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ECONOMIC ORDER QUANTITY MODELS FOR PRICE DEPENDENT DEMAND AND DIFFERENT HOLDING COST FUNCTIONS

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Abstract. Demand for any type of item depends on its nature like; sensitivity for the price and degree of freshness. Previous inventory models usually assumed that the demand of the commodities was constant or stock-dependent. This paper develops an EOQ models for items whose demand is a decreasing function of selling price. The first model assumes holding cost is non-linear multiplicative function of selling price and time. In the second model holding cost is considered to be non-linear multiplicative function of selling price and level of current inventory. Under these assumptions, we first formulate mathematical models and then some useful theoretical results have been discussed to characterize the optimal solutions. Numerical examples are provided to illustrate the proposed model and optimal solution. The sensitivity analysis is performed and managerial insights are proposed.

1. Introduction

In year 1915 classical inventory model was developed in which demand rate was constant. However, in real life demand rate of any product is not always constant, it is price dependent, time-dependent or stock-dependent. The demand for a particular product made by manufacturer depends on some internal factor such as selling price and availability. The demand elasticity plays an important role in of inventory system

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for any business transaction. In this paper, demand rate is considered to be price dependent and downward slopping function of selling price. Goh [16] established the continuous, infinite horizon, single inventory system and deterministic inventory model, in which the demand rate is inventory level dependent. Alfares [9] developed the inventory model for an item containing stock-level dependent demand rate and a storage time dependent holding cost by considering two time-dependent holding cost functions: (i) Retroactive holding cost increase and (ii) Incremental holding cost increase.Teng *et al.* [13] established an appropriate pricing and lot-sizing model for a retailer when the supplier provides a permissible delay in payment by considering demand is decreasing function of selling price. Roy [1] presented a deterministic inventory model for time proportion deterioration rate in which demand rate is a function of selling price and holding cost is linearly time dependent. Burwell *et al.* [27] established economic lot size model for price-dependent demand under quantity and freight discount. Mondal *et al.* [2] presented an inventory model for ameliorating items in which demand rate is price dependent. Inventory model with price and time-dependent demand was developed by You [26]. Tripathi *et al.* [21] considered optimal ordering policy for price dependent demand under permissible delay in payments. Tripathi [20] studied an inventory model for deteriorating item with linearly time-dependent demand rate under permissible delay in payments. Tripathi [19] developed an inventory model for deteriorating items with linearly time-dependent demand rate under trade credits. Goyal and Chang [24] presented an ordering – transfer inventory model to determine the retailer's optimal order quantity and the number of transfers per order from the warehouse to the display area. Soni and Shah [8] developed optimal ordering policy for retailer when customer demand is stock-dependent and supplier offers progressive credit periods. Inventory model for deteriorating item and quadratic time dependent demand under trade credits was developed by Khanra *et al.* [25]. Teng *et al.* [15] developed an EOQ (Economic Ordering Quantity) model under trade credit financing with increasing demand. Many articles related to variable demand (price dependent, time-dependent, stock-level dependent) established by Teng *et al.* [14], Mandal and Phaujder [5], Chang *et al.* [7], Yang *et al.* [12], Urban [28] and their quotation. Recently, Sarker *et al.* [4] developed an economic order quantity (EOQ) model for finite production rate and deteriorating items with time dependent demand.

In most inventory models holding cost per unit time is considered as constant. However, holding cost of commodities is not always constant. Freshness of an item is dependent on increasing holding cost. Tripathi [23] presented an EOQ (Economic Oder Quantity) model in which demand rate is time-dependent and carrying cost per unit is allowed to vary. Recently Yang [6] developed an inventory model under stock-dependent demand rate and stock-dependent holding cost under shortages. Tripathi [22] established an inventory model for time varying demand, constant demand; and time-dependent holding cost, constant holding cost. Weiss [11] presented an inventory model in which holding cost per item is a convex potential function of time. Pando *et al.* [30] considered an inventory model in which both the demand rate and holding cost are stock-dependent. Other studies related to stock dependent demand are Hwang and Hahn [10],Valliathal and Uthayakumar [17], Pando *et al.* [29], Giri and Chaudhuri [3] and Naddor [18].

The selling price of some products decreases with increase of time as in perishable products, such as bread (due to loss of freshness with each passing day). In such cases, there is revenue loss due to holding the item. On the other hand, when stock-level of items such as jewelry increases, holding cost increases markedly.

In this paper, the mathematical model for inventory system with two different circumstances (i) holding cost per cycle is the integral of $h(pt)^n dI(t)$ and (ii) $h(pI(t))^n dI(t)$ is formulated. The main objective is to minimize the total inventory cost. The effect of different parameters on total inventory cost is studied.

The remainder of the paper is organized as follows: In the next section 2, we provide the assumptions and notations used in the whole manuscript. Mathematical models are presented in section 3. In section 4 numerical examples are provided. The sensitivity analysis of the optimal solution for different parameters is explained in section 5. Finally, we provide conclusion and future research directions in the last section.

2. Assumptions and Notation

The following assumptions are being made to develop the mathematical model:

- The demand for the item is price dependent and downward sloping function of the price
- No backorders are allowed.
- Lead time is negligible.
- The cost does not change with order size.
- The inventory system involves only one item.

In addition, the following notations are used through the manuscript.

k	:	Replenishment cost
T	:	Cycle time
Q	:	Order quantity of item
$I(t)$	:	On-hand inventory level
t	:	Length of time spent in inventory
HC	:	Holding cost per cycle
TIC	:	Total relevant inventory cost per unit time
D	:	The demand as a decreasing function of price, we set $D(p) = \alpha p^{-\beta}$, $\alpha > 0, \beta > 1$.
TIC_1^*	:	Optimal total relevant inventory cost per unit time for model I
TIC_2^*	:	Total relevant inventory cost per unit time for model II
p	:	Selling price of the item (unit)

3. Mathematical Models

Model I: Instantaneous replenishment with non-linear time and price dependent holding cost

The inventory level $I(t)$ decreases gradually to meet price dependent demand only. Thus, the change of inventory with respect to time can be expressed by the following differential equation:

$$\frac{dI(t)}{dt} = -D(p) = -\alpha p^{-\beta} \tag{3.1}$$

In this model, we now assume, that the cost on holding an item $dI(t)$ up to and including time t is given by $h(t \cdot p)^n dI(t)$, where, $n > 1$ is an integer, $h > 0$.

The solution of (3.1) with the initial condition $I(0) = Q$ is given by

$$I(t) = Q - Dt \tag{3.2}$$

Also $I(T) = 0$, from (3.2) we get

$$T = \frac{Q}{D} \tag{3.3}$$

The order quantity Q which minimizes the total relevant inventory cost per unit time can be derived from the inventory cost equation

$$TIC = \frac{k}{T} + \frac{HC}{T} \tag{3.4}$$

where the holding cost per cycle is written as the integral of $h(p \cdot t)^n dI(t)$, from $t = 0$ to $t = T$. So

$$HC = \int_0^T h(p \cdot t)^n dI(t) = \frac{hDp^n T^{n+1}}{(n+1)}. \tag{3.5}$$

From (3.4) and (3.5), we have

$$TIC = \frac{kD}{Q} + \frac{hp^n Q^n}{(n+1)D^{n-1}} \tag{3.6}$$

Differentiating equation (3.6) partially with respect to Q and p respectively two times, we get

$$\frac{\partial(TIC)}{\partial Q} = -\frac{kD}{Q^2} + \frac{nhp^nQ^{n-1}}{(n+1)D^{n-1}}. \tag{3.7}$$

$$\frac{\partial(TIC)}{\partial p} = -\frac{k\beta D}{pQ} + \frac{h(n+n\beta-\beta)p^nQ^n}{(n+1)pD^{n-1}}. \tag{3.8}$$

$$\frac{\partial^2(TIC)}{\partial Q^2} = \frac{2kD}{Q^3} + \frac{n(n-1)hp^nQ^{n-2}}{(n+1)D^{n-1}} > 0. \tag{3.9}$$

$$\frac{\partial^2(TIC)}{\partial p^2} = \frac{k\beta(\beta+1)D}{Qp^2} + \frac{h(n+n\beta-\beta-1)(n+n\beta-\beta)\,(pQ)^n}{(n+1)D^{n-1}p^2} > 0. \tag{3.10}$$

$$\frac{\partial^2(TIC)}{\partial p\partial Q} = \frac{k\beta D}{pQ^2} + \frac{hn(n+n\beta-\beta)}{(n+1)}\left(\frac{pQ}{D}\right)^{n-1} > 0. \tag{3.11}$$

The optimal solution is obtained by putting $\frac{\partial(TIC)}{\partial Q} = 0$ and $\frac{\partial(TIC)}{\partial p} = 0$, from (3.7) and (3.8), we get

$$-k(n+1)D^n + nhp^nQ^{n+1} = 0 \tag{3.12}$$

and

$$-k\beta(n+1)D^n + h(n+n\beta-\beta)Q^{n+1}p^n = 0. \tag{3.13}$$

Solving (3.12) and (3.13) simultaneously, we get

$$n = \beta \tag{3.14}$$

Since $\frac{\partial^2(TIC)}{\partial Q^2} > 0$, $\frac{\partial^2(TIC)}{\partial p^2} > 0$ and $\left(\frac{\partial^2(TIC)}{\partial Q^2}\right)\left(\frac{\partial^2(TIC)}{\partial p^2}\right) - \left(\frac{\partial^2(TIC)}{\partial p\partial Q}\right)^2 > 0$. For validation of $\left(\frac{\partial^2(TIC)}{\partial Q^2}\right)\left(\frac{\partial^2(TIC)}{\partial p^2}\right) - \left(\frac{\partial^2(TIC)}{\partial p\partial Q}\right)^2 > 0$ (see Appendix I). Thus for any $n = \beta$, TIC gives the minimum value. That is the curve between TIC_1, p and Q will be convex (see in the appendix)

$$TIC = TIC_1^* = \frac{k\alpha}{Qp^n} + \frac{hQ^np^{n^2}}{(n+1)\alpha^{n-1}}. \tag{3.15}$$

Using (3.12), (3.15) becomes

$$TIC = TIC_1^* = \left\{ h\alpha k^n \left(1 + \frac{1}{n}\right)^n \right\}^{\frac{1}{n+1}} \tag{3.16}$$

From (3.16), we observe that the option of the non- linearity factor 'n' determines the degree to which the $TIC = TIC_1^*$ value will be affected. As 'n' tends to infinity TIC_1^* tends to 'k' (the replenishment cost); i.e. $TIC_1^* \to k$ as $n \to \infty$.

The total relevant inventory profit is obtained by subtracting TIC with Sales Revenue (SR) is calculated as follows:

(a) $$SR = p \int_0^T \alpha p^{-\beta} dt = \alpha p^{1-\beta} T.$$

Total relevant inventory profit $(TIP) = SR - TIC$

(b) $$TIP = \alpha p^{1-\beta} - \frac{kD}{Q} - \frac{hp^n Q^n}{(n+1)D^{n-1}}.$$

Differentiating (b) w.r.t. 'Q' and 'p' two times partially, we get

(c) $$\frac{\partial(TIP)}{\partial Q} = \frac{kD}{Q^2} - \frac{nhp^n Q^{n-1}}{(n+1)D^{n-1}}.$$

(d) $$\frac{\partial(TIP)}{\partial p} = \alpha(1-\beta)p^{-\beta} + \frac{k\beta D}{pQ} - \frac{(n + n\beta - \beta)hp^n Q^n}{(n+1)pD^{n-1}}.$$

(e) $$\frac{\partial^2(TIP)}{\partial Q^2} = -\frac{2kD}{Q^3} - \frac{n(n-1)hp^n Q^{n-2}}{(n+1)D^{n-1}} < 0, \text{ (A say)}$$

(f) $$\frac{\partial^2(TIP)}{\partial p^2} = -\alpha\beta(1-\beta)p^{-\beta-1} - \frac{k\beta(\beta+1)D}{p^2 Q}$$

(B say)

(g) $$\frac{\partial(TIP)}{\partial p \partial Q} = -\frac{k\beta D}{pQ^2} - \frac{n(n + n\beta - \beta)h)hp^{n-1}Q^{n-1}}{(n+1)pD^{n-1}} < 0, \text{ (C say)}$$

It is clear from Eqs. (e) ,(f) and (g) $AB - C^2 > 0, A < 0$ and $B < 0$. Which shows that the TIP is maximum in this case.

Model II: Instantaneous replenishment with non linear price and inventory dependent carrying cost. In this case, we consider the carrying cost rate as a power of $\{pI(t)\}^n$, namely

$$\frac{d(HC)}{dt} = h\{pI(t)\}^n, \quad n > 1 \tag{3.17}$$

Integrating equation (3.17) with respect to t from $t = 0$ to $t = T$, we get

$$HC = \frac{hp^n D^n T^{n+1}}{(n+1)} \tag{3.18}$$

From (3.3), (3.4) and (3.18), we get

$$TIC = \frac{kD}{Q} + \frac{hp^n Q^n}{(n+1)}. \tag{3.19}$$

Differentiating (3.19), partially with respect to Q and p respectively two times, we get

$$\frac{\partial(TIC)}{\partial Q} = -\frac{kD}{Q^2} + \frac{nhp^n Q^{n-1}}{(n+1)}. \tag{3.20}$$

$$\frac{\partial(TIC)}{\partial p} = -\frac{k\beta D}{pQ} + \frac{nhp^n Q^n}{p(n+1)}. \tag{3.21}$$

$$\frac{\partial^2(TIC)}{\partial Q^2} = \frac{2kD}{Q^3} + \frac{n(n-1)hp^n Q^{n-2}}{(n+1)} > 0. \tag{3.22}$$

$$\frac{\partial^2(TIC)}{\partial p \partial Q} = \frac{k\beta D}{pQ^2} + \frac{nhp^{n-1} Q^{n-1}}{(n+1)} > 0. \tag{3.23}$$

$$\frac{\partial^2(TIC)}{\partial p^2} = \frac{2k\beta(\beta+1)D}{p^2 Q} + \frac{n(n-1)hp^{n-2} Q^n}{(n+1)} > 0. \tag{3.24}$$

To find the optimal solution, we set $\frac{\partial(TIC)}{\partial Q} = 0$ and $\frac{\partial(TIC)}{\partial p} = 0$, from (3.20) and (3.21), we get

$$k(n+1)D = nhp^n Q^{n+1} \tag{3.25}$$

and

$$k\beta(n+1)D = nhp^n Q^{n+1} \tag{3.26}$$

Solving (3.25) and (3.26), we get,

$$\beta = 1 \tag{3.27}$$

Since $\frac{\partial^2(TIC)}{\partial Q^2} > 0$, $\frac{\partial^2(TIC)}{\partial p^2} > 0$ and $\left(\frac{\partial^2(TIC)}{\partial Q^2}\right)\left(\frac{\partial^2(TIC)}{\partial p^2}\right) - \left(\frac{\partial^2(TIC)}{\partial p \partial Q}\right)^2 > 0$.
For validation of $\left(\frac{\partial^2(TIC)}{\partial Q^2}\right)\left(\frac{\partial^2(TIC)}{\partial p^2}\right) - \left(\frac{\partial^2(TIC)}{\partial p \partial Q}\right)^2 > 0$ (see Appendix II)
That is the curve between TIC_2, p and Q will be convex (see in the appendix).

Thus for any $\beta = 1$, TIC gives the minimum value. That is

$$TIC = TIC_2* = \frac{k\alpha}{pQ} + \frac{hp^nQ^n}{(n+1)} \tag{3.28}$$

Using (3.25), (3.28) becomes

$$TIC = TIC_2^* = \left\{hk^n\alpha^n\left(1+\frac{1}{n}\right)^n\right\}^{\frac{1}{(n+1)}} \tag{3.29}$$

In this model II, the holding cost rate as a power function of the product of selling price (p) and on-hand inventory ($I(t)$). Also from (3.29) TIC_2^* tends to '$k\alpha$' as n tends to infinity
i.e. $TIC_2^* \to k\alpha$,
as $n \to \infty$. If $\alpha = 1$, (3.29) approaches to same asymptotic result as in model I discussed above. From (3.16) and (3.29), we see that TIC_1^* and TIC_2^* both are independent of selling price 'p' and order quantity 'Q'.

In model II the total relevant inventory profit (TIP) is calculated as follows:

$$TIP = \alpha p^{1-\beta} - \frac{kD}{Q} - \frac{hp^nQ^n}{(n+1)}. \tag{h}$$

Differentiating (h) w.r.t. 'Q' and 'p' two times partially, we get

$$\frac{\partial(TIP)}{\partial Q} = \frac{kD}{Q^2} - \frac{nhp^nQ^{n-1}}{(n+1)}. \tag{i}$$

$$\frac{\partial(TIP)}{\partial p} = \alpha(1-\beta)p^{-\beta} + \frac{k\beta D}{pQ} - \frac{nhp^nQ^n}{(n+1)p}. \tag{j}$$

$$\frac{\partial^2(TIP)}{\partial Q^2} = -\frac{2kD}{Q^3} - \frac{n(n-1)hp^nQ^{n-2}}{(n+1)} < 0, \text{ (A1 say)} \tag{k}$$

(l) $$\frac{\partial^2(TIP)}{\partial p^2} = -\alpha\beta(1-\beta)p^{-\beta-1} - \frac{k\beta(\beta+1)D}{p^2 Q} - \frac{n(n-1)hp^{n-2}Q^n}{(n+1)} < 0,$$

(B1 say)

(m) $$\frac{\partial(TIP)}{\partial p \partial Q} = -\frac{k\beta D}{pQ^2} - \frac{n^2 hp^{n-1}Q^{n-1}}{(n+1)} < 0, \text{ (C1 say)}$$

It is clear from Eqs. (k) ,(l) and (m) $A1B1 - C1^2 > 0, A1 < 0$ and $B1 < 0$. Which shows that the TIP is maximum in this case also.

4. Numerical Examples

In this section, numerical examples are provided to illustrate the models developed in section 3:

Example 4.1. *Let us consider the parameters for a given model I are assigned the following values:* $\alpha = 1.0$, $k = 10.0$, $h = 0.5$ *and* $n = 2$. *From* (3.16), *we obtain* $TIC = TIC_1^* = 5.31329$.

Example 4.2. *In this example, let us take the same parameter values as in Example 1 except* α, *taking* $\alpha = 2.0$. *From* (3.29), *we obtain* $TIC = TIC_2^* = 8.43433$.

The following managerial insights are obtained from Tables 1 and 2:

(i) The total inventory cost is asymptotically convex for model I and asymptotically constant for model II for large value of n.

(ii) The increase of demand parameter (α), holding cost parameter (h) and replenishment cost (k), show increase in total inventory cost TIC. That is, change in α, h, and k will lead to positive change in TIC.

5. Sensitivity Analysis

5.1. **Sensitivity analysis for Model I.** (i) Sensitivity analysis is performed by changing n, α, h, k and keeping the remaining system parameters at their original values (as in Example 1). The corresponding variations in the total inventory cost are presented in Table 1.

Table 1. Variation of Total Inventory cost (TIC_1^*) with the variation of n, α, h and k.

n	TIC_1^*	α	TIC_1^*	h	TIC_1^*	k	TIC_1^*
4	7.57858	2.0	6.69433	0.6	5.64622	12	6.00000
6	8.6073	3.0	7.66309	1.0	6.69433	15	6.96238
8	9.15099	4.0	8.43433	1.5	7.66309	20	8.43433
10	9.47102	10.0	11.4471	2.0	8.43433	25	9.78717
20	10.0233	20.0	14.4225	3.0	9.65489	30	11.0521
30	10.1424	30.0	16.5096	5.0	11.4471	50	15.5362
40	10.1766	40.0	18.1712	10.0	14.4225	100	24.6621
100	10.1616	50.0	19.5743	20.0	18.1712	150	32.3165
200	10.1155	100.0	24.6621	50.0	24.6621	200	39.1487
300	10.0905	200.0	31.0723	100.0	31.0723	250	45.4280
500	10.0645	300.0	35.5689	300.0	44.814	500	72.1125
1000	10.0392	500.0	42.1716	500.0	53.1329	1000	114.471
2000	10.0230	1000.0	53.1329	1000.0	66.9433	1500	150.000
2500	10.0193	2000.0	66.9433	2000.0	84.3433	2000	181.712

5.2. **Sensitivity analysis for Model II.** (ii) Sensitivity analysis is performed by changing n, α, h, k and keeping the remaining system parameters at their original values (as in Example 2). The corresponding variations in the total inventory cost are exhibited in Table 2.

Table 2. Variation of Total Inventory cost (TIC_2^*) with the variation of n, α, h and k.

n	TIC_2^*	α	TIC_2^*	h	TIC_2^*	k	TIC_2^*
4	13.1951	4.0	13.3887	0.6	8.96281	12	9.52441
6	15.5917	5.0	15.5362	1.0	10.6266	15	11.0521
8	16.9453	6.0	17.5441	1.5	12.1644	20	13.3887
10	17.7853	10.0	24.6621	2.0	13.3887	25	15.5362
20	19.3956	20.0	39.1487	3.0	15.3262	30	17.5441
30	19.8362	30.0	51.2993	5.0	18.1712	50	24.6621
40	20.0120	40.0	62.1447	10.0	22.8943	100	39.1487
100	20.1843	50.0	72.1125	20.0	28.8450	150	51.2993
200	20.1613	100.0	114.471	50.0	39.1487	200	62.1447
300	20.1346	200.0	181.712	100.0	49.3242	250	72.1125
500	20.1012	300.0	238.110	300.0	71.1379	500	114.471
1000	20.0644	500.0	334.716	500.0	84.3433	1000	181.712
2000	20.0391	1000.0	531.329	1000.0	106.266	1500	238.110
2500	20.0330	2000.0	843.433	2000.0	133.887	2000	288.450

Limitations:

(i) The models discussed in the paper are valid, if the selling price is small. The models will not valid for large values of selling price.

(ii) The models are valid, if the order quantity is not large. The models will not valid for large values of order quantities.

Note: Figures for variation of total inventory cost with respect to the variation of n, α, h and k are represented in the A. Appendix for model I and II respectively.

6. Conclusion and Future Research

A model has been provided for determination of total inventory cost under two types of holding cost functions: (i) non-linear function of the product of selling price and time and (ii) non-linear function of selling price and on hand inventory. If n tends to infinity (very large) total inventory cost TIC tends to replenishment cost (k) for model I and total inventory cost tends to $k\alpha$ (the product of replenishment cost and α) for model II. Numerical examples are provided to illustrate the results. The sensitivity analysis of the optimal solution is provided with the variation of different system parameters. Some managerial phenomenon are derived from Table 1 and 2: (i) higher value of non linearity factor n caused higher value of total inventory cost TIC at certain level, for large value of non linearity factor n, the total inventory cost become constant (k and $k\alpha$ for model I and II respectively) and (ii) higher values of α, h and k will lead higher value of total inventory cost. It is observed that the optimal total inventory costs are independent of the selling price for both model I and II. If α is equal to one both models (I and II) will provide the same optimal total inventory costs.

The model developed in this paper provides some extensions. We may extend the models including cost due to freight charges. The model will be generalized to allowable shortages, inflation and time value of money. The other possible extension of the models is to find total inventory profits.

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Appendix-I

At $n = \beta$ equations (3.9), (3.10) and (3.11) become

(n) $$\frac{\partial^2(TIC)}{\partial Q^2} = \frac{2kD}{Q^3} + \frac{n(n-1)hp^n Q^{n-2}}{(n+1)D^{n-1}} \ (= \text{r say})$$

(o) $$\frac{\partial^2(TIC)}{\partial p^2} = \frac{2kn(n+1)D}{Qp^2} + \frac{hn^2(n^2-1)\,(pQ)^n}{(n+1)D^{n-1}p^2} \ (= \text{t say})$$

(p) $$\frac{\partial^2(TIC)}{\partial p \partial Q} = \frac{knD}{pQ^2} + \frac{hn^3\,(pQ)^{n-1}}{(n+1)D^{n-1}} \ (= \text{s say})$$

Let $A_1 = \dfrac{knd}{pQ^2}, A_2 = \dfrac{hn^3\gamma}{(n+1)}$, where $\gamma = \left(\dfrac{pQ}{D}\right)^{n-1}$. On substituting these values in (n), (o) and (p), we obtain

$$r = \frac{\partial^2(TIC)}{\partial Q^2} = \frac{p}{n^2 Q}\{2nA_1 + (n-1)A_2\}$$

$$t = \frac{\partial^2(TIC)}{\partial p^2} = \frac{(n+1)Q}{np}\{nA_1 + (n-1)A_2\}$$

$$s = \frac{\partial^2(TIC)}{\partial p \partial Q} = A_1 + A_2$$

Thus

$$rt - s^2 = \frac{\partial^2(TIC)}{\partial Q^2}\frac{\partial^2(TIC)}{\partial p^2} - \left\{\frac{\partial^2(TIC)}{\partial p \partial Q}\right\}^2$$

$$= \frac{n+1}{n^3}\{2nA_1 + (n-1)A_2\}\{nA_1 + (n-1)A_2\} - (A_1 + A_2)^2$$

$$= \frac{1}{n^3}\left\{(n+1)(2n^2-1)A_1^2 + \left\{3n^3 - 2(n+1)\right\}A_1A_2 + n(n-2)A_2^2\right\} > 0$$

Since A_1 contains k, A_2 contains h, k is much greater than $h(i.e. k = 10$ and $h = 0.5)$, also $Q = 0.5, n \geq 2$, which shows that $A_1 > A_2$. For example, let us consider $k = 10, n = 2, p = 1, Q = 0.5$ and $\alpha = 2$ then $A_1 = 160$ and $A_2 = \dfrac{1}{3}$, thus $A_1 > A_2$.

Appendix-II

At $\beta = 1$ equations (3.22), (3.23) and (3.24) become

(q) $$\frac{\partial^2(TIC)}{\partial Q^2} = \frac{2kD}{Q^3} + \frac{n(n-1)hp^nQ^{n-2}}{(n+1)} \text{ (= r say)}$$

(r) $$\frac{\partial^2(TIC)}{\partial p^2} = \frac{4kD}{Qp^2} + \frac{hn(n-1)p^{n-2}Q^n}{(n+1)} \text{ (= t say)}$$

(s) $$\frac{\partial^2(TIC)}{\partial p \partial Q} = \frac{kD}{pQ^2} + \frac{hn\,(pQ)^{n-1}}{(n+1)} \text{ (= s say)}$$

Let $B_1 = \dfrac{knd}{pQ^2}, B_2 = \dfrac{hn\gamma}{(n+1)}$, where $\gamma = (pQ)^{n-1}$. On substituting these values in (q), (r) and (s), we obtain

$$r = \frac{\partial^2(TIC)}{\partial Q^2} = \frac{p}{Q}\left\{2B_1 + (n-1)B_2\right\}$$

$$t = \frac{\partial^2(TIC)}{\partial p^2} = \frac{Q}{p}\left\{4B_1 + (n-1)B_2\right\}$$

$$s = \frac{\partial^2(TIC)}{\partial p \partial Q} = B_1 + B_2$$

Therefore,

$$rt - s^2 = \frac{\partial^2(TIC)}{\partial Q^2}\frac{\partial^2(TIC)}{\partial p^2} - \left\{\frac{\partial^2(TIC)}{\partial p \partial Q}\right\}^2$$

$$= \left\{2B_1 + (n-1)B_2\right\}\left\{4B_1 + (n-1)B_2\right\} - (B_1 + B_2)^2$$

$$= \left\{7B_1^2 + n(n-2)B_2^2 + 2(3n-4)B_1B_2\right\} > 0,$$

(since $n \geq 2$).

7. Appendix

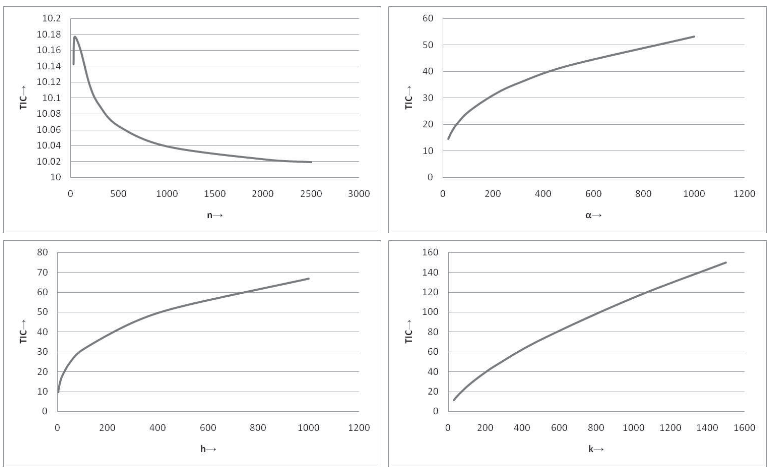

Figure 1. Graph between n, α, h, k and TIC for Model I

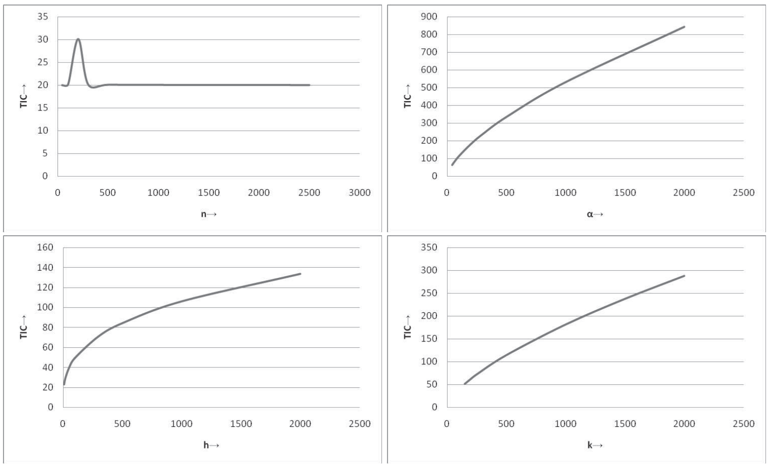

FIGURE 2. Graph between n, $alpha$, h, k and TIC for Model II

Consider the numerical data: $k = 10$, $\alpha = 1$. $H = 0.5$, $n = 2$, $Q = 0.5$ in appropriate units. The graph between TIC_1 and selling price p is given below.

Consider the numerical date $k = 10$, $\alpha = 2$. $h = 0.5$, $p = 1$, $n = 2$, in appropriate units. The graph between TIC_1 and order quantity Q is given below.

Similarly we can show the curve between TIC_2, p and Q will be convex.

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