REMARKS ON FUZZY MINIMAL GROUPS

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ABSTRACT. In this paper, we introduce and investigate some properties of fuzzy minimal groups. It is shown that, the right and the left translations are relatively fuzzy minimal continuous. Moreover, we prove that the inverse image of a fuzzy minimal group under a fuzzy minimal homeomorphism is also a fuzzy minimal group.

1. INTRODUCTION

After the discovery of the fuzzy sets by Zadeh [17], many attempts have been made to extend various branches of mathematics to the fuzzy setting. Fuzzy topological spaces as a very natural generalization of topological spaces were first put forward in the literature by Chang [5] in 1968. He studied a number of the basic concepts including interior and closure of a fuzzy set, fuzzy continuous mapping and fuzzy compactness. Many authors used Chang's definition in many directions to obtain some results which are compatible with results in general topology. In 1976, Lowen [8] suggested an alternative and more natural definition for achieving more results which are compatible to the general case in topology. For example with Chang's definition, constant functions between fuzzy topological spaces are not necessarily fuzzy continuous but in Lowen's sense, all of the constant functions are fuzzy continuous.

In 1985, Sostak [16] introduced the smooth fuzzy topology as an extension of Chang's fuzzy topology.

The concepts of minimal structures and minimal spaces, as generalizations of topology and topological spaces were introduced in [10]. Alimohammady et. al. [1–3] introduced and studied the notions of fuzzy minimal structures and fuzzy minimal spaces. Nematollahi and Roohi [13] introduced and investigated some properties of induced fuzzy minimal structures, fuzzy minimal subspaces and relatively fuzzy minimal continuous functions. In this paper, the concept of fuzzy minimal group and its properties is introduced and studied.

To ease understanding of the material incorporated in this paper, we recall some basic definitions and results. For details on the following notions we refer to [1–3], [5, 8, 11, 12, 14] and the references cited therein.

A *fuzzy set* in (on) a universe set X is a function with domain X and values in $\mathbf{I} = [0, 1]$. The class of all fuzzy sets on X will be denoted by $\mathbf{I}^X$ and symbols $A, B, \ldots$ are used for fuzzy sets on X. For two fuzzy sets A and B in X, we say that A is *contained in* B, denoted by $A \leq B$, provided $A(x) \leq B(x)$ for all $x \in X$. The *complement* of A, denoted by A^c, is defined by $A^c(x) = 1 - A(x)$. 0_X is called *empty fuzzy set* while 1_X denotes the characteristic function on X.

A fuzzy set in X is called a *fuzzy point* if it takes the value 0 for all $x \in X$ except one, say $a \in X$. If its value at a is λ ($0 < \lambda \leq 1$), we will denote this fuzzy point by a_λ, where the point x is called its *support*.

Definition 1.1. Suppose $\{A_\gamma : \gamma \in \Gamma\}$ is a family of fuzzy sets in X. The

(a) *union* of $\{A_\gamma : \gamma \in \Gamma\}$, denoted by $\bigvee_{\gamma \in \Gamma} A_\gamma$ or $\bigvee \{A_\gamma : \gamma \in \Gamma\}$, for each $x \in X$ is defined by

$$\left(\bigvee_{\gamma \in \Gamma} A_\gamma \right)(x) := \sup\{A_\gamma(x) : \gamma \in \Gamma\},$$

(b) *intersection* of $\{A_\gamma : \gamma \in \Gamma\}$, denoted by $\bigwedge_{\gamma \in \Gamma} A_\gamma$ or $\bigwedge \{A_\gamma : \gamma \in \Gamma\}$, is defined by

$$\left(\bigwedge_{\gamma \in \Gamma} A_\gamma\right)(x) := \inf\{A_\gamma(x) : \gamma \in \Gamma\}$$

for all $x \in X$.

A family τ of fuzzy sets in X is called a *fuzzy topology* for X if

(a) $\alpha_X \in \tau$ for each $\alpha \in \mathbf{I}$,

(b) $A \wedge B \in \tau$, where $A, B \in \tau$ and

(c) $\bigvee_{\alpha \in \mathcal{A}} A_\alpha \in \tau$ whenever, $A_\alpha \in \tau$ for all α in $\mathcal{A}$.

The pair (X, τ) is called a *fuzzy topological space* [8]. Every member of τ is called a *fuzzy open set* and its complement is called a *fuzzy closed set* [8]. In a fuzzy topological space X, the *interior* and the *closure* of a fuzzy set A (denoted by $\text{Int}(A)$ and $\text{Cl}(A)$ respectively) are defined by

$$\text{Int}(A) = \bigvee\{U \ : \ U \leq A, \ U \text{ is fuzzy open set}\} \text{ and}$$

$$\text{Cl}(A) = \bigwedge\{F \ : \ A \leq F, \ F \text{ is fuzzy closed set}\}.$$

Let f be a function from X to Y. f induces a fuzzy function defined by

$$f(A)(y) = \begin{cases} \bigvee\limits_{x \in f^{-1}(\{y\})} A(x) & f^{-1}(\{y\}) \neq \emptyset \\ 0 & f^{-1}(\{y\}) = \emptyset, \end{cases}$$

for all y in Y, where A is an arbitrary fuzzy set in X and also $f^{-1}(B) = B \circ f$ for any fuzzy set B in Y [17].

2. FUZZY MINIMAL SPACES AND FUZZY MINIMAL SUBSPACES

In this section we gather some results on fuzzy minimal spaces and fuzzy minimal subspaces which all of them with details and proofs can be found in [1–3] and specially in [13].

Definition 2.1. A family $\mathcal{M}$ of fuzzy sets in a nonempty set X is said to be a

(a) *fuzzy minimal structure in the sense of Chang* on X if $0_X, 1_X \in \mathcal{M}$ ([2]),

(b) *fuzzy minimal structure in the sense of Lowen* on X if $\alpha_X \in \mathcal{M}$, for all $\alpha \in \mathbf{I}$ ([3]).

In the first case, $(X, \mathcal{M})$ is called a *fuzzy minimal space in Chang's sense* and in the second case, $(X, \mathcal{M})$ is called a *fuzzy minimal space in Lowen's sense*. In the sequel, unless otherwise stated, we shall consider fuzzy minimal structures and fuzzy minimal spaces in Lowen's sense and for simplicity we call them fuzzy minimal structures and fuzzy minimal spaces. A fuzzy set $A \in \mathbf{I}^X$ is said to be *fuzzy m-open* if $A \in \mathcal{M}$, and a *fuzzy m-closed* set if $A^c \in \mathcal{M}$. Let

$$(2.1) \qquad m\text{-Int}(A) = \bigvee \{U : U \le A, U \in \mathcal{M}\} \quad \text{and}$$

$$(2.2) \qquad m\text{-Cl}(A) = \bigwedge \{F : A \le F, F^c \in \mathcal{M}\}.$$

Example 2.1. *Let X be a nonempty set. Then*

(a) $\{\alpha_X : \alpha \in \mathbf{I}\}$ *is a fuzzy minimal structure on X,*

(b) $\mathbf{I}^X$ *is a fuzzy minimal structure on X,*

(c) *the family of all fuzzy open (semi open, preopen, α-open and β-open) sets on X is a fuzzy minimal structure on X.*

In [3], authors introduced the fuzzy minimal continuous functions as following.

Definition 2.2. Let $(X, \mathcal{M})$ and $(Y, \mathcal{N})$ be two fuzzy minimal spaces. We say that $f : (X, \mathcal{M}) \to (Y, \mathcal{N})$ is *fuzzy minimal continuous* (briefly, *fuzzy m-continuous*) if $f^{-1}(B) \in \mathcal{M}$, for each $B \in \mathcal{N}$.

Theorem 2.1. *Let $(X_\gamma, \mathcal{M}_\gamma)$ be fuzzy minimal spaces for all $\gamma \in \Gamma$ and let $\{f_\gamma : X \to (X_\gamma, \mathcal{M}_\gamma) : \gamma \in \Gamma\}$ be a family of fuzzy functions. Equip X by the fuzzy minimal structure $\mathcal{M}$ generated by $\{f_\gamma : \gamma \in \Gamma\}$. Suppose $f : (Y, \mathcal{N}) \to (X, \mathcal{M})$ is a fuzzy*

function. Then f is fuzzy m-continuous if and only if $f_\gamma \circ f$ is fuzzy m-continuous for each $\gamma \in \Gamma$.

In [3], it is shown that for a family of fuzzy functions, there exists a weakest fuzzy minimal structure for which all members of it are fuzzy m-continuous. As a consequence, *fuzzy product minimal structure* for an arbitrary family $\{(X_\gamma, \mathcal{M}_\gamma) : \gamma \in \Gamma\}$ of fuzzy minimal spaces can be introduced. In fact, fuzzy product minimal structure on $X = \prod_{\gamma \in \Gamma} X_\gamma$ is the weakest fuzzy minimal structure on X (denoted by $\mathcal{M} = \prod_{\gamma \in \Gamma} \mathcal{M}_\gamma$) such that for each $\gamma \in \Gamma$ the canonical projection $\pi_\gamma : X \to X_\gamma$ is fuzzy m-continuous. It should be noticed that fuzzy product minimal structure for two fuzzy minimal spaces $(X, \mathcal{M})$ and $(Y, \mathcal{N})$ is the family of fuzzy sets

$$\mathcal{M} \times \mathcal{N} = \{1_X \times V : V \in \mathcal{N}\} \cup \{U \times 1_Y : U \in \mathcal{M}\}.$$

Similarly one can verify that fuzzy product minimal structure of $\{(X_j, \mathcal{M}_j) : j = 1, 2, \ldots, n\}$ is

$$\prod_{j=1}^{n} \mathcal{M}_j = \bigcup_{j=1}^{n} \left\{ \prod_{l=1}^{n} F_l : F_l = \left\{ \begin{array}{ll} 1_{X_l} & l \neq j \\ U_j & l = j, \end{array} \right. \text{ where } U_j \in \mathcal{M}_j \right\}.$$

We use $\mathcal{M}_1 \times \mathcal{M}_2 \times \cdots \times \mathcal{M}_n$ instead of $\prod_{i=1}^{n} \mathcal{M}_i$ and specially $\mathcal{M}_1 \times \mathcal{M}_2$ instead of $\prod_{i=1}^{2} \mathcal{M}_i$.

Definition 2.3. [13] Let A be a fuzzy set in X and $\mathcal{M}$ be a fuzzy minimal space on X. Then $\mathcal{M}_A = \{U \wedge A : U \in \mathcal{M}\}$ is called an *induced fuzzy minimal structure on A* and $(A, \mathcal{M}_A)$ is called *fuzzy minimal subspace* of $(X, \mathcal{M})$.

Definition 2.4. [13] Suppose $(A, \mathcal{M}_A)$ and $(B, \mathcal{N}_A)$ are fuzzy minimal subspaces of fuzzy minimal spaces $(X, \mathcal{M})$ and $(Y, \mathcal{N})$ respectively. Also, suppose that $f : (X, \mathcal{M}) \to (Y, \mathcal{N})$ is a mapping. We say that f is a mapping from $(A, \mathcal{M}_A)$ into $(B, \mathcal{N}_A)$ if $f(A) \leq B$.

The mapping f from $(A, \mathcal{M}_A)$ into $(B, \mathcal{N}_B)$ is said to be

(a) *relatively fuzzy minimal continuous* (briefly, (rfm)-continuous), if $f^{-1}(W) \wedge A \in \mathcal{M}_A$ for every fuzzy set W in $\mathcal{N}_B$,

(b) *relatively fuzzy minimal open* (briefly, (rfm)-open), if $f(V) \in \mathcal{N}_B$ for every fuzzy set V in $\mathcal{M}_A$.

Theorem 2.2. [13] *Suppose $(A, \mathcal{M}_A)$ and $(B, \mathcal{N}_B)$ are fuzzy minimal subspaces of fuzzy minimal spaces $(X, \mathcal{M})$ and $(Y, \mathcal{N})$ respectively. If $f : (X, \mathcal{M}) \to (Y, \mathcal{N})$ is fuzzy m-continuous with $f(A) \le B$, then $f : (A, \mathcal{M}_A) \to (B, \mathcal{N}_B)$ is (rfm)-continuous.*

Theorem 2.3. [13] *The composition of two (rfm)-continuous functions is (rfm)-continuous.*

Theorem 2.4. [13] *Suppose $\{(X_j, \mathcal{M}_j) : j \in \{1, \ldots, n\}\}$ is a family of fuzzy minimal spaces, $(X, \mathcal{M})$ is the corresponding fuzzy product minimal space, A_j is a fuzzy set in X_j for each $j \in \{1, \ldots, n\}$ and $A = \prod_{j=1}^{n} A_j$. Let $(B, \mathcal{N}_B)$ be a fuzzy minimal subspace of the fuzzy minimal space $(Y, \mathcal{N})$. Then $f : (B, \mathcal{N}_B) \to (A, \mathcal{M}_A)$ is (rfm)-continuous if and only if $\pi_j o f : (B, \mathcal{N}_B) \to (A_j, \mathcal{M}_{j_{A_j}})$ is (rfm)-continuous for all $j \in \{1, \ldots, n\}$.*

Theorem 2.5. [13] *Suppose $(X, \mathcal{M})$, $(Y, \mathcal{N})$ are fuzzy minimal spaces, $C = A \times B$, $\mathcal{Q} = \mathcal{M} \times \mathcal{N}$ and also A and B are fuzzy sets in X and Y respectively. Then for each $y_0 \in Y$ with $B(y_0) \ge A(x)$ for all $x \in X$, the mapping $i_{y_o} : (A, \mathcal{M}_A) \to (C, \mathcal{Q}_C)$ defined by $i_{y_0}(x) = (x, y_0)$ is (rfm)-continuous.*

Theorem 2.6. [13] *Suppose $(X, \mathcal{M})$, $(Y, \mathcal{N})$ are fuzzy minimal spaces, $C = A \times B$, $\mathcal{Q} = \mathcal{M} \times \mathcal{N}$ and also A and B are fuzzy sets in X and Y respectively. Then for each $x_0 \in X$ with $A(x_0) \ge B(y)$ for all $y \in Y$, the mapping $j_{x_o} : (B, \mathcal{N}_B) \to (C, \mathcal{Q}_C)$ defined by $j_{x_0}(y) = (x_0, y)$ is (rfm)-continuous.*

3. Fuzzy minimal groups

The concept of fuzzy subgroupoid and fuzzy subgroup were introduced and studied by Rosenfeld in [15]. Anthony and Sherwood [4] redefined the notion of fuzzy group. Foster [7] in 1979 introduced the concept of fuzzy topological group using the Lowen's definition of a fuzzy topological space. Ma and Yu [9] changed the definition of fuzzy topological group in order to make sure that an ordinary topological group is a special case of a fuzzy topological group.

Definition 3.1. [6, 15] A fuzzy set G in a group X is called a

(a) *fuzzy semi group*, if $G(xy) \geq \min\{G(x), G(y)\}$ for all $x, y \in X$,

(b) *fuzzy group* in X if $G(xy) \geq \min\{G(x), G(y)\}$ for all $x, y \in X$ and $G(x^{-1}) \geq G(x)$ for all $x \in X$.

Proposition 3.1. [15] *Suppose G is a fuzzy group in a group X and e is the identity element of G. Then $G(x^{-1}) = G(x)$ and $G(e) \geq G(x)$ for all $x \in X$.*

Proposition 3.2. [15] *A fuzzy set G in a group X is a fuzzy group if and only if $G(xy^{-1}) \geq \min\{G(x), G(y)\}$ for all $x, y \in X$.*

Definition 3.2. Suppose X is a group, $(X, \mathcal{M})$ is a fuzzy minimal space and G is a fuzzy group in X endowed with induced fuzzy minimal structure $\mathcal{M}_G$. Then G is said to be *fuzzy minimal group*, if the functions $\varphi : X \times X \to X$ and $\psi : X \to X$ defined by $\varphi(x, y) = xy$ and $\psi(x) = x^{-1}$ respectively, are (rfm)-continuous. This fuzzy minimal group is denoted by $(G, \mathcal{M}_G)$.

Theorem 3.1. *Suppose that X is a group and $(X, \mathcal{M})$ is a minimal space. A fuzzy group G in X is a fuzzy minimal group if and only if the mapping $\eta : (G, \mathcal{M}_G) \times (G, \mathcal{M}_G) \to (G, \mathcal{M}_G)$ defined by $\eta(x, y) = xy^{-1}$ is (rfm)-continuous.*

Proof. Suppose G is a fuzzy minimal group. Since $id_G : (G, \mathcal{M}_G) \to (G, \mathcal{M}_G)$ and $\psi : (G, \mathcal{M}_G) \to (G, \mathcal{M}_G)$ are (rfm)-continuous, so it follows from Theorem 2.4 that the mapping $\alpha : (G, \mathcal{M}_G) \times (G, \mathcal{M}_G) \to (G, \mathcal{M}_G) \times (G, \mathcal{M}_G)$ defined by $\alpha(x, y) = (x, y^{-1})$ is (rfm)-continuous. That η is (rfm)-continuous, follows from Theorem 2.3 and the fact that $\eta = \varphi o \alpha$. Conversely, suppose the mapping $\eta : (G, \mathcal{M}_G) \times (G, \mathcal{M}_G) \to (G, \mathcal{M}_G)$ defined by $\eta(x, y) = xy^{-1}$ is (rfm)-continuous. It follows from Proposition 3.1 that $G(e) \geq G(x)$ for all $x \in X$ and so by Theorem 2.6 the mapping j_e is (rfm)-continuous. Clearly $\psi = \eta o j_e$, then by Theorem 2.3, ψ is (rfm)-continuous. On the other hand, since $id_G : (G, \mathcal{M}_G) \to (G, \mathcal{M}_G)$ and $\psi : (G, \mathcal{M}_G) \to (G, \mathcal{M}_G)$ are (rfm)-continuous, it follows from Theorem 2.4 that the mapping $\alpha : (G, \mathcal{M}_G) \times (G, \mathcal{M}_G) \to (G, \mathcal{M}_G) \times (G, \mathcal{M}_G)$ defined by $\alpha(x, y) = (x, y^{-1})$ is (rfm)-continuous. It is easy to see that $\varphi = \eta o \alpha$ and so by Theorem 2.3, φ is (rfm)-continuous. $\square$

Definition 3.3. [7] For a fuzzy group G in a group X with identity e, set $G_e := \{x \in X : G(x) = G(e)\}$. Clearly, G_e is a subgroup of X. For $a \in X$, the *right and left translations* $r_a, l_a : X \to X$ are defined by $r_a(x) = xa$ and $l_a(x) = ax$ respectively.

Proposition 3.3. [7] *Suppose G is a fuzzy group in a group X. Then for all $a \in G_e$ we have $r_a(G) = l_a(G) = G$.*

Definition 3.4. Suppose $(X, \mathcal{M})$ and $(Y, \mathcal{N})$ are fuzzy minimal spaces. The bijection mapping f from $(X, \mathcal{M})$ into $(Y, \mathcal{N})$ is said to be *fuzzy minimal homeomorphism* if f and f^{-1} are fuzzy minimal continuous; i.e., f is both fuzzy minimal continuous and fuzzy minimal open. Suppose $(A, \mathcal{M}_A)$ and $(B, \mathcal{N}_A)$ are fuzzy minimal subspaces of fuzzy minimal spaces $(X, \mathcal{M})$ and $(Y, \mathcal{N})$ respectively. The bijection mapping f from $(A, \mathcal{M}_A)$ into $(B, \mathcal{N}_B)$ is said to be *relatively fuzzy minimal homeomorphism* (briefly, (rfm)-homeomorphism), if $f(A) = B$ and f and f^{-1} are (rfm)-continuous; i.e., $f(A) = B$ and f is both (rfm)-continuous and (rfm)-open.

Theorem 3.2. *Suppose G is a fuzzy minimal group in X and $a \in G_e$. Then r_a and l_a are (rfm)-homeomorphism.*

Proof. It follows from Proposition 3.3 that $r_a(G) = l_a(G) = G$. Obviously, $r_a = \varphi o i_a$ where φ and i_a are defined in Definition 3.2 and Theorem 2.5 respectively. By Proposition 3.1 and Definition 3.3, $G(a) \geq G(x)$ for all $x \in X$ and hence Theorem 2.5 implies that i_a is (rfm)-continuous. That r_a is (rfm)-continuous follows from Theorem 2.3. On the other hand, it is easy to see that $(r_a)^{-1} = r_{a^{-1}}$ and so (rfm)-continuity of $(r_a)^{-1}$ is achieved similarly. Therefore, r_a is (rfm)-homeomorphism.

Finally, by using Theorem 2.6, in a similar manner, one can prove that l_a is (rfm)-homeomorphism. $\square$

Corollary 3.1. *Suppose G is a fuzzy minimal group in X and $a \in G_e$. Then the inner automorphism $T : G \rightarrow G$ defined by $T(x) = axa^{-1}$ is (rfm)-homeomorphism.*

Proof. It is an immediate consequence of Theorem 3.2, Theorem 2.3 and the facts that $T = l_a o r_{a^{-1}}$ and $T^{-1} = r_a o l_{a^{-1}}$. $\square$

Definition 3.5. [6] Suppose A and B are two fuzzy sets in X and $a \in X$. Then, the fuzzy sets AB and A^{-1} in X are defined by $AB(x) = \sup\{\min\{A(y), B(z)\} : x = yz\}$ and $A^{-1}(x) = A(x^{-1})$. Also, aA means $a_1 A$.

Corollary 3.2. *Suppose G is a fuzzy minimal group in X, A is a fuzzy set in X and $a \in G_e$. If U is (rfm)-open, then aU, Ua and U^{-1} are (rfm)-open sets.*

Proof. Suppose U is a (rfm)-open set. It is not hard to verify that $l_a(U) = aU$ and $r_a(U) = Ua$. That aU and Ua are (rfm)-open sets, follows from Theorem 3.2. Clearly, $\psi(U) = U^{-1}$ and ψ is (rfm)-homeomorphism, which imply that U^{-1} is (rfm)-open. $\square$

Corollary 3.3. *Suppose G is a fuzzy minimal group in X and $a \in G_e$. If F is (rfm)-closed, then aF, Fa, F^{-1} are fuzzy (rfm)-closed sets.*

Proof. Similar to the Corollary 3.2, one can deduce the result. □

Definition 3.6. Suppose $(X, \mathcal{M})$ is a fuzzy minimal space and $x \in X$. A fuzzy set W in X is called *fuzzy minimal neighborhood* of x if there exists $U \in \mathcal{M}$ for which $U \leq W$ and $U(x) = W(x) > 0$.

Corollary 3.4. *Suppose G is a fuzzy minimal group in X, W is a fuzzy minimal neighborhood of e with $W(e) = 1$ and $a \in G_e$. Then aW is a fuzzy minimal neighborhood of a such that $aW(a) = 1$.*

Proof. Since W is a fuzzy minimal neighborhood of e with $W(e) = 1$, there exists a fuzzy m-open set $U \leq W$ such that $U(e) = W(e) = 1$. On the other hand, $aU(a) = U(a^{-1}a) = U(e) = 1$, $aW(a) = W(a^{-1}a) = W(e) = 1$ and

$$aW(x) = W(a^{-1}x) \geq U(a^{-1}x) = aU(x)$$

for all $x \in X$. Then there exits a fuzzy minimal open set aU such that $aU \leq aW$ and $aU(a) = aW(a) = 1$. □

Proposition 3.4. [7] *Suppose X and Y are two groups and $T : X \to Y$ is a homomorphism. Then, for a fuzzy group G in Y, the inverse image $T^{-1}(G)$ of G is a fuzzy group in X.*

Theorem 3.3. *Suppose X and Y are two groups and $T : X \to Y$ is a homomorphism. Equip Y and X with fuzzy minimal structures $\mathcal{N}$ and $\mathcal{M} = T^{-1}(\mathcal{N})$ respectively. If G is a fuzzy minimal group in Y, then the inverse image $T^{-1}(G)$ of G is a fuzzy minimal group in X too.*

Proof. It follows from Proposition 3.4 that $T^{-1}(G)$ is a fuzzy group in X. In accordance with Theorem 3.1 it is sufficient to prove that

$$\eta : (T^{-1}(G), \mathcal{M}_{T^{-1}(G)}) \times (T^{-1}(G), \mathcal{M}_{T^{-1}(G)}) \to (T^{-1}(G), \mathcal{M}_{T^{-1}(G)})$$

defined by $\eta(x,y) = xy^{-1}$ is (rfm)-continuous. To see this, suppose U is an relatively fuzzy minimal open set in $(T^{-1}(G), \mathcal{M}_{T^{-1}(G)})$. Therefore, there exists $V \in \mathcal{N}_G$ for which $T^{-1}(V) = U$. Since $T : (X, \mathcal{M}) \to (Y, \mathcal{N})$ is fuzzy m-continuous, it follows from Theorem 2.2 that $T : (T^{-1}(G), \mathcal{M}_{T^{-1}(G)}) \to (T^{-1}(G), \mathcal{M}_{T^{-1}(G)})$ is (rfm)-continuous. We have

$$\eta^{-1}(U)(x_1, x_2) = U(x_1, x_2^{-1}) = T^{-1}(V)(x_1, x_2^{-1})$$

$$= V(T(x_1)(T(x_2))^{-1})$$

for all $(x_1, x_2) \in X$. Then

$$\eta^{-1}(U) \wedge (T^{-1}(G) \times T^{-1}(G)) = (T \times T)^{-1}(\eta^{-1}(U)) \wedge (T^{-1}(G) \times T^{-1}(G))$$

is relatively fuzzy minimal open in $T^{-1}(G) \times T^{-1}(G)$. We are done. $\square$

Definition 3.7. [15] A fuzzy set A in X is said to have *sup property* if, for any subset $\Omega \subseteq X$, there exists $\omega_0 \in \Omega$ such that $A(\omega_0) = \sup_{\omega \in \Omega} A(\omega)$.

Proposition 3.5. [7] *Suppose X and Y are two groups and $T : X \to Y$ is a homomorphism. Also suppose that G is a fuzzy group in X with sup property. Then the image $T(G)$ of G is a fuzzy group in Y.*

Definition 3.8. [15] A fuzzy minimal group G in X is called *f-invariant*, if for all $x, y \in X$, $f(x) = f(y)$ implies that $G(x) = G(y)$.

Conjecture 3.1. Suppose X and Y are two groups and $T : X \to Y$ is a homomorphism. Equip X and Y with fuzzy minimal structures $\mathcal{M}$ and $T(\mathcal{M})$ respectively. If the fuzzy minimal group G in X is T-invariant, then the image $T(G)$ of G is a fuzzy minimal group in Y too.

Remark 1. It follows from Proposition 3.5 that $T(G)$ is a fuzzy group in Y. In accordance with Theorem 3.1 it is sufficient to prove that

$$\eta : (T(G), \mathcal{M}_{T(G)}) \times (T(G), \mathcal{M}_{T(G)}) \to (T(G), \mathcal{M}_{T(G)})$$

defined by $\eta(x, y) = xy^{-1}$ is (rfm)-continuous. We think that Conjecture 3.1 without any additional conditions is not true. But, may be with the property U for the minimal space (i.e., arbitrary union of fuzzy m-open sets is also fuzzy m-open), one can prove Conjecture 3.1.

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