safer than the rest of the groups. The aim is to find out which group can be used for safer transmission.

6.1. **Procedure for solving the problem.** Consider a collection of codewords $\mathfrak{C}$ = {C_1, C_2,..., C_n}. This collection has codewords of varying length.

Step 1: Modify the group of codewords into separate collection, so that each collection has codewords of all length given in the collection.

Step 2: Compute the relative weights of each codeword.

Step 3: Arrange the codewords in each collection, so that the sequence of relative weights is a super increasing fuzzy sequence of fuzzy codewords.

Step 4: Generate the n x n matrix. For example , for $\mathfrak{C} = \{C_1, C_2, C_3, C_4\}$ the entries are arranged as

$$\begin{pmatrix} C_1 & C_2 \\ C_3 & C_4 \end{pmatrix}$$

Likewise, generate the relative weights of each C_i in the n x n matrix.

Step 5: Calculate the measure of each super increasing sequence of fuzzy codes.

Step 6: Calculate the fuzzy intersection of entries of each row and the fuzzy union of the entries of each column.

Step 7: If the fuzzy intersection of fuzzy union equals the fuzzy union of fuzzy intersection then the collection of codewords which corresponds to that measure is the required group of codewords.

Step 8: If not, then the minor value of each collection is computed. The resulting sequence must also be a fuzzy super increasing sequence.

Step 9: Find the magnitude of each fuzzy codeword and form a square matrix by using all the magnitudes. The highest entry of the resultant matrix corresponds to the required collection of fuzzy codes.

Step 10: If more than one entry in the matrix has the same magnitude, then compute the sum of relative weights in the respective collection. The collection with the highest sum is the required collection of fuzzy codewords.

6.2. **Numerical Example.** Let the collection of codewords be $\mathfrak{C} = \{$11000, 01100, 11100, 11010, 00011, 10011, 11011, 10111, 01111, 1000, 0100, 1100, 0110, 1001, 0101, 0011, 1011, 0111, 010000, 111000, 110100, 101100, 010110, 101110, 011110, 011011, 001111$\}$

Step 1: Grouping these codewords we get, C_1 = {0100, 00011, 011011, 0011111}; C_2 = {1000, 11010, 110100, 0001110}; C_3 = {1100, 11000, 010110, 1000110}; C_4 = {1001, 11100, 010000, 1010000}; C_5 = { 0110, 01111, 011110, 1011100,}; C_6 = {0111, 01100, 001111, 0101011}; C_7 = { 0011, 10111, 101110, 1001100}; C_8 = {1011, 10011, 111000, 1110000}; C_9={0101, 11011, 101100, 0111000}.

Step 2: The relative weights of the codewords are:
J(C_1) = {0.2, 0.6, 0.76, 0.89}; J(C_2) = {0.1, 0.47, 0.33, 0.54}; J(C_3) = {0.3, 0.2, 0.52, 0.43}; J(C_4) = {0.5, 0.4, 0.09, 0.14}; J(C_5) = {0.5, 0.93, 0.67, 0.46}; J(C_6) = {0.9, 0.33, 0.86, 0.68}; J(C_7) = {0.7, 0.87, 0.62, 0.36}; J(C_8) = {0.8, 0.67, 0.28, 0.11}; J(C_9) = {0.6, 0.8, 0.38, 0.32}

Step 3: Arranging these codewords as super increasing sequence, we get,
C_1 = {0100, 00011, 011011, 0011111} ; C_2 = {1000, 110100, 11010, 0001110}; C_3 = {11000, 1100, 1000110, 010110}; C_4 = {010000, 1010000, 11100, 1001}; C_5 = {1011100, 0110, 011110, 01111}; C_6 = {01100, 0101011, 001111, 0111}; C_7 = {1001100, 101110, 0011, 10111}; C_8 = {1110000, 111000, 10011, 1011}; C_9={0111000, 101100, 0101, 11011} and hence

J(C_1) = {0.2, 0.6, 0.76, 0.89}; J(C_2) = {0.1, 0.33, 0.47, 0.54}; J(C_3) = {0.2, 0.3, 0.43, 0.52}; J(C_4) = {0.09, 0.14, 0.4, 0.5}; J(C_5) = {0.46, 0.5, 0.67, 0.93}; J(C_6) = {0.33, 0.68, 0.86, 0.9}; J(C_7) = {0.36, 0.62, 0.7, 0.87}; J(C_8) = {0.11, 0.28, 0.67, 0.8}; J(C_9) = {0.32, 0.38, 0.6, 0.8}

Step 4: The 3x3 matrix is now generated as shown below:

$$\begin{pmatrix} C_1 & C_2 & C_3 \\ C_4 & C_5 & C_6 \\ C_7 & C_8 & C_9 \end{pmatrix}$$

Thus, the computed matrix of fuzzy codewords are:

$$\begin{pmatrix} (0.2, 0.6, 0.76, 0.89) & (0.1, 0.33, 0.47, 0.54) & (0.2, 0.3, 0.43, 0.52) \\ (0.09, 0.14, 0.4, 0.5) & (0.46, 0.5, 0.67, 0.93) & (0.33, 0.68, 0.86, 0.9) \\ (0.36, 0.62, 0.7, 0.87) & (0.11, 0.28, 0.67, 0.8) & (0.32, 0.38, 0.6, 0.8) \end{pmatrix}$$

Step 5: The measure of the fuzzy codes are tabulated below:

Fuzzy Codewords	**Measure of the fuzzy codes**
{0.2, 0.6, 0.76, 0.89}	0.43
{0.1, 0.33, 0.47, 0.54}	0.23
{0.2, 0.3, 0.43, 0.52}	0.27
{0.09, 0.14, 0.4, 0.5}	0.14
{0.46, 0.5, 0.67, 0.93}	0.55
{0.33, 0.68, 0.86, 0.9}	0.52
{0.36, 0.62, 0.7, 0.87}	0.53
{0.21, 0.28, 0.67, 0.8}	0.23
{0.32, 0.38, 0.6, 0.8}	0.4

The entries are now tabulated as

$$\begin{pmatrix} 0.43 & 0.23 & 0.27 \\ 0.14 & 0.55 & 0.52 \\ 0.53 & 0.28 & 0.4 \end{pmatrix}$$

Steps 6 & 7: It is clear that The fuzzy union of intersection $(0.28) \neq$ The fuzzy intersection of union (0.52).

Since they are not equal, we proceed to the next step.

Steps 8: The magnitude of all fuzzy codewords are calculated as follows:

Magnitude of {0.2, 0.6, 0.76, 0.89}:

$$\begin{pmatrix} (0.46, 0.5, 0.67, 0.93) & (0.33, 0.68, 0.86, 0.9) \\ (0.21, 0.28, 0.67, 0.8) & (0.32, 0.38, 0.6, 0.8) \end{pmatrix}$$

=[(0.46, 0.5, 0.67, 0.93)*(0.32, 0.38, 0.6, 0.8)−(0.33, 0.68, 0.86, 0.9)*(0.21, 0.28, 0.67, 0.8)] =(0.15, 0.19, 0.4, 0.74) + (-0.72, -0.6, -0.2, -0.07) = 0

Magnitude of {0.1, 0.33, 0.47, 0.54}:

$$\begin{pmatrix} (0.09, 0.14, 0.4, 0.5) & (0.33, 0.68, 0.86, 0.9) \\ (0.36, 0.62, 0.7, 0.87) & (0.32, 0.38, 0.6, 0.8) \end{pmatrix}$$

= (0.03, 0.05, 0.24, 0.4) + (-0.78, -0.6, -0.4, -0.12) = 0

Similarly, the other magnitudes are calculated. Thus the magnitude of all the codewords are as follows:

$$\begin{pmatrix} 0 & 0 & 0 \\ 0.0075 & 0.15 & 0.10 \\ 0.03 & 0.4 & 0.3 \end{pmatrix}$$

From this, we can see that the maximum magnitude 0.4 corresponds to the fuzzy codewords J(C_8) = {0.11, 0.28, 0.67, 0.8} which corresponds to the collection {1110000, 111000, 10011, 1011} and hence the user can use these codewords for transmitting his message.

Earlier existing methods on fuzzy codes contains decoding measures that revolves around the field of fuzzy codes alone. But the proposed method contains multi-dimensional ideas related to areas of Fuzzy Coding Theory, Fuzzy Game Theory and also Fuzzy Matrices. Since the decoding contains ideas from the above mentioned fields the breaking of message by any intruder is impossible.

Delays due to natural causes are always unavoidable. But the arrangement of the fuzzy codes in the form of a super increasing sequence does not provide a chance for any further higher transmission delay. This arrangement thus ensures more safer transmission.

7. Conclusion

The transmission of messages in the form of codes minimizes third party interference in viewing a confidential message. Delays in transmission and receival occurs when a message is broken due to many causes that may be both natural and man-made. Natural causes are unavoidable whereas man-made intrusion can be stopped if proper secrecy is maintained. In this paper we have considered 3 x 3 fuzzy matrix for fuzzy codes. The magnitude determines how strong the codewords are. The direct summation of the relative weights are always trivial and another user can easily interpret the message. Hence, the concept of magnitude is introduced about which, only the sender and receiver are aware of.

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