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m EXTENSION OF LUCAS p-NUMBERS IN INFORMATION THEORY

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Abstract. In this paper, we introduced a new Lucas $Q_{p,m}$ matrix for m-extension of Lucas p-numbers where $p(>0)$ is integer and $m(>0)$. Thereby, we discuss various properties of $Q_{p,m}$ matrix, coding and decoding theory followed from the $Q_{p,m}$ matrix.

1. Introduction

The Lucas p-numbers [2] are defined by the following recurrence relation

$$L_p(n) = L_p(n-1) + L_p(n-p-1)$$

with $n > p+1$ and initial terms

$$L_p(1) = L_p(2) = \cdots = L_p(p) = 1, L_p(p+1) = p+2$$

where $p = 0, 1, 2, \cdots$.

For $p = 1$, $L_1(n) = L_n$ are known as classical Lucas numbers.

The Fibonacci numbers, F_n are defined by the following recurrence relation

$$F_n = F_{n-1} + F_{n-2}$$

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with $n > 2$ and initial terms

$$F_1 = F_2 = 1.$$

The ratio of two consecutive Lucas numbers or Fibonacci numbers converges to the irrational number, $\mu = x_1 = \frac{1+\sqrt{5}}{2}$ which is known as golden mean or golden ratio or golden proportion. The Fibonacci numbers, Lucas numbers and golden mean have widely identified in physical science, biological sciences, architectures and art [5,6,10]. Now a days, the golden proportion widely used in modern sciences. It is impossible to imagine the further research work in almost all conceivable physical, mathematical, chemical, biological disciplines and even in human body. It also plays an profound role in various areas of art. Prof. El. Naschie [8,9] is one of the well known followers of golden proportion with a special interest. He [7] shows its uses in $\varepsilon^{(\infty)}$ Cantorian-fractal space-time or E-infinity theory. The practical applications of recent scientific discoveries based on the golden proportion are El Nashie's theory of E-infinity theory [7], Fibonacci matrices and a new coding theory [4], a new class of the hyperbolic functions and the improved method of golden cryptography [10], the golden genomatrices etc.

E. Kocer EG et al.[3] introduced the m-extension of Lucas p-numbers which satisfy the recurrence relation

$$L_{p,m}(n) = mL_{p,m}(n-1) + L_{p,m}(n-p-1) \tag{1.1}$$

with initial terms

$$L_{p,m}(1) = a_1, L_{p,m}(2) = a_2, L_{p,m}(3) = a_3, \cdots, L_{p,m}(p+1) = a_{p+1} \tag{1.2}$$

where $p(\geq 0)$ is integer, $m(> 0)$, $n > p+1$ and a_1, a_2, a_3, ..., a_{p+1} are arbitrary real or complex numbers.

In this paper, we consider initial terms as

$$L_{p,m}(n) = m^{n-1}, n = 1, 2, 3, 4, \cdots, p + 1. \tag{1.3}$$

In this paper, we established the relations among the code elements for all values of $p(> 0)$ is integer and $m(> 0)$. The relation among the code matrix elements for $p(> 0)$ is integer and $m = 1$, coincides with the relation among the code matrix elements [1] and correction ability of this method increases as p increases but it is independent of m.

2. Golden (p, m)-proportion(mean), $\mu_{p,m}$

The characteristic equation of the m-extension of the Lucas p-numbers is

$$x^{p+1} - mx^p - 1 = 0 \tag{2.1}$$

where $x = \lim_{n\longrightarrow\infty} \frac{L_p(n)}{L_p(n-1)}$. The only one positive root $\mu_{p,m}$ of the equation (2.1) is called golden (p, m)-proportion(mean).

TABLE 1: $\mu_{p,m}$

p=1	m=1	$\mu_{1,1} = 1.6180$	p=2	m=1	$\mu_{2,1} = 1.4656$
	m=2	$\mu_{1,2} = 2.4142$		m=2	$\mu_{2,2} = 2.2056$
	m=3	$\mu_{1,3} = 3.3028$		m=3	$\mu_{2,3} = 3.1038$
	m=4	$\mu_{1,4} = 4.2361$		m=4	$\mu_{2,4} = 4.0606$
	m=5	$\mu_{1,5} = 5.1926$		m=5	$\mu_{2,5} = 5.0394$
	m=6	$\mu_{1,6} = 6.6123$		m=6	$\mu_{2,6} = 6.0275$

p=3	m=1	$\mu_{3,1} = 1.3803$	p=4	m=1	$\mu_{4,1} = 1.3247$
	m=2	$\mu_{3,2} = 2.1069$		m=2	$\mu_{4,2} = 2.0560$
	m=3	$\mu_{3,3} = 3.0357$		m=3	$\mu_{4,3} = 3.0121$
	m=4	$\mu_{3,4} = 4.0154$		m=4	$\mu_{4,4} = 4.0039$
	m=5	$\mu_{3,5} = 5.0080$		m=5	$\mu_{4,5} = 5.0016$
	m=6	$\mu_{3,6} = 6.0046$		m=6	$\mu_{4,6} = 6.0008$
p=5	m=1	$\mu_{5,1} = 1.2852$	p=6	m=1	$\mu_{6,1} = 1.2554$
	m=2	$\mu_{5,2} = 2.0291$		m=2	$\mu_{6,2} = 2.0150$
	m=3	$\mu_{5,3} = 3.0041$		m=3	$\mu_{6,3} = 3.0014$
	m=4	$\mu_{5,4} = 4.0010$		m=4	$\mu_{6,4} = 4.0002$
	m=5	$\mu_{5,5} = 5.0003$		m=5	$\mu_{6,5} = 5.0001$
	m=6	$\mu_{5,6} = 6.0001$		m=6	$\mu_{6,6} = 6.0000$

$\mu_{p,m}$ numbers are of theoretical interest for discrete mathematics and open new perspectives for the development of theoretical physics and information sciences.

3. Relation among $\mu_{p,m}$, $\mu_{p,1}$ and $\mu_{1,m}$

The characteristic equation of the m-extension of the Lucas p-numbers is

$$x^{p+1} - mx^p - 1 = 0 \tag{3.1}$$

Whereas the characteristic equation of the Lucas p-numbers is

$$x^{p+1} - x^p - 1 = 0 \tag{3.2}$$

Both the equations (3.1) and (3.2) have $(p+1)$ roots. The only one positive root $x_3 = \mu_{p,m}$ of the equation (3.1) is called golden (p, m)-proportion. Also the only one positive root $x_2 = \mu_p$, golden p-proportion, of the equation (3.2) coincides with $\mu_{p,1}$, golden $(p, 1)$-proportion. $\mu_{1,m}$, golden $(1, m)$-proportion is the positive root x_4 of the characteristic equation $x^2 - mx - 1 = 0$.

It is obvious that x_2 , x_3, x_4 satisfy the equation

$$\frac{logx_3}{logx_2} = \frac{log(1 + x_4(x_3 - x_4)) - logx_4}{log(x_2 - 1)}$$

4. Lucas $Q_{p,m}$ matrix

In this paper, we define a new matrix called Lucas $Q_{p,m}$ matrix (4.1) of order $(p+1)$ on the m-extension of the Lucas p-numbers where p (> 0) is integer and $m > 0$.

$$Q_{p,m} = \begin{pmatrix} m & 1 & 0 & . & . & . & . & 0 & 0 \\ 0 & 0 & 1 & . & . & . & . & 0 & 0 \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ 0 & 0 & 0 & . & . & . & . & 0 & 1 \\ 1 & 0 & 0 & . & . & . & . & 0 & 0 \end{pmatrix} \tag{4.1}$$

Using (1.3) we can write

$$Q_{p,m} = \begin{pmatrix} L_{p,m}(2) & L_{p,m}(1) & . & . & . & . & L_{p,m}(3-p) & L_{p,m}(2-p) \\ L_{p,m}(2-p) & L_{p,m}(1-p) & . & . & . & . & L_{p,m}(3-2p) & L_{p,m}(2-2p) \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ L_{p,m}(0) & L_{p,m}(-1) & . & . & . & . & L_{p,m}(1-p) & L_{p,m}(-p) \\ L_{p,m}(1) & L_{p,m}(0) & . & . & . & . & L_{p,m}(2-p) & L_{p,m}(1-p) \end{pmatrix} \tag{4.2}$$

So that,

$$Q_{1,m} = \begin{pmatrix} m & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} L_{1,m}(2) & L_{1,m}(1) \\ L_{1,m}(1) & L_{1,m}(0) \end{pmatrix}$$

$$Q_{2,m} = \begin{pmatrix} m & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} L_{2,m}(2) & L_{2,m}(1) & L_{2,m}(0) \\ L_{2,m}(0) & L_{2,m}(-1) & L_{2,m}(-2) \\ L_{2,m}(1) & L_{2,m}(0) & L_{2,m}(-1) \end{pmatrix}$$

$$Q_{3,m} = \begin{pmatrix} m & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} L_{3,m}(2) & L_{3,m}(1) & L_{3,m}(0) & L_{3,m}(-1) \\ L_{3,m}(-1) & L_{3,m}(-2) & L_{3,m}(-3) & L_{3,m}(-4) \\ L_{3,m}(0) & L_{3,m}(-1) & L_{3,m}(-2) & L_{3,m}(-3) \\ L_{3,m}(1) & L_{3,m}(0) & L_{3,m}(-1) & L_{3,m}(-2) \end{pmatrix}$$

$$Q_{4,m} = \begin{pmatrix} m & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} L_{4,m}(2) & L_{4,m}(1) & L_{4,m}(0) & L_{4,m}(-1) & L_{4,m}(-2) \\ L_{4,m}(-2) & L_{4,m}(-3) & L_{4,m}(-4) & L_{4,m}(-5) & L_{4,m}(-6) \\ L_{4,m}(-1) & L_{4,m}(-2) & L_{4,m}(-3) & L_{4,m}(-4) & L_{4,m}(-5) \\ L_{4,m}(0) & L_{4,m}(-1) & L_{4,m}(-2) & L_{4,m}(-3) & L_{4,m}(-4) \\ L_{4,m}(1) & L_{4,m}(0) & L_{4,m}(-1) & L_{4,m}(-2) & L_{4,m}(-3) \end{pmatrix}$$

and so on.

Theorem 4.1. *For a given integer n ($n = 0, \pm 1, \pm 2, \pm 3, \ldots$) the nth power of the $Q_{p,m}$ matrix is given by*

$Q_{p,m}^n =$

$$\begin{pmatrix} L_{p,m}(n+1) & L_{p,m}(n) & \cdot\ \cdot\ \cdot\ \cdot\ \cdot & L_{p,m}(n-p+2) & L_{p,m}(n-p+1) \\ L_{p,m}(n-p+1) & L_{p,m}(n-p) & \cdot\ \cdot\ \cdot\ \cdot\ \cdot & L_{p,m}(n-2p+2) & L_{p,m}(n-2p+1) \\ \cdot & \cdot & \cdot\ \cdot\ \cdot\ \cdot\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot\ \cdot\ \cdot\ \cdot\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot\ \cdot\ \cdot\ \cdot\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot\ \cdot\ \cdot\ \cdot\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot\ \cdot\ \cdot\ \cdot\ \cdot & \cdot & \cdot \\ L_{p,m}(n-1) & L_{p,m}(n-2) & \cdot\ \cdot\ \cdot\ \cdot\ \cdot & L_{p,m}(n-p) & L_{p,m}(n-p-1) \\ L_{p,m}(n) & L_{p,m}(n-1) & \cdot\ \cdot\ \cdot\ \cdot\ \cdot & L_{p,m}(n-p+1) & L_{p,m}(n-p) \end{pmatrix}$$

where $L_{p,m}(n) = m^{n-1}$ is given in (1.3).

Proof. When $p = 1$, we have to prove

$$Q_{1,m}^n = \begin{pmatrix} L_{1,m}(n+1) & L_{1,m}(n) \\ L_{1,m}(n) & L_{1,m}(n-1) \end{pmatrix}. \tag{4.3}$$

We will prove it by mathematical induction.

For $n = 1$

$$Q_{1,m} = \begin{pmatrix} m & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} L_{1,m}(2) & L_{1,m}(1) \\ L_{1,m}(1) & L_{1,m}(0) \end{pmatrix} \text{ by (1.3)}$$

which is true for $n = 1$.

For $n = 2$

$$Q_{1,m}^2 = \begin{pmatrix} m^2+1 & m \\ m & 1 \end{pmatrix} = \begin{pmatrix} L_{1,m}(3) & L_{1,m}(2) \\ L_{1,m}(2) & L_{1,m}(1) \end{pmatrix} \text{ by (1.3)}$$

which is true for $n = 2$.

Suppose (4.3) is true for integer $n = k$, then

$$Q_{1,m}^{k} = \begin{pmatrix} L_{1,m}(k+1) & L_{1,m}(k) \\ L_{1,m}(k) & L_{1,m}(k-1) \end{pmatrix}$$

Now, we can write

$$Q_{1,m}^{k+1} = (Q_{1,m}^{k})(Q_{1,m}) = \begin{pmatrix} L_{1,m}(k+1) & L_{1,m}(k) \\ L_{1,m}(k) & L_{1,m}(k-1) \end{pmatrix} \begin{pmatrix} m & 1 \\ 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} L_{1,m}(k+2) & L_{1,m}(k+1) \\ L_{1,m}(k+1) & L_{1,m}(k) \end{pmatrix} \text{ by } (1.3)$$

Hence by induction, we can write

$$Q_{1,m}^{n} = \begin{pmatrix} L_{1,m}(n+1) & L_{1,m}(n) \\ L_{1,m}(n) & L_{1,m}(n-1) \end{pmatrix}.$$

When $p = 2$, we have to prove

$$Q_{2,m}^{n} = \begin{pmatrix} L_{2,m}(n+1) & L_{2,m}(n) & L_{2,m}(n-1) \\ L_{2,m}(n-1) & L_{2,m}(n-2) & L_{2,m}(n-3) \\ L_{2,m}(n) & L_{2,m}(n-1) & L_{2,m}(n-2) \end{pmatrix}. \tag{4.4}$$

We will prove it by mathematical induction. For $n = 1$

$$Q_{2,m} = \begin{pmatrix} m & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} L_{2,m}(2) & L_{2,m}(1) & L_{2,m}(0) \\ L_{2,m}(0) & L_{2,m}(-1) & L_{2,m}(-2) \\ L_{2,m}(1) & L_{2,m}(0) & L_{2,m}(-1) \end{pmatrix} \text{ by } (1.3)$$

which is true for $n = 1$.

For $n = 2$

$$Q_{2,m}^2 = \begin{pmatrix} m^2 & m & 1 \\ 1 & 0 & 0 \\ m & 1 & 0 \end{pmatrix} = \begin{pmatrix} L_{2,m}(3) & L_{2,m}(2) & L_{2,m}(1) \\ L_{2,m}(1) & L_{2,m}(0) & L_{2,m}(-1) \\ L_{2,m}(2) & L_{2,m}(1) & L_{2,m}(0) \end{pmatrix} \text{ by (1.3)}$$

which is true for $n = 2$.

Suppose (4.4) is true for integer $n = k$, then

$$Q_{2,m}^k = \begin{pmatrix} L_{2,m}(k+1) & L_{2,m}(k) & L_{2,m}(k-1) \\ L_{2,m}(k-1) & L_{2,m}(k-2) & L_{2,m}(k-3) \\ L_{2,m}(k) & L_{2,m}(k-1) & L_{2,m}(k-2) \end{pmatrix}$$

Now, we can write

$$Q_{2,m}^{k+1} = (Q_{2,m}^k)(Q_{2,m}) = \begin{pmatrix} L_{2,m}(k+1) & L_{2,m}(k) & L_{2,m}(k-1) \\ L_{2,m}(k-1) & L_{2,m}(k-2) & L_{2,m}(k-3) \\ L_{2,m}(k) & L_{2,m}(k-1) & L_{2,m}(k-2) \end{pmatrix} \begin{pmatrix} m & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} L_{2,m}(k+2) & L_{2,m}(k+1) & L_{2,m}(k) \\ L_{2,m}(k) & L_{2,m}(k-1) & L_{2,m}(k-2) \\ L_{2,m}(k+1) & L_{2,m}(k) & L_{2,m}(k-1) \end{pmatrix} \text{ by (1.3)}$$

Hence by induction, we can write

$$Q_{2,m}^n = \begin{pmatrix} L_{2,m}(n+1) & L_{2,m}(n) & L_{2,m}(n-1) \\ L_{2,m}(n-1) & L_{2,m}(n-2) & L_{2,m}(n-3) \\ L_{2,m}(n) & L_{2,m}(n-1) & L_{2,m}(n-2) \end{pmatrix}.$$

Similarly, by induction it can be proved for all values of p.

Hence the theorem. □

Theorem 4.2. $Q^n_{p,m} = mQ^{n-1}_{p,m} + Q^{n-(p+1)}_{p,m}$

Proof. By theorem 1

$Q^n_{p,m}$=

$$\begin{pmatrix} L_{p,m}(n+1) & L_{p,m}(n) & . & . & . & . & . & L_{p,m}(n-p+2) & L_{p,m}(n-p+1) \\ L_{p,m}(n-p+1) & L_{p,m}(n-p) & . & . & . & . & . & L_{p,m}(n-2p+2) & L_{p,m}(n-2p+1) \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ L_{p,m}(n-1) & L_{p,m}(n-2) & . & . & . & . & . & L_{p,m}(n-p) & L_{p,m}(n-p-1) \\ L_{p,m}(n) & L_{p,m}(n-1) & . & . & . & . & . & L_{p,m}(n-p+1) & L_{p,m}(n-p) \end{pmatrix}$$

When $p = 1$

$$Q^n_{1,m} = \begin{pmatrix} L_{1,m}(n+1) & L_{1,m}(n) \\ L_{1,m}(n) & L_{1,m}(n-1) \end{pmatrix}$$

$$= \begin{pmatrix} mL_{1,m}(n) + L_{1,m}(n-1) & mL_{1,m}(n-1) + L_{1,m}(n-2) \\ mL_{1,m}(n-1) + L_{1,m}(n-2) & mL_{1,m}(n-2) + L_{1,m}(n-3) \end{pmatrix}$$

$$= \begin{pmatrix} mL_{1,m}(n) & mL_{1,m}(n-1) \\ mL_{1,m}(n-1) & mL_{1,m}(n-2) \end{pmatrix}$$

$$+ \begin{pmatrix} L_{1,m}(n-1) & L_{1,m}(n-2) \\ L_{1,m}(n-2) & L_{1,m}(n-3) \end{pmatrix}$$

$$= mQ^{n-1}_{1,m} + Q^{n-2}_{1,m}.$$

When $p = 2$

$$Q_{2,m}^n = \begin{pmatrix} L_{2,m}(n+1) & L_{2,m}(n) & L_{2,m}(n-1) \\ L_{2,m}(n-1) & L_{2,m}(n-2) & L_{2,m}(n-3) \\ L_{2,m}(n) & L_{2,m}(n-1) & L_{2,m}(n-2) \end{pmatrix}$$

$$= \begin{pmatrix} mL_{2,m}(n) + L_{2,m}(n-2) & mL_{2,m}(n-1) + L_{2,m}(n-3) & mL_{2,m}(n-2) + L_{2,m}(n-4) \\ mL_{2,m}(n-2) + L_{2,m}(n-4) & mL_{2,m}(n-3) + L_{2,m}(n-5) & mL_{2,m}(n-4) + L_{2,m}(n-6) \\ mL_{2,m}(n-1) + L_{2,m}(n-3) & mL_{2,m}(n-2) + L_{2,m}(n-4) & mL_{2,m}(n-3) + L_{2,m}(n-5) \end{pmatrix}$$

$$= \begin{pmatrix} mL_{2,m}(n) & mL_{2,m}(n-1) & mL_{2,m}(n-2) \\ mL_{2,m}(n-2) & mL_{2,m}(n-3) & mL_{2,m}(n-4) \\ mL_{2,m}(n-1) & mL_{2,m}(n-2) & mL_{2,m}(n-3) \end{pmatrix}$$

$$+ \begin{pmatrix} L_{2,m}(n-2) & L_{2,m}(n-3) & L_{2,m}(n-4) \\ L_{2,m}(n-4) & L_{2,m}(n-5) & L_{2,m}(n-6) \\ L_{2,m}(n-3) & L_{2,m}(n-4) & L_{2,m}(n-5) \end{pmatrix}$$

$$= mQ_{2,m}^{n-1} + Q_{2,m}^{n-3}.$$

Similarly, we can show that,

$$Q_{p,m}^n = mQ_{p,m}^{n-1} + Q_{p,m}^{n-(p+1)}.$$

□

5. Lucas $Q_{p,m}$ coding and decoding method

Lucas $Q_{p,m}$ matrix allows developing the following applications to the coding theory. Let us represent the initial message in the form of the nonsingular square matrix, M of order $(p+1)$ where $p = 1, 2, 3, \ldots$. We take $Q_{p,m}^n$ matrix of order $(p+1)$ as a coding matrix. We name a transformation $M \times Q_{p,m}^n = E$ as coding and E is known as code matrix.

For example, consider the case for $p = 1$, we represent the initial message, M in the form of nonsingular square matrix of order 2

$$M = \begin{pmatrix} m_1 & m_2 \\ m_3 & m_4 \end{pmatrix} \tag{5.1}$$

where all elements of the matrix are positive integers. i.e. m_1, m_2, m_3, $m_4 > 0$. Let us select for any value of n, the $Q^n_{1,m}$ matrix treated as the coding matrix. For $n = 4$ we have

$$Q^4_{1,m} = \begin{pmatrix} m^4 + 3m^2 + 1 & m^3 + 2m \\ m^3 + 2m & m^2 + 1 \end{pmatrix} \tag{5.2}$$

Then the coding of the message (5.1) consists of the multiplication by the initial matrix (5.2) that is

$$M \times Q^4_{1,m} = \begin{pmatrix} m_1 & m_2 \\ m_3 & m_4 \end{pmatrix} \begin{pmatrix} m^4 + 3m^2 + 1 & m^3 + 2m \\ m^3 + 2m & m^2 + 1 \end{pmatrix}$$

$$= \begin{pmatrix} m_1m^4 + 3m_1m^2 + m_1 + m_2m^3 + 2m_2m & m_1m^3 + 2m_1m + m_2m^2 + m_2 \\ m_3m^4 + 3m_3m^2 + m_3 + m_4m^3 + 2m_4m & m_3m^3 + 2m_3m + m_4m^2 + m_4 \end{pmatrix}$$

$$= \begin{pmatrix} e_1 & e_2 \\ e_3 & e_4 \end{pmatrix} = E \tag{5.3}$$

where $e_1 = m_1m^4 + 3m_1m^2 + m_1 + m_2m^3 + 2m_2m$, $e_2 = m_1m^3 + 2m_1m + m_2m^2 + m_2$, $e_3 = m_3m^4 + 3m_3m^2 + m_3 + m_4m^3 + 2m_4m$, $e_4 = m_3m^3 + 2m_3m + m_4m^2 + m_4$.

The decoding of Lucas $Q_{p,m}$ matrix is defined by the following way. The initial message, M is a square nonsingular matrix of order $(p + 1)$ where $p = 1, 2, 3, \cdots$. We take the inverse of coding matrix $Q^n_{p,m}$ as decoding matrix $(Q^n_{p,m})^{-1}$ so that the transformation $E \times (Q^n_{p,m})^{-1}$ is called decoding and $E \times (Q^n_{p,m})^{-1} = M$ where E is the code matrix.

For example,

The inverse matrix of (5.2) is given by

$$= (Q^4_{1,m})^{-1} = \begin{pmatrix} m^2+1 & -m^3-2m \\ -m^3-2m & m^4+3m^2+1 \end{pmatrix} \tag{5.4}$$

The decoding of the code message, E (5.3) is

$$E \times (Q^4_{p,m})^{-1} = \begin{pmatrix} e_1 & e_2 \\ e_3 & e_4 \end{pmatrix} \begin{pmatrix} m^2+1 & -m^3-2m \\ -m^3-2m & m^4+3m^2+1 \end{pmatrix}$$

$$= \begin{pmatrix} e_1m^2+e_1-e_2m^3-2e_2m & -e_1m^3-2e_1m+e_2m^4+3e_2m^2+e_2 \\ e_3m^2+e_3-e_4m^3-2e_4m & -e_3m^3-2e_3m+e_4m^4+3e_4m^2+e_4 \end{pmatrix}$$

$$= \begin{pmatrix} m_1 & m_2 \\ m_3 & m_4 \end{pmatrix} = M.$$

The code matrix E is defined by the following formula $E = M \times Q^n_{p,m}$. According to the matrix theory, we have

$$Det\ E = Det\ (M \times Q^n_{p,m}) = Det\ M \times Det\ Q^n_{p,m} = Det\ M \times (-1)^{pn} = (-1)^{pn} \times Det\ M$$

6. Relations among the code matrix elements for m (>0)

Case 1: For $p = 1$, Similar to [1] , we obtain $\frac{e_1}{e_2} \approx \mu_{1,m}$; $\frac{e_3}{e_4} \approx \mu_{1,m}$ where $\mu_{1,m} = \frac{m+\sqrt{m^2+4}}{2}$, e_1, e_2, e_3, e_4 are given in (5.3).

Case 2: For $p = 2$, In this case, let the message

$M = \begin{pmatrix} m_1 & m_2 & m_3 \\ m_4 & m_5 & m_6 \\ m_7 & m_8 & m_9 \end{pmatrix}$ then the $Q^n_{2,m}$ coding of the message M is

$$M \times Q^n_{2,m} = \begin{pmatrix} e_1 & e_2 & e_3 \\ e_4 & e_5 & e_6 \\ e_7 & e_8 & e_9 \end{pmatrix} = E.$$

Similar to [1], we obtain $\frac{e_1}{e_2} \approx \mu_{2,m}$; $\frac{e_2}{e_3} \approx \mu_{2,m}$ and $\frac{e_1}{e_3} \approx \mu^2_{2,m}$

$\frac{e_4}{e_5} \approx \mu_{2,m}$; $\frac{e_5}{e_6} \approx \mu_{2,m}$ and $\frac{e_4}{e_6} \approx \mu^2_{2,m}$

$\frac{e_7}{e_8} \approx \mu_{2,m}$; $\frac{e_8}{e_9} \approx \mu_{2,m}$ and $\frac{e_7}{e_9} \approx \mu^2_{2,m}$

where $\mu_{2,m} = \frac{h^2+2hm+4m^2}{6h}$ and $h = \sqrt{108 + 8m^3 + 12\sqrt{81 + 12m^3}}$.

In general, like [1], when $p = t$ and $n > p + 1 = t + 1$, The generalized relations among the code matrix elements are

$\frac{e_1}{e_2} \approx \mu_{t,m}$; $\frac{e_2}{e_3} \approx \mu_{t,m}$; $\cdots$; $\frac{e_t}{e_{t+1}} \approx \mu_{t,m}$

$\frac{e_1}{e_3} \approx \mu^2_{t,m}$; $\frac{e_2}{e_4} \approx \mu^2_{t,m}$; $\cdots$; $\frac{e_{t-1}}{e_{t+1}} \approx \mu^2_{t,m}$

$\cdots$

$\cdots$

$\frac{e_1}{e_{t+1}} \approx \mu^t_{t,m}$

where e_1, e_2, e_3, $\cdots$, e_t, e_{t+1} are the first row elements of the code matrix, E. We also obtain similar type of relations among the elements of the second row, third row, $\cdots$, $(t + 1)$th row of the code matrix, E where $\mu_{t,m}$ is golden (t, m)-proportion.

7. Error detection and correction

For the simplest case $p = 1$ the correction ability of the method is 93.33% [1] which exceeds the essentially all well known correcting codes. The correction ability of the method for $p = 2$ is 99.80% [1]. In general, for $p = t$ and $n > p + 1 = t + 1$ the correction ability of the method is $\frac{2^{(t+1)^2}-2}{2^{(t+1)^2}-1}$ which depends on p but not on m. Hence, for large value of p the correction ability of the method is $\frac{2^{(p+1)^2}-2}{2^{(p+1)^2}-1} \approx 1 = 100\%$.

8. Conclusion

The Lucas coding and decoding method is the main application of the Lucas $Q_{p,m}$ matrix. There lies a difference between the classical algebraic coding and Lucas $Q_{p,m}$ coding method. The accuracy of Lucas $Q_{p,m}$ coding method is given below:

(1) This coding and decoding method converts to matrix multiplication. Now a days it can be done very quickly by computer for large values of p.

(2) The correction ability of the method increases as p increases and it is independent of m.

(3) Lucas $Q_{p,m}$ matrix coincides with golden matrix for $p = 1$, $m = 1$ which develops a new kind of cryptography [10].

(4) In future, based on the works of Prof. EL Naschie, Stakhov, etc. we hope that the Lucas $Q_{p,m}$ matrix can also have wide applications in matrix theory, cryptography and information and coding theory.

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