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RATIO-TO-PRODUCT EXPONENTIAL-TYPE ESTIMATORS UNDER NON-RESPONSE

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ABSTRACT. The aim of the present note is to estimate the population mean $\bar{Y}$ of study variable y using information on an auxiliary variable x in the presence of non response. We have proposed a general family of exponential type estimators concerning two different cases of non response and studied their properties under large sample approximation. In the efficiency comparison, we have shown that the proposed class of estimators perform better than usual unbiased estimator, traditional ratio and product estimator, classical ratio-type and product-type exponential estimator in each case. An empirical study consisting four data sets is also examined to judge the merits of the proposed class of estimators.

1. Introduction and Notation

In literature, the statisticians have proved that the use of auxiliary information in the sample which posses a mathematical correlation with the study variable, often results efficient estimators to estimate the population mean $\bar{Y}$. Ratio type estimators and product type estimators are traditionally considered by survey researchers when the correlation between study variable and auxiliary variable is either positively (high) or negatively (high), respectively. It is very common in most of the surveys

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that there are incomplete information in the sample due to occurrence of non response on some of the sampled units. The sample data with missing information are treated as non-respondents whose responses may be differ from respondents and the inferences about the population parameter can be spoiled owing to non response. Therefore, considering the issues of non response as unavoidable, many authors including Cochran (1977), Rao (1986, 1987), Khare and Srivastava (1993, 1995, 1997), Okafor and Lee (2000), have used the sub sample technique of non respondents introduced by Hansen and Hurwitz (1946) to tackle the threat of incompleteness in estimating the population mean $\bar{Y}$ of study variable y. Moreover, Sarndal and Lundstrom (2005), Tabasum and Khan (2004, 2006), Singh and Kumar (2008a,b, 2009a,b, 2010a,b), Khare and Sinha (2007, 2011), Kumar and Viswanathaiah(2014), Singh et. al. (2016) and many others have advocated the same technique in survey sampling and studied their properties.

Consider a finite population $U = \{U_1, U_2, ..., U_N\}$ of size N where we wish to estimate the population mean $\bar{Y}$ of the study variable y which is correlated with auxiliary variable x. Let y_i and x_i be the observations of y and x respectively which seek the values on the unit $U_i = (y_i, x_i)$ in U. Let a sample S of size n is drawn by the simple random sampling without replacement($SRSWOR$) from the population U. Unfortunately, the response is obtained only on $n_1(= n - n_2)$ units. So the remaining n_2 units are treated as group of non-response. Following Hansen and Hurwitz (1946) method, it is tried to cover more information than n_1 units by making extra effort. They have considered two attempts in their model to collect the information which are specified as follows:

(i) In the first attempt, only n_1 units responded followed by the mailing questionnaire.

(ii) The second attempt was made for sub-sampled units $r = n_2/k$ $(k > 1)$ chosen from non response group and interviewed personally.

Now combine the data obtained from both attempts, yield an unbiased estimator.

In the Hansen and Hurwitz's (1946) procedure, the population is assumed to be divide in two groups, response group of size N_1 and non response group of size $N_2 = (N - N_1)$. Let $\bar{Y} = \sum_{i=1}^{N} y_i/N$, $\bar{Y}_1 = \sum_{i=1}^{N_1} y_i/N_1$ and $\bar{Y}_2 = \sum_{i=1}^{N_2} y_i/N_2$ denote the means of overall population, response group and non response group respectively. Let $S_y^2 = \sum_{i=1}^{N}(y_i - \bar{Y})^2/(N-1)$, $S_{y(1)}^2 = \sum_{i=1}^{N_1}(y_i - \bar{Y})^2/(N_1 - 1)$ and $S_{y(2)}^2 = \sum_{i=1}^{N_2}(y_i - \bar{Y})^2/(N_2 - 1)$ denote the variances of overall population, response group and non response group of the population, respectively. Moreover, let $\{\bar{y}, \bar{y}_1, \bar{y}_2, \bar{y}_{r2}\}$ and $\{s_y^2, s_{y(1)}^2, s_{y(2)}^2, s_{yr(2)}^2\}$ be the sets of means and variances based on the units n, n_1, n_2 and r respectively, defined by

$$\bar{y} = \sum_{i=1}^{n} y_i/n, \quad \bar{y}_1 = \sum_{i=1}^{n_1} y_i/n_1, \quad \bar{y}_2 = \sum_{i=1}^{n_2} y_i/n_2, \quad \text{and} \quad \bar{y}_{r(2)} = \sum_{i=1}^{r} y_i/r$$

$$s_y^2 = \sum_{i=1}^{n}(y_i - \bar{y})^2/(n-1), \quad s_{y(1)}^2 = \sum_{i=1}^{n_1}(y_i - \bar{y}_{(1)})^2/(n_1 - 1),$$

$$s_{y(2)}^2 = \sum_{i=1}^{n_2}(y_i - \bar{y}_2)^2/(n_2 - 1), \quad \text{and} \quad s_{yr(2)}^2 = \sum_{i=1}^{r}(y_i - \bar{y}_{r2})^2/(r-1)$$

The population mean $\bar{Y}$ can also be discussed as

$$\bar{Y} = W_1\bar{Y}_1 + W_2\bar{Y}_2$$

where $W_1 = N_1/N$ and $W_2 = N_2/N$ are the proportions of responding and non responding units in the population U. The stratum sizes N_1 and N_2 can be estimated by $\hat{N}_1 = (n_1/n)N$ and $\hat{N}_2 = (n_2/n)N$, respectively.

Hansen and Hurwitz (1946) proposed an unbiased estimator for the population mean $\bar{Y}$ which is extensively used by researchers, defined by

$$\bar{y}^* = \frac{n_1}{n}\bar{y}_1 + \frac{n_2}{n}\bar{y}_{r2}. \tag{1.1}$$

The variance of the estimator $\bar{y}^*$ is given by

$$V(\bar{y}^*) = \bar{Y}^2\left[\phi C_y^2 + \phi' C_{y(2)}^2\right] \tag{1.2}$$

where $\phi = (N-n)/nN,\ \phi' = W_2(k-1)/n$. Here, $C_y = \frac{S_y}{\bar{Y}}$ and $C_{y(2)} = \frac{S_{y(2)}}{\bar{Y}}$ are the coefficient of variations of whole population and non-response group, respectively. Similarly, Hansen and Hurwitz's (1946) estimator for auxiliary variable x in order to improve the precision of estimates is given by

$$\bar{x}^* = \frac{n_1}{n}\bar{x}_1 + \frac{n_2}{n}\bar{x}_{r2} \tag{1.3}$$

where $\bar{x}_1 = \sum_{i=1}^{n_1} y_i/n_1$ and $\bar{x}_{r(2)} = \sum_{i=1}^{r} y_i/r$.

The variance of the estimator $\bar{x}^*$ is given by

$$V(\bar{x}^*) = \bar{X}^2\left[\phi C_x^2 + \phi' C_{x(2)}^2\right] \tag{1.4}$$

where $\bar{X} = \sum_{i=1}^{N} x_i/N$, $C_x = \frac{S_x}{\bar{X}}$ and $C_{x(2)} = \frac{S_{x(2)}}{\bar{X}}$. We also define $S_{yx} = \sum_{i=1}^{N}(y_i - \bar{Y})(x_i - \bar{X})/(N-1)$, $S_{yx(2)} = \sum_{i=1}^{N_2}(y_i - \bar{Y})(x_i - \bar{X})/(N_2 - 1)$, $C_{yx} = \frac{S_{yx}}{\bar{Y}\bar{X}}$ and $C_{yx(2)} = \frac{S_{yx(2)}}{\bar{Y}\bar{X}}$. It is to be mentioned that S_x^2 and $S_{x(2)}^2$ are the variances of auxiliary variable based on N and N_2 units.

The goal of this paper is to develop an improved family of estimators of population mean using the single auxiliary variable in two different cases of non response. Expressions for the bias and mean squared error (MSE) of the proposed class of estimators are obtained up to first-order approximation. It has been empirically shown that the proposed family of exponential estimators in each case are better than usual unbiased estimator, traditional ratio and product estimator, classical ratio-type and product-type exponential estimator. The paper in distinct cases of non response is organized as follows: In Section 2 some ratio and product type estimators have been discussed along with the bias and MSE expressions. In Section 3 we provide theoretical comparisons to asses the performance of proposed and existing estimators. An empirical study is conducted in support of present study in Section 4. The concluding remarks are given in Section 6.

2. Ratio and Product Type Estimators

In this section, we have discussed usual ratio and product estimators and classical ratio and product-type exponential estimators in two different cases of non response given in Case I and Case II, respectively.

2.1. Case I*(Non-Response occurred on study variable as well as auxiliary variable)*: We assume that the non-response occurred on both study variable y as well as auxiliary variable x and the population mean $\bar{X}$ is already known. In this situation, we define the error terms as

$$\varepsilon_0 = \frac{\bar{y}^* - \bar{Y}}{\bar{Y}}, \quad \varepsilon_1 = \frac{\bar{x}^* - \bar{X}}{\bar{X}}$$

then

$$E(\varepsilon_0) = E(\varepsilon_1) = 0$$

To obtain the bias and mean square error (MSE) of the proposed class of estimators, we need the following lemma.

Lemma 2.1: The variance of the error terms ε_0 and ε_1 are given by

$$V(\varepsilon_0) = E(\varepsilon_0^2) = [\phi C_y^2 + \phi' C_{y(2)}^2],$$

and

$$V(\varepsilon_1) = E(\varepsilon_1^2) = [\phi C_x^2 + \phi' C_{x(2)}^2].$$

The covariance between ε_0 and ε_1 is given by

$$Cov(\varepsilon_0\varepsilon_1) = E(\varepsilon_0\varepsilon_1) = [\phi C_{yx} + \phi' C_{yx(2)}].$$

Proof. The variance of ε_0 can be derived as

$$\begin{aligned} V(\varepsilon_0) &= E[\varepsilon_0 - E(\varepsilon_0)]^2 \\ &= E(\varepsilon_0)^2 = E\left(\frac{\bar{y}^* - \bar{Y}}{\bar{Y}}\right)^2 \\ &= \frac{1}{\bar{Y}^2} V(\bar{y}^*) \end{aligned}$$

Similarly

$$V(\varepsilon_1) = \frac{1}{\bar{X}^2} V(\bar{x}^*)$$

Then we have the required variances of ε_0 and ε_1.

and

$$\begin{aligned} Cov(\varepsilon_0 \varepsilon_1) &= E[\varepsilon_0 - E(\varepsilon_0)][\varepsilon_1 - E(\varepsilon_1)] \\ &= E(\varepsilon_0)\varepsilon_1) = E\left(\frac{\bar{y}^* - \bar{Y}}{\bar{Y}} \frac{\bar{x}^* - \bar{X}}{\bar{X}}\right) \\ &= \frac{1}{\bar{Y}\bar{X}} Cov(\bar{y}^* \bar{x}^*) \end{aligned}$$

Then we have the required covariance between ε_0 and ε_1. Hence the lemma.

The traditional ratio and product estimators and classical ratio and product-type exponential estimators for population mean $\bar{Y}$ are respectively defined by

(2.1) $$t_R^* = \bar{y}^* \frac{\bar{X}}{\bar{x}^*}$$

(2.2) $$t_P^* = \bar{y}^* \frac{\bar{x}^*}{\bar{X}}$$

(2.3) $$t_{Re}^* = \bar{y}^* exp\left(\frac{\bar{X} - \bar{x}^*}{\bar{X} + \bar{x}^*}\right)$$

(2.4) $$t_{Pe}^* = \bar{y}^* exp\left(\frac{\bar{x}^* - \bar{X}}{\bar{x}^* + \bar{X}}\right)$$

We note that t_R^* is due to Cochran (1977) and t_{Re}^*, t_{Pe}^* are owing to Singh et al. (2009).

The biases and MSE of the estimators in (2.1)-(2.4) to the first degree of approximation are discussed in the following theorems.

Theorem 2.1: The bias and MSE of the estimator in (2.1) are given by

$$\begin{aligned} B(t_R^*) &= \bar{Y}\left[\phi C_x^2(1-C) + \phi' C_{x(2)}^2(1-C_{(2)})\right] \\ &= \bar{Y}\left[\phi C_x^2 + \phi' C_{x(2)}^2\right](1-\lambda^*) \end{aligned} \tag{2.5}$$

and

$$\begin{aligned} MSE(t_R^*) &= \bar{Y}^2\left[\phi\{C_y^2 + C_x^2(1-2C)\} + \phi'\{C_{y(2)}^2 + C_{x(2)}^2(1-2C_{(2)})\}\right] \\ &= \bar{Y}^2\left[(\phi C_y^2 + \phi' C_{y(2)}^2) + (\phi C_x^2 + \phi' C_{x(2)}^2)(1-2\lambda^*)\right] \end{aligned} \tag{2.6}$$

where $C = \rho_{yx}\frac{C_y}{C_x}$, $C_{(2)} = \rho_{yx(2)}\frac{C_{y(2)}}{C_{x(2)}}$ and $\lambda^* = (\frac{\phi C C_x^2 + \phi' C_{(2)} C_{x(2)}^2}{\phi C_x^2 + \phi' C_{x(2)}^2})$. Here, ρ_{yx} and $\rho_{yx(2)}$ are the correlation coefficients between y and x of the whole population and non response class.

Proof. Expressing (2.1) in terms $\varepsilon_i's$ $(i = 0, 1)$, we get

$$t_R^* = \bar{Y}(1+\varepsilon_0)(1+\varepsilon_1)^{-1} \tag{2.7}$$

We assume that $|\varepsilon_1| < 1$, so that $(1+\varepsilon_1)^{-1}$ is expandable in terms of ε_1. Expanding the right hand side (r.h.s.) of (2.7) and neglecting the terms having the power of $\varepsilon's$ greater than two, we have

$$t_R^* - \bar{Y} = \bar{Y}(\varepsilon_0 - \varepsilon_1 + \varepsilon_1^2 - \varepsilon_0\varepsilon_1) \tag{2.8}$$

or

$$(t_R^* - \bar{Y})^2 \approx \bar{Y}^2(\varepsilon_0^2 - \varepsilon_1^2 - 2\varepsilon_0\varepsilon_1) \tag{2.9}$$

Now, taking the expectations of (2.8) and (2.9), we have the results in (2.5) and (2.6) respectively. Hence, the theorem.

Theorem 2.2: The bias and MSE of the estimator in (2.2) are given by

$$\begin{aligned} B(t_P^*) &= \bar{Y}\left[\phi C C_x^2 + \phi' C_{(2)} C_{x(2)}^2\right] \\ &= \bar{Y}\left[\phi C_x^2 + \phi' C_{x(2)}^2\right]\lambda^* \end{aligned} \tag{2.10}$$

and

$$\begin{aligned} MSE(t_P^*) &= \bar{Y}^2\left[\phi\{C_y^2 + C_x^2(1+2C)\} + \phi'\{C_{y(2)}^2 + C_{x(2)}^2(1+2C_{(2)})\}\right] \\ &= \bar{Y}^2\left[(\phi C_y^2 + \phi' C_{y(2)}^2) + (\phi C_x^2 + \phi' C_{x(2)}^2)(1+2\lambda^*)\right] \end{aligned} \tag{2.11}$$

Theorem 2.3: The bias and MSE of the estimator in (2.3) are given by

$$\begin{aligned} B(t_{Re}^*) &= \bar{Y}\left[\phi C_x^2\left(\frac{3}{8} - \frac{C}{2}\right) + \phi' C_{x(2)}^2\left(\frac{3}{8} - \frac{C_{(2)}}{2}\right)\right] \\ &= \bar{Y}\left[\phi C_x^2 + \phi' C_{x(2)}^2\right]\left(\frac{3}{8} - \frac{\lambda^*}{2}\right) \end{aligned} \tag{2.12}$$

and

$$\begin{aligned} MSE(t_{Re}^*) &= \bar{Y}^2\left[\phi\{C_y^2 + C_x^2(\frac{1}{4} - C)\} + \phi'\{C_{y(2)}^2 + C_x^2(\frac{1}{4} - C)\}\right] \\ &= \bar{Y}^2\left[(\phi C_y^2 + \phi' C_{y(2)}^2) + (\phi C_x^2 + \phi' C_{x(2)}^2)\left(\frac{1}{4} - \lambda^*\right)\right] \end{aligned} \tag{2.13}$$

Theorem 2.4: The bias and MSE of the estimator in (2.4) are given by

$$\begin{aligned} B(t_{Pe}^*) &= \bar{Y}\left[\phi C_x^2\left(\frac{C}{2} - \frac{1}{8}\right) + \phi' C_{x(2)}^2\left(\frac{C_{(2)}}{2} - \frac{1}{8}\right)\right] \\ &= \bar{Y}\left[\phi C_x^2 + \phi' C_{x(2)}^2\right]\left(\frac{\lambda^*}{2} - \frac{1}{2}\right) \end{aligned} \tag{2.14}$$

and

$$\begin{aligned} MSE(t_{Pe}^*) &= \bar{Y}^2\left[\phi\{C_y^2 + C_x^2(\frac{1}{4} + C)\} + \phi'\{C_{y(2)}^2 + C_x^2(\frac{1}{4} + C)\}\right] \\ &= \bar{Y}^2\left[(\phi C_y^2 + \phi' C_{y(2)}^2) + (\phi C_x^2 + \phi' C_{x(2)}^2)\left(\frac{1}{4} + \lambda^*\right)\right] \end{aligned} \tag{2.15}$$

It is easy to prove the Theorems 2.2, 2.3 and 2.4 by following the proof steps of Theorem 2.1.

Theorem 2.5: The estimators t_R^*, t_P^*, t_{Re}^* and t_{Pe}^* are more efficient than the estimator $\bar{y}^*$, respectively if

$$[C > (1/2) \text{ and } C_{(2)} > (1/2)] \quad \text{or} \quad \lambda^* > (1/2) \tag{2.16}$$

$$[C < -(1/2) \text{ and } C_{(2)} < -(1/2)] \quad \text{or} \quad \lambda^* < -(1/2) \tag{2.17}$$

$$[C > (1/4) \text{ and } C_{(2)} > (1/4)] \quad \text{or} \quad \lambda^* > (1/4) \tag{2.18}$$

and

$$[C < -(1/4) \text{ and } C_{(2)} < -(1/4)] \quad \text{or} \quad \lambda^* < -(1/4) \tag{2.19}$$

Proof. From (1.2) and (2.6), we have

$$V(\bar{y}^*) - MSE(t_R^*) \approx \bar{Y}^2 \left[\phi C_x^2(2C-1) + \phi' C_{x(2)}^2(2C_{(2)}-1)\right]$$
$$\approx \bar{Y}^2 \left[\phi C_x^2 + \phi' C_{x(2)}^2\right](2\lambda^* - 1)$$

It is clear that $V(\bar{y}^*) > MSE(t_R^*)$ if the condition (2.16) is satisfied.
Similarly, from (1.2), (2.11), (2.13) and (2.15), the conditions (2.17), (2.18) and (2.19) can be proved. Hence the theorem.

2.2. Case II *(Non-Response occurred only on study variable)*: Now, suppose that the non response only on the study variate y while complete response is obtained for auxiliary variate x and the population mean $\bar{X}$ thereto is known. In this situation, the error terms are defined as

$$\varepsilon_0 = \frac{\bar{y}^* - \bar{Y}}{\bar{Y}}, \quad \varepsilon_2 = \frac{\bar{x} - \bar{X}}{\bar{X}}$$

then

$$E(\varepsilon_0) = E(\varepsilon_2) = 0$$

To obtain the bias and MSE of the proposed class of estimators under this case, we need the following lemma.

Lemma 2.2: The variance of the error terms ε_0 and ε_2 are given by

$$V(\varepsilon_0) = E(\varepsilon_0^2) = [\phi C_y^2 + \phi' C_{y(2)}^2],$$

and

$$V(\varepsilon_2) = E(\varepsilon_2^2) = \phi C_x^2.$$

The covariance between ε_0 and ε_2 is given by

$$Cov(\varepsilon_0 \varepsilon_2) = E(\varepsilon_0 \varepsilon_2) = \phi C_{yx}.$$

Proof. It follows the proof of Lemma 2.1.

The traditional ratio and product estimators and classical ratio and product-type exponential estimators for population mean $\bar{Y}$ are defined as

$$t_R = \bar{y}^* \frac{\bar{X}}{\bar{x}} \tag{2.20}$$

$$t_P = \bar{y}^* \frac{\bar{x}}{\bar{X}} \tag{2.21}$$

$$t_{Re} = \bar{y}^* exp\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right) \tag{2.22}$$

$$t_{Pe} = \bar{y}^* exp\left(\frac{\bar{x} - \bar{X}}{\bar{x} + \bar{X}}\right) \tag{2.23}$$

To the first order approximation the bias and MSE of the estimators in (2.20)-(2.23) are, respectively, given in the following theorems.

Theorem 2.6: The bias and MSE of the estimator in (2.20) are given by

$$B(t_R) = \bar{Y}\left[\phi C_x^2 (1 - C)\right] \tag{2.24}$$

and

$$MSE(t_R) = \bar{Y}^2 \left[\phi\{C_y^2 + C_x^2(1-2C)\} + \phi' C_{y(2)}^2\right] \tag{2.25}$$

Proof. Expressing (2.20) in terms $\varepsilon_i's$ $(i = 0, 2)$, we get

$$t_R = \bar{Y}(1+\varepsilon_0)(1+\varepsilon_2)^{-1} \tag{2.26}$$

We assume that $|\varepsilon_2| < 1$, so that $(1+\varepsilon_2)^{-1}$ is expandable in terms of ε_1. Expanding the r.h.s. of (2.26) and neglecting the terms having the power of $\varepsilon' s$ greater than two, we have

$$t_R - \bar{Y} = \bar{Y}(\varepsilon_0 - \varepsilon_2 + \varepsilon_2^2 - \varepsilon_0\varepsilon_2) \tag{2.27}$$

or

$$(t_R - \bar{Y})^2 \approx \bar{Y}^2(\varepsilon_0^2 - \varepsilon_2^2 - 2\varepsilon_0\varepsilon_2) \tag{2.28}$$

Now, taking the expectations of (2.27) and (2.28), we have (2.5) and (2.6) respectively. Hence, the theorem.

Theorem 2.7: The bias and MSE of the estimator (2.21) are given by

$$B(t_P) = \bar{Y}\phi C C_x^2 \tag{2.29}$$

and

$$MSE(t_P) = \bar{Y}^2 \left[\phi\{C_y^2 + C_x^2(1+2C)\} + \phi' C_{y(2)}^2\right] \tag{2.30}$$

Theorem 2.8: The bias and MSE of the estimator (2.22) are given by

$$B(t_{Re}) = \bar{Y}\left[\phi C_x^2 \left(\frac{3}{8} - \frac{C}{2}\right)\right] \tag{2.31}$$

and

$$MSE(t_{Re}) = \bar{Y}^2 \left[\phi\{C_y^2 + C_x^2(\frac{1}{4} - C)\} + \phi' C_{y(2)}^2\right] \tag{2.32}$$

Theorem 2.9: The bias and MSE of the estimator in (2.23) are given by

$$B(t_{Pe}) = \bar{Y}\left[\phi C_x^2\left(\frac{C}{2} - \frac{1}{8}\right)\right] \tag{2.33}$$

and

$$MSE(t_{Pe}) = \bar{Y}^2\left[\phi\{C_y^2 + C_x^2(\frac{1}{4} + C)\} + \phi' C_{y(2)}^2\right] \tag{2.34}$$

Theorems 2.7, 2.8 and 2.9 can be easily proved following the proof steps of Theorem 2.6.

Theorem 2.10: The estimators t_R, t_P, t_{Re} and t_{Pe} are more efficient than the estimator $\bar{y}^*$, respectively if

$$C > (1/2) \tag{2.35}$$

$$C < -(1/2) \tag{2.36}$$

$$C > (1/4) \tag{2.37}$$

and

$$C < -(1/4) \tag{2.38}$$

Proof. Comparing (2.25), (2.30), (2.32) and (2.34) with (1.2), we have (2.35)-(2.38) respectively. Hence the theorem.

3. The Proposed Family of Exponential Estimators

Motivated by Srivastava (1967), using the information on a single auxiliary variable x with known population mean $\bar{X}$, we have proposed a family of estimators of population mean $\bar{Y}$ in two well defined cases of non response which are as follows.

3.1. Case I *(Non-Response occurred on study variable as well as auxiliary variable)*: Suppose the non response observed on the $n_2 < (n)$ units for study variable y as well as for auxiliary variable x in the sample and the population means $\bar{X}$ is known from the previous census. In this situation, using Hansen and Hurwitz (1946) technique of sub-sample to deal with the group of non response, we have proposed a class of estimators for population mean $\bar{Y}$ of the study variable y which is defined as

$$t^*_{RP} = \bar{y}^* \left(\frac{\bar{X}}{\bar{x}^*}\right)^{\theta} exp\left(a\frac{\bar{X}-\bar{x}^*}{\bar{X}+\bar{x}^*}\right) \tag{3.1}$$

where θ is an arbitrary constant whose optimum value is determined later. Here a is a fixed constant which takes the values 1, -1, 0.

Some special members of the proposed class of Estimators t^*_{RP}

In (3.1), if we set

(i) (θ, a)=(0, 0), we get usual unbiased estimator $\bar{y}^*$.

(ii) (θ, a)=(1, 0) we get traditional ratio estimator t^*_R.

(iii)(θ, a)=(-1, 0) we get traditional product estimator t^*_P.

(iv)(θ, a)=(0, 1) we get classical ratio-type exponential estimator t^*_{Re}.

(v)(θ, a)=(0,-1) we get classical product-type exponential estimator t^*_{Pe}.

Theorem 3.1: The bias and MSE of the estimator in (3.1) to the first order approximation is given by

(3.2)
$$B(t^*_{RP}) = \bar{Y}\left[\left\{\frac{3a}{4}+\frac{a^2}{8}+\theta(\theta+1)\right\}(\phi C_x^2+\phi' C_{x(2)}^2) - \left(\frac{a}{2}+\theta\right)(\phi C_{yx}+\phi' C_{yx(2)})\right]$$

and

(3.3)
$$MSE(t^*_{RP}) = \bar{Y}^2\left[\phi C_y^2+\phi' C_{y(2)}^2+\left(\frac{a}{2}+\theta\right)\left\{\phi C_x^2\left(\frac{a}{2}+\theta-2C\right)+\phi' C_{x(2)}^2\left(\frac{a}{2}+\theta-2C_{(2)}\right)\right\}\right]$$

Proof. Expressing (3.1) in terms of $\varepsilon_i's (i=0,1)$, we have

$$t^*_{RP} = \bar{Y}\left[1+\varepsilon_0(1+\varepsilon_1)^{-\theta} exp\left(\frac{-a\varepsilon_1}{2+\varepsilon_1}\right)\right] \tag{3.4}$$

We know that $(1+\varepsilon_1)^{-\theta}$ is expandable in terms of ε_1. Expanding the r.h.s. of (3.4) binomially and exponentially and neglecting the terms having the power of $\varepsilon's$ greater than two, we get

$$(3.5)\ \ (t^*_{RP} - \bar{Y}) = \bar{Y}\left[\varepsilon_0 - \left(\frac{1}{2} + \theta\right)\varepsilon_1 + \left\{\frac{3a}{4} + \frac{a^2}{8} + \theta(\theta+1)\right\}\varepsilon_1^2 - \left(\frac{a}{2} + \theta\right)\varepsilon_0\varepsilon_1\right]$$

or

$$(t^*_{RP} - \bar{Y})^2 \approx \bar{Y}^2\left[\varepsilon_0^2 + \left(\frac{a}{2} + \theta\right)^2\varepsilon_1^2 - 2\left(\frac{a}{2} + \theta\right)\varepsilon_0\varepsilon_1\right] \tag{3.6}$$

Taking the expectation of both sides of (3.5) and (3.6), we have (3.2) and (3.3) respectively. Hence the theorem.

Theorem 3.2: The minimum MSE of the estimator (3.1) is given by

$$\begin{aligned} min.MSE(t^*_{RP}) &= V(\bar{y}^*) - \bar{Y}^2\frac{(\phi C_{yx} + \phi' C_{yx(2)})^2}{(\phi C_x^2 + \phi' C_{x(2)}^2)} \\ &= \bar{Y}^2(\phi C_x^2 + \phi' C_{x(2)}^2)\lambda^{*2} \end{aligned} \tag{3.7}$$

at the optimum value

$$\theta = \left[\frac{(\phi C_{yx} + \phi' C_{yx(2)})}{(\phi C_x^2 + \phi' C_{x(2)}^2)} - \frac{a}{2}\right] = \theta_0(say). \tag{3.8}$$

Proof. Differentiating (3.3) with respect to θ, equating to zero and solving, we get (3.8). By putting (3.8) in (3.3) we have (3.7). Hence the theorem.

3.2. Case II*(Non-Response occurred only on study variable)*: Let there is complete response obtained on n units for the auxiliary variable x while n_2 units are observed as non-respondents for study variable y, that is, the non response occurs only on study variable y. In this situation, we have proposed a class of exponential type estimators using Hansen and Hurwitz (1946) technique when the population means $\bar{X}$ is known defined as

$$t_{RP} = \bar{y}^*\left(\frac{\bar{X}}{\bar{x}}\right)^{\theta^*} exp\left(a\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right) \tag{3.9}$$

where θ^* is a suitably chosen constant whose optimum value is determined later in (3.16) and a is same as described above in (3.1). It is easy to see that the estimators $\bar{y}^*$, t_R, t_P, t_{Re} and t_{Pe} are the special members of the proposed class of estimators t_{RP}.

Theorem 3.3: The bias and MSE of the estimator in (3.9) to the first order approximation is given by

$$B(t_{RP}) = \bar{Y}\left[\left\{\frac{3a}{4}+\frac{a^2}{8}+\theta^*(\theta^*+1)\right\}(\phi C_x^2)-\left(\frac{a}{2}+\theta^*\right)(\phi C_{yx})\right] \tag{3.10}$$

$$MSE(t_{RP}) = \bar{Y}^2\left[\phi C_y^2+\phi' C_{y(2)}^2+\left(\frac{a}{2}+\theta^*\right)^2\phi C_x^2-2\left(\frac{a}{2}+\theta^*\right)\phi C_{yx}\right] \tag{3.11}$$

Proof. Expressing (3.9) in terms of errors, we have

$$t_{RP} = \bar{Y}\left[1+\varepsilon_0(1+\varepsilon_2)^{-\theta^*}exp\left(\frac{-a\varepsilon_2}{2+\varepsilon_2}\right)\right] \tag{3.12}$$

Expanding the r.h.s. of (3.12) and neglecting the terms of $\varepsilon's$ having the power greater than two, we get

(3.13)
$$(t_{RP}-\bar{Y}) = \bar{Y}\left[\varepsilon_0-\left(\frac{a}{2}+\theta^*\right)\varepsilon_2+\left\{\frac{3a}{4}+\frac{a^2}{8}+\theta^*(\theta^*+1)\right\}\varepsilon_2^2-\left(\frac{a}{2}+\theta^*\right)\varepsilon_0\varepsilon_2\right]$$

or

$$(t_{RP}-\bar{Y})^2 \approx \bar{Y}^2\left[\varepsilon_0^2+\left(\frac{a}{2}+\theta^*\right)^2\varepsilon_2^2-2\left(\frac{1}{2}+\theta^*\right)\varepsilon_0\varepsilon_2\right] \tag{3.14}$$

Taking expectation of both sides of (3.13) and (3.14), we respectively have (3.10) and (3.11). Hence the theorem.

Theorem 3.4: The minimum MSE of the estimator in (3.9) is given by

$$min.MSE(t_{RP}) = V(\bar{y}^*)-\bar{Y}^2\frac{\phi C_{yx}^2}{C_x^2} \tag{3.15}$$

at the optimum value

$$\theta^* = \left[\frac{C_{yx}}{C_x^2}-\frac{a}{2}\right] = \theta_0^*(say). \tag{3.16}$$

Proof. Differentiating (3.11) with respect to θ^*, equating to zero and solving, we get (3.16). By putting (3.16) in (3.11) we have (3.15). Hence the theorem.

4. Efficiency Comparison

In this section, we compare the proposed class of estimators t^*_{RP} and t_{RP} with other considered estimators in both well defined cases of non response. We have discussed the optimal condition and the general condition of the estimators t^*_{RP} and t_{RP}, respectively, given in Case-I and Case-II.

4.1. Case-I *(Non-Response on both y and x)*:

Theorem 4.1: At the optimum value $\theta = \theta_0$, the proposed class of estimators t^*_{RP} is always better than the estimators $\bar{y}^*$, t^*_R, t^*_P, t^*_{Re} and t^*_{Pe}.

Proof. Comparing (1.2), (2.6), (2.11), (2.13) and (2.15) with (3.7), we respectively have

$$MSE(\bar{y}^*) - min.MSE(t^*_{RP}) = \bar{Y}^2[\phi C_x^2 + \phi' C^2_{x(2)}]\lambda^{*2} > 0, \tag{4.1}$$

$$MSE(t^*_R) - min.MSE(t^*_{RP}) = \bar{Y}^2[\phi C_x^2 + \phi' C^2_{x(2)}](1 - \lambda^*)^2 > 0, \tag{4.2}$$

$$MSE(t^*_P) - min.MSE(t^*_{RP}) = \bar{Y}^2[\phi C_x^2 + \phi' C^2_{x(2)}](1 + \lambda^*)^2 > 0, \tag{4.3}$$

$$MSE(t^*_{Re}) - min.MSE(t^*_{RP}) = \bar{Y}^2[\phi C_x^2 + \phi' C^2_{x(2)}]\left(\frac{1}{2} - \lambda^*\right)^2 > 0, \tag{4.4}$$

and

$$MSE(t^*_{Pe}) - min.MSE(t^*_{RP}) = \bar{Y}^2[\phi C_x^2 + \phi' C^2_{x(2)}]\left(\frac{1}{2} + \lambda^*\right)^2 > 0 \tag{4.5}$$

Observing (4.1)-(4.5) carefully, we have the required theorem.

If the value of θ does not coincide exactly with it's optimum value i.e.; $\theta \neq \theta_0$ then we state the following theorems.

Theorem 4.2: The proposed estimator t^*_{RP} is better than unbiased estimator $\bar{y}^*$ if

$$-(a/2) < \theta < (2C - a/2) \quad \text{and} \quad -(a/2) < \theta < (2C_{(2)} - a/2)$$

$$\text{or} \quad -(a/2) > \theta > (2C - a/2) \quad \text{and} \quad -(a/2) > \theta > (2C_{(2)} - a/2).$$

Theorem 4.3: The estimator t^*_{RP} is better than the estimator t^*_R if

$$\{1-a/2\} < \theta < \{2C-(1+a/2)\} \quad \text{and} \quad \{1-a/2\} < \theta < \{2C_{(2)}-(1+a/2)\}$$

$$\text{or} \quad \{1-a/2\} > \theta > \{2C-(1+a/2)\} \quad \text{and} \quad \{1-a/2\} > \theta > \{2C_{(2)}-(1+a/2)\}.$$

Theorem 4.4: t^*_{RP} is better than t^*_P if

$$-\{1+a/2\} < \theta < \{2C+(1-a/2)\} \text{ and } -\{1+a/2\} < \theta < \{2C_{(2)}+(1-a/2)\}$$

$$\text{or } -\{1+a/2\} > \theta > \{2C+(1-a/2)\} \text{ and } -\{1+a/2\} > \theta > \{2C_{(2)}+(1-a/2)\}.$$

Theorem 4.5: t^*_{RP} is better than t^*_{Re} if

$$-1/2\{1-a\} < \theta < \{2C-1/2(1+a)\} \text{ and } -1/2\{1-a\} < \theta < \{2C_2-1/2(1+a)\}$$

$$\text{or } -1/2\{1-a\} > \theta > \{2C-1/2(1+a)\} \text{ and } -1/2\{1-a\} > \theta > \{2C_2-1/2(1+a)\}.$$

Theorem 4.6: t^*_{RP} is better than t^*_{Pe} if

$$-1/2\{1+a\} < \theta < \{2C-1/2(1-a)\} \text{ and } -1/2\{1+a\} < \theta < \{2C_2+1/2(1-a)\}$$

$$\text{or } -1/2\{1+a\} > \theta > \{2C-1/2(1-a)\} \text{ and } -1/2\{1+a\} > \theta > \{2C_2+1/2(1-a)\}.$$

Comparing (1.2), (2.6), (2.11), (2.13) and (2.15) with (3.3), the Theorems 4.2-4.6 can be easily proved.

4.2. Case-II *(Non-Response only on y)*:

Theorem 4.7: At the optimum value $\theta^* = \theta^*_0$, the proposed estimator t_{RP} is always better than $\bar{y}^*$, t_R, t_P, t_{Re} and t_{Pe}.

Proof. It can be proved by comparing (1.2), (2.25), (2.30), (2.32) and (2.34) with (3.15).

If $\theta \neq \theta_0$(ie; optimum value) then we have the following theorems.

Theorem 4.8: The estimator t_{RP} is better than estimator $\bar{y}^*$ if

$$-(a/2) < \theta < (2C-a/2) \text{ or } -(a/2) > \theta > (2C-a/2).$$

Theorem 4.9: t_{RP} is better than t_R if

$$\{1-a/2\} < \theta < \{2C-(1+a/2)\} \text{ or } \{1-a/2\} > \theta > \{2C-(1+a/2)\}$$

Theorem 4.10: t_{RP} is better than t_P if

$$-\{1+a/2\} < \theta < \{2C+(1-a/2)\} \text{ or } -\{1+a/2\} > \theta > \{2C+(1-a/2)\}$$

Theorem 4.11: t_{RP} is better than t_{Re} if

$$-1/2\{1-a\} < \theta < \{2C-1/2(1+a)\}$$

$$\text{or } -1/2\{1-a\} > \theta > \{2C-1/2(1+a)\}$$

Theorem 4.12: the usual unbiased estimator t_{Pe} if

$$-1/2\{1+a\} < \theta < \{2C-1/2(1-a)\}$$

$$\text{or } -1/2\{1+a\} > \theta > \{2C-1/2(1-a)\}$$

Theorems 4.8-4.12 can be proved by comparing (1.2), (2.25), (2.30), (2.32), (2.34) with (3.11).

5. Empirical Study

To demonstrate the results numerically regarding the proposed class of estimators t^*_{RP} and t_{RP} in Case-I and Case-II, we have taken four (4) data sets as Data Set-1, Data Set-2, Data Set-3 and Data Set-4. These are exemplified as follows.

Data Set-1: [Sinha and Kumar (2017)]: Let y be total population of the village and x be the area of the village. The desription of the parameters for this data are: $N = 109$, $n = 35$, $\bar{Y} = 485.92$, $\bar{X} = 255.97$, $W_2 = 0.2$, $S^2_y = 101593.31$, $S^2_x = 23881.57$, $\rho = 0.857$, $S^2_{y(2)} = 127050.35$, $S^2_{x(2)} = 31172.04$, $\rho_2 = 0.834$.

Data Set-2: [Sinha and Kumar (2017)]: Let y be total population of the village and

x be the agricultural labours of the village. The description of the parameters for this data are: $N = 109$, $n = 35$, $\bar{Y} = 485.92$, $\bar{X} = 41.24$, $W_2 = 0.2$, $S_y^2 = 101593.31$, $S_x^2 = 2156.05$ $\rho = 0.451$, $S^2_{y(2)} = 54070.78$, $S^2_{x(2)} = 2314.59$ $\rho_2 = 0.714$.

Data Set-3: [Khare and Rehman (2014)]: Let y be The number of agricultural labors in the village and x be the area (in hectares) of the village. The description of the parameters for this data are: $N = 96$, $n = 40$, $\bar{Y} = 137.9271$, $\bar{X} = 144.8720$, $W_2 = 0.25$, $S_y^2 = 33306.69$, $S_x^2 = 13821.21$, $\rho = 0.773$, $S^2_{y(2)} = 82610.37$, $S^2_{x(2)} = 18353.53$, $\rho_2 = 0.724$.

Data Set-4: [Khare and Kumar (2011)]: Let y be the average value of products sold ($ thousand) and x be the average size of farms (hundred of acres). The description of the parameters for this data are: $N = 56$, $n = 15$, $\bar{Y} = 61.59$, $\bar{X} = 75.79$, $W_2 = 0.2$, $S_y^2 = 577.4409$, $S_x^2 = 155.5009$, $\rho = -0.508$, $S^2_{y(2)} = 193.4881$, $S^2_{x(2)} = 110.25$, $\rho_2 = -0.379$.

We have computed the percentage relative efficiencies ($PREs$) of various estimators of population mean $\bar{Y}$ with respect to unbiased estimator $\bar{y}^*$ using above four data sets (1 to 4) at different values of k using the following formulas.

$$PRE(\bullet) = \frac{MSE(\bullet)}{MSE(\bar{y}^*)} \times 100$$

Findings in Case-I and Case-II are clearly presented in Table 1.

TABLE 1. PRE of the various estimators with respect to unbiased estimator $\bar{y}^*$ for different values of k in Case-I and Case-II.

		Case-I						**Case-II**					
		Estimators						Estimators					
Data Sets	1/k	$\bar{y}^*$	t_R^*	t_P^*	t_{Re}^*	t_{Pe}^*	t_{RP}^*	$\bar{y}^*$	t_R	t_P	t_{Re}	t_{Pe}	t_{RP}
1	1/5	100.00	333.89	*****	231.50	*****	343.47	100.00	134.59	*****	125.47	*****	135.00
	1/4	100.00	337.43	*****	232.00	*****	346.59	100.00	144.23	*****	131.97	*****	145.00
	1/3	100.00	342.81	*****	232.75	*****	351.34	100.00	161.34	*****	142.93	*****	162.00
	1/2	100.00	351.95	*****	233.99	*****	359.48	100.00	200.04	*****	165.30	*****	201.15
2	1/5	100.00	34.890	*****	115.34	*****	150.85	100.00	56.063	*****	102.15	*****	112.86
	1/4	100.00	35.817	*****	113.38	*****	146.18	100.00	53.179	*****	102.42	*****	114.69
	1/3	100.00	37.069	*****	110.95	*****	140.61	100.00	49.890	*****	102.77	*****	117.11
	1/2	100.00	38.874	*****	107.89	*****	133.86	100.00	46.104	*****	103.24	*****	120.49
3	1/5	100.00	189.80	*****	141.99	*****	213.79	100.00	112.22	*****	107.80	*****	113.00
	1/4	100.00	191.95	*****	143.01	*****	215.04	100.00	115.81	*****	109.97	*****	116.63
	1/3	100.00	195.68	*****	144.76	*****	217.41	100.00	122.39	*****	113.84	*****	123.63
	1/2	100.00	203.70	*****	148.45	*****	223.15	100.00	138.36	*****	122.58	*****	140.77
4	1/5	100.00	*****	126.13	*****	119.24	127.23	100.00	*****	122.46	*****	114.19	123.28
	1/4	100.00	*****	127.50	*****	119.47	128.51	100.00	*****	124.47	*****	115.36	125.39
	1/3	100.00	*****	129.12	*****	119.74	130.18	100.00	*****	126.87	*****	116.75	127.89
	1/2	100.00	*****	131.06	*****	120.06	132.46	100.00	*****	129.80	*****	118.42	130.96

From Table 1, it is observed

- in Data set-1 and Data Set-3, that the PRE of the estimators t_R^*, t_{Re}^*, t_{RP}^* and t_R, t_{Re}, t_{RP} are increasing as the value of $1/k$ increasing in both Case-I and Case-II.
- in Data Set-2 that

(a) the ratio estimators t_R^* and t_R are less efficient than unbiased estimator $\bar{y}^*$ in both cases of non response because of it is well known condition [see (1.16) and (1.32)] in both cases.

(b) the PRE of the classical ratio-type exponential estimator t_{Re}^* decreases as the value of $1/k$ increases in Case-I while in Case-II it is completely inverted.

(c) the PRE of the estimators t_{RP}^* in Case-I and t_{RP} in Case-II are increasing as the value of $1/k$ is increasing.

- in Data set-4, that the PRE of the estimators t_P^*, t_{Pe}^*, t_{RP}^* and t_P, t_{Pe}, t_{RP} are increasing as the value of $1/k$ increasing in the Case-I and Case-II.
- that the proposed class of estimators t_{RP}^* and t_{RP} are more efficient than the estimators $\bar{y}^*$, t_R^*, t_P^*, t_{Re}^*, t_{Pe}^*, t_{RP}^* and t_R, t_P, t_{Re}, t_{Pe}, t_{RP} in case-I and case-II, respectively.

Thus, the proposed class of estimators t_{RP}^* and t_{RP} are most efficient estimators than other considered estimators in each case of non response.

6. Conclusions

We have proposed a general family of exponential type estimators using an auxiliary variable in two well defined cases of non response. In the efficiency comparison, it has been found that the minimum MSE of proposed class of estimators t_{RP}^* and t_{RP} are always less than the MSEs of the usual unbiased estimator, traditional ratio and product estimators, classical ratio-type and product type exponential estimators in each case *ie;* Case-I and Case-II. The results obtained in empirical study also advocate our theoretical statement. Thus, the proposed class of estimators are preferable in estimating the population mean $\bar{Y}$ under any amount of correlation.

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